

$\mathbb{R}, +$

$$p + p = 2p$$

$$p + p + p = 3p$$

$$\underbrace{p + p + \dots + p}_n = np$$

$$0 \cdot p = 0$$

$$(0 + 0) \cdot p \stackrel{\text{dist}}{=} 0 \cdot p + 0 \cdot p$$

0 is the additive idem $\Rightarrow 0 \cdot p$

$$\Rightarrow 0 \cdot p + (-0 \cdot p)$$

$$\begin{aligned} & \underbrace{(0 \cdot p + 0 \cdot p)}_{\text{def}} + (-0 \cdot p) \stackrel{\text{assoc}}{=} 0 \cdot p + (0 \cdot p + (-0 \cdot p)) \\ & \stackrel{\text{def}}{=} 0 \cdot p + 0 = 0 \cdot p \end{aligned}$$

$\mathbb{R}_{>0}, \times$

$$a \cdot a = a^2$$

$$a \cdot a \cdot a = a^3$$

$$\underbrace{a \cdot a \cdot a \cdot \dots \cdot a}_n = a^n$$

$$a^0 = 1$$

0

~~set~~

$$(p-q) + (r-s) \\ = (p+r) - (q+s)$$

$$(p-q) - (r-s) \\ \parallel \\ (p+s) - (q+r) \\ -(-p) = p$$

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc} \\ = \frac{a}{b} \times \frac{d}{c}$$

$$\frac{1}{\frac{1}{a}} = a$$

