1. (a) Find all solutions in the complex numbers to the equation $z^3 = \frac{1}{8}$.

(b) Let $A = \begin{pmatrix} 3 - i & \frac{1}{1+i} \\ 2i & 2 \end{pmatrix}$. Show that $A$ is invertible; let $A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Find the largest and smallest of the numbers $|a|, |b|, |c|$ and $|d|$.

(c) Solve the system of linear equations

\[
\begin{align*}
(3 - i)x_1 + \frac{1}{1+i}x_2 &= 2 \\
2ix_1 + 2x_2 &= 1 - i
\end{align*}
\]
2. Let \( A = \begin{pmatrix}
0 & 0 & 2 & 6 & -1 \\
-1 & -2 & 1 & 4 & -1 \\
0 & 0 & 1 & 3 & 0 \\
2 & 4 & 0 & -2 & 0
\end{pmatrix} \).

(a) Find a basis for each of the column space, row space, and null space of \( A \).

(b) Find a system of linear equations (possibly just a single equation) such that the set of solutions to the system is just the column space of \( A \).
3. (a) Let \( V \) be the vector space \( K^3 \) over \( K \), where \( K \) is either \( \mathbb{R} \) (the reals) or \( \mathbb{C} \) (the complexes) as the case may be. For each subset \( U \subseteq V \), determine whether or not \( U \) is a subspace of \( V \); justify your answers.

i. \( U = \{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R}, xz = 0 \}; \ K = \mathbb{R} \).

ii. \( U = \{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{C}, x^2 + y^2 + z^2 = 0 \}; \ K = \mathbb{C} \).

iii. \( U = \{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R}, x^2 + y^2 + z^2 = 0 \}; \ K = \mathbb{R} \).

(b) Let \( V \) be a finite-dimensional vector space and \( U \subseteq V \) a subspace of \( V \). Show that \( \dim(U) \leq \dim(V) \). Show further that if \( \dim(U) = \dim(V) \), then \( U = V \).
4. Let \( V = P_2(t) \), the space of polynomials of degree \( \leq 2 \). Then \( B = (p_1, p_2, p_3) \) is an ordered basis for \( V \), where \( p_1(t) = (t - 1)^2 \), \( p_2(t) = t - 1 \) and \( p_3(t) = 1 \). Let \( q_1(t) = (t + 1)^2 \), \( q_2(t) = (t - 1)^2 + t \) and \( q_3(t) = t - 1 \).

(a) Show that \( B' = (q_1, q_2, q_3) \) is also a basis for \( V \), and find the change of basis matrix from \( B \) to \( B' \) and from \( B' \) to \( B \).

(b) Compute \([p]_B'\), where \( p(t) = a_0 + a_1(t - 1) + a_2(t - 1)^2 \).
5. Let $V = M_{2,2}(\mathbb{R})$ be the vector space of real $2 \times 2$ matrices. Let $C = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$. Define the transformation $L : V \rightarrow V$ by $L(A) = CA - 2A^T$, where $A^T$ is the transpose of $A$.

(a) Show that $L$ is linear.

(b) Let $B = \{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \}$; you may assume that $B$ is a basis for $V$. Find $[L]_B$.

(c) Is $L$ singular or nonsingular? Justify your answer.
6. (a) Suppose that \( \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = 5 \) and \( \det \begin{pmatrix} a' & b' \\ c & d \end{pmatrix} = -2 \). Find

\[
\det \begin{pmatrix} 3a + 2a' & 2c - 9a - 6a' \\ 3b + 2b' & 2d - 9b - 6b' \end{pmatrix}.
\]

(b) Let \( A \) be a 4 \times 4 matrix of the form \( \begin{pmatrix} B & C \\ 0 & D \end{pmatrix} \), where \( B, C \) and \( D \) are 2 \times 2 matrices, and 0 represents the 2 \times 2 zero matrix. Show that

\[ \det(A) = \det(B)\det(D). \]