Real functions

Def: A real function is any function whose domain & range are subsets of $\mathbb{R}$.

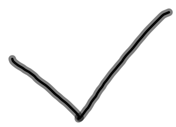
* These fns. are the emphasis of pre-calc, calc.

Q: Why focus on these fns?

* Because we can graph them in cartesian plane!

Typically, HS math deals with a small list of "types of function"

1. Linear functions... polynomial functions.
2. rational functions: $f(x)=\frac{p(x)}{q(x)}$, p, q polys.
3. Exponentials $y=b^x$, $b\neq 1$, $b>0$
4. Logs: $y=\log_b(x)$, $b\neq 1$, $b>0$
5. Trig, fns, inv. trig.
6. "Piecewise" fns of types 1–5



$|x|$

Analyzing real functions

In order to understand props/behavior of real fns, we analyze:

1. Domain
2. Range
3. Singularities/asymptotes
4. Zeroes
5. Critical pts / max mins
6. Increasing/decreasing
   Concavity, points of inflection.
7. "extremal behavior"
   $x \to \pm\infty$, asymptote ...
8. General props
   - cts
   - differentiable
   - "integrable"
   - taylor series
9. Special properties
   - even/odd
   - symmetries
   - periodic
10. Where/how does fn. occur in life?

Ex: $f(x) = mx + b$, $m > 0$.

1. domain $= \mathbb{R}$
2. range $= \mathbb{R}$
3. No sing/asymp.
4. Zeroes: $x = -b/m$
5. Crit. pts: no max/mins
6. Increasing, $f'(x) = m > 0$
   Concavity $= 0$, $f''(x) = 0$
7. $\lim_{x \to \pm\infty} f(x) = \pm\infty$
8. Cts, diff, Integrable
   $\int f(x)\,dx = \frac{mx^2}{2} + bx + C$
9. Linear fn = constant rate of change
10. Modelling any situation w/ constant change

.... Calculus!

approximate cts fns. by linear fns.

Ex: $f(x) = \frac{x^2 - x}{x^2 - 3x + 2} = \frac{x(x-1)}{(x-1)(x-2)} \underset{\substack{\uparrow \\ x \neq 1}}{=} \frac{x}{x-2}$

1. Domain: $\mathbb{R} - \{1, 2\}$

2. Range:

3. Asymptotes: $x = 2$ (vertical), $y = 1$ (horiz. asymp)
 Removable singularity $x = 1$

4. Zeroes: $x = 0$

5. Critical points: $f'(x) = \frac{(x-2)\cdot 1 - x(1)}{(x-2)^2} = \frac{-2}{(x-2)^2}$
 - $f'(x) \neq 0 \ \forall x \rightarrow$ no local mins/max
 - $f'(x) < 0 \ \forall x \rightarrow f$ is always decreasing

6. f is decreasing, $f''(x) = \frac{4}{(x-2)^3}$
 - $f''(x) \neq 0$
 - $f''(x) > 0$ if $x > 2$
  < 0 if $x < 2$
 - f has no points of infl in its domain.

