MATH 223, Linear Algebra
Fall, 2007
Assignment 1, due *in class* Friday September 14, 2007

1. Let $z = 2 - 7i$ and $w = 3 + 4i$. Find $\bar{z}$, $\bar{w}$, $z + w$, $z - w$, $z \cdot w$ and $\frac{z}{w}$ (all in the form $a + bi$ with a and b real numbers). Find the absolute value of each of these 6 numbers.

2. Show that if z and w are any two complex numbers, then $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$. Use this to show that if A and B are any complex matrices, then $\overline{A \cdot B} = \bar{A} \cdot \bar{B}$. [N.B. The conjugate $\bar{A}$ of a matrix A is the most obvious thing — you just replace each entry of A by its conjugate. Also, we of course assume here that $A \cdot B$ is defined.]

3. Solve each of the following systems of equations. That is, find the unique solution if there is one, the general solution in vector parametric form if there is more than one solution, or explain why there is no solution if that is the case. Use augmented matrices.

   (a) This one's over the field $\mathcal{R}$, the reals. $\begin{array}{rrrrrcr} x_1 & -3x_2 & & +2x_4 & +5x_5 & = & 7 \\ 3x_1 & -6x_2 & +2x_3 & +x_4 & -2x_5 & = & 1 \\ 5x_1 & -12x_2 & +2x_3 & +5x_4 & +8x_5 & = & 15 \end{array}$

   (b) This one's also over the field $\mathcal{R}$. $\begin{array}{rrrrrcr} x_1 & -3x_2 & & +2x_4 & +5x_5 & = & 4 \\ 3x_1 & -6x_2 & +2x_3 & +x_4 & -2x_5 & = & -3 \\ 5x_1 & -12x_2 & +2x_3 & +5x_4 & +8x_5 & = & 7 \end{array}$

   (c) This one's over the field $\mathcal{C}$, the complex numbers. $\begin{array}{rrcr} (1+i)x_1 & +(3-i)x_2 & = & 6+i \\ (2+2i)x_1 & +(1-5i)x_2 & = & -2i \end{array}$

   (d) This one's over the two-element field $\mathcal{Z}_2$. $\begin{array}{rrrrrcr} x_1 & & +x_3 & & +x_5 & = & 1 \\ x_1 & +x_2 & +x_3 & +x_4 & +x_5 & = & 1 \\ & x_2 & & +x_4 & +x_5 & = & 1 \\ x_1 & & +x_3 & & & = & 0 \end{array}$

   In this case, explicitly list all the solutions.

4. (a) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be any 2×2 matrix. Show that there is a nonzero vector $\vec{v}$ with $A\vec{v} = \vec{0}$ if and only if $ad - bc = 0$.

   (b) Find all complex numbers λ (if any) such that $\begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} \vec{v} = \lambda\vec{v}$ has a nonzero solution $\vec{v}$.

   (c) For each λ you found in the previous part, find all vectors $\vec{v}$ such that $\begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} \vec{v} = \lambda\vec{v}$.