MATH 593
Assignment # 8
(Due Wednesday, November 13)

Hand in: #'s 2,3,4

Recall: The second hour exam will be on Monday, Novemeber 11.

(#1). Find the rational canonical form and Jordan canonical form of the matrix

$$\begin{pmatrix} 0 & -1 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}.$$

Do some other similar problems from Chapter 12 of Dummit and Foote.

(#2). (a). List all isomorphism classes of additive abelian groups of order 500.

(b). How many similarity classes are there of 10×10 matrices over a field k whose characteristic polynomial is $(x^2 - x)^5$? Write the minimal polynomial for each class.

(c). Let k be a field. Classify up to similarity all 4×4 matrices with entries in k satisfying the equation

$$T^3 - 2T^2 + T = 0.$$

(#3) Let $A = \{a_{ij}\}$ be an $n \times n$ matrix of integers. Denote by M the (additive) abelian group generated by n generators $e_1, \ldots, e_n$ subject to the relations

$$\begin{gathered} a_{11}e_1 + a_{12}e_2 + \ldots + a_{1n}e_n = 0 \\ a_{21}e_1 + a_{22}e_2 + \ldots + a_{2n}e_n = 0 \\ \ldots \\ a_{n1}e_1 + a_{n2}e_2 + \ldots + a_{nn}e_n = 0. \end{gathered}$$

Prove that M is finite if and only if $\det(A) \neq 0$, in which case $\#M = |\det(A)|$.

(#4). Let T be an $n \times n$ invertible matrix over a field k, and let $P_T(x)$ be the characteristic polynomial of T.

(i). Find $P_{T^{-1}}(x)$ in terms of $P_T(x)$.

(ii). Let V_T denote the $k[x]$-module corresponding to T in the usual way. Given a decomposition of V_T into cyclic modules, explain how to decompose $V_{T^{-1}}$ into cyclic modules.