7. $\lim_{x \to 2^+} f(x) = +\infty$ $\lim_{x \to \pm\infty} f(x) = 1$

 $\lim_{x \to 2^-} f(x) = -\infty$

8. Cts except at $X=2$, $X=1$,
diff ______________

Taylor series $\frac{X}{X-2} = -\frac{X}{2}\left(\frac{1}{1-\frac{X}{2}}\right)$

$= -\frac{X}{2}\left(1+\frac{X}{2}+\left(\frac{X}{2}\right)^2+\cdots\right)$

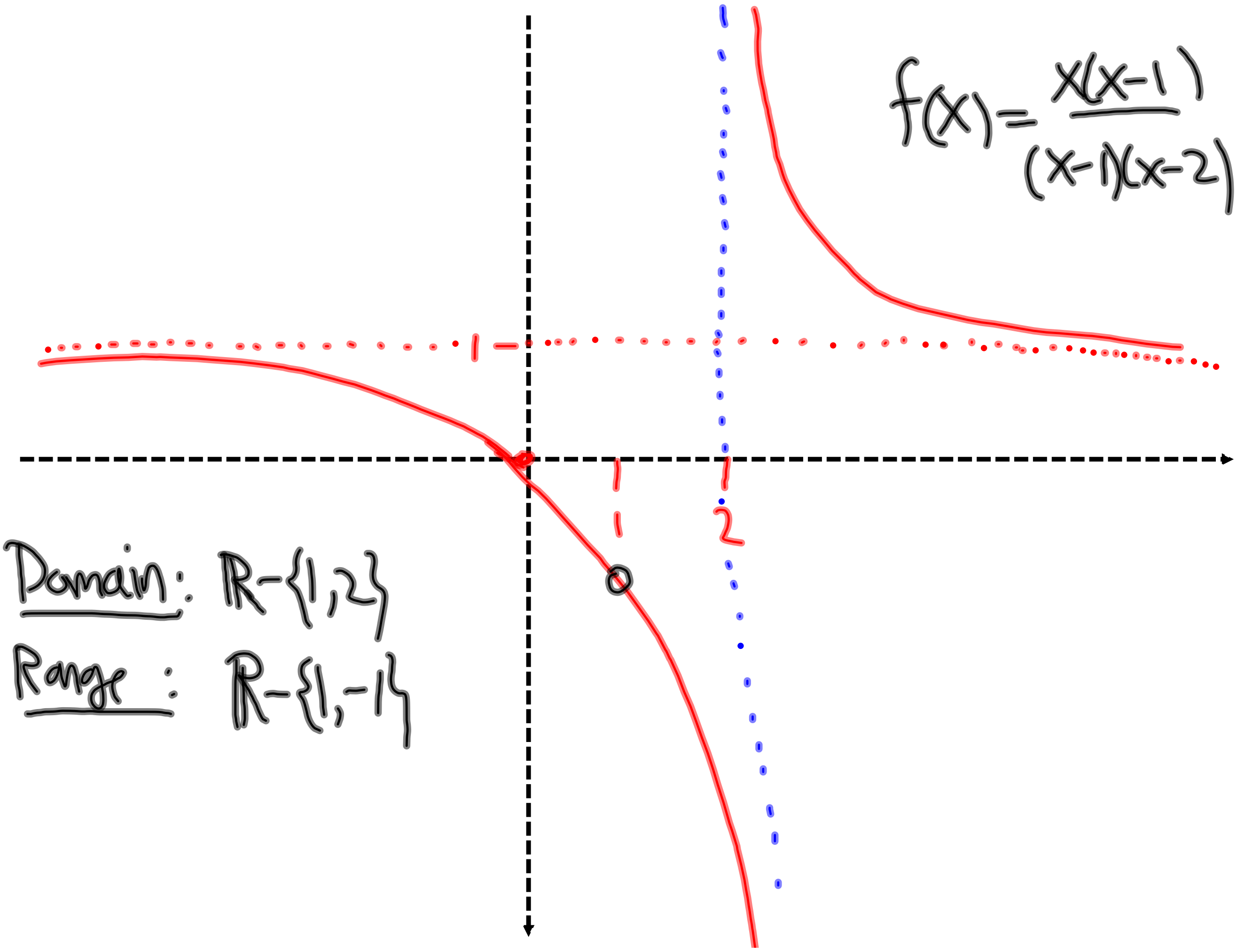
for $|x|<2$

$= -\frac{X}{2}-\frac{X^2}{4}-\frac{X^3}{8}-\frac{X^4}{16}-\cdots$

9. Special props: 1-1

10. ?

$$f(x) = \frac{x(x-1)}{(x-1)(x-2)}$$



Domain: $\mathbb{R} - \{1, 2\}$

Range: $\mathbb{R} - \{1, -1\}$

Fibbonacci numbers:

$F_0 = 0$ $\quad F_n = F_{n-1} + F_{n-2} \quad n \geq 2$

$F_1 = 1$

n	0	1	2	3	4	5	6
F_n	0	1	1	2	3	5	8

<u>Q</u>: Is there a <u>formula</u> for F_n?

<u>A</u> Yes! $F_n = \dfrac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{-1+\sqrt{5}}{2}\right)^n}{\sqrt{5}}$

<u>Q</u>: Where does this come from?