p-adic Hodge Theory
MATH 847, Topics in Algebra
Spring, 2011
TR 11:00–12:15

One of the central aims of modern algebraic number theory is to understand the absolute Galois group G of the rational numbers. The main approach to this end is the study of (continuous) representations of G, an important class of which consists of representations on vector spaces over a p-adic field (henceforth called "p-adic representations"). Since G is (topologically) generated by the decomposition subgroups G_ℓ at primes ℓ, it is enough to restrict attention to (continuous) p-adic representations of these simpler groups. For $\ell \neq p$, such representations are mostly understood as in these cases the continuity requirement drastically limits the kinds of representations one can have. For $\ell = p$, however, the situation turns out to be much more subtle and interesting.

The goal of p-adic Hodge theory is to classify and study p-adic representations of G_p (i.e. $\ell = p$ above) and to this end p-adic Hodge theory is astonishingly successful. Much of the theory has been motivated by Galois representations coming from geometry (via étale cohomology) so it should be no surprise that p-adic Hodge theory has many spectacular applications in number theory. Indeed, the recent proof of Serre's conjecture on modular forms and Kisin's refinements of the Taylor-Wiles method for proving Fermat's Last Theorem both rely heavily on p-adic Hodge theory, and many questions about p-adic L-functions have been fruitfully analyzed via the comparison isomorphisms of p-adic Hodge theory.

In this course, we will develop p-adic Hodge theory from the beginning and will provide complete proofs of many of the key results and theorems. Time permitting, we will discuss other topics such as (φ, Γ)-modules, norm fields, p-adic differential equations and integral p-adic Hodge theory. We assume only a good understanding of linear algebra and familiarity with p-adic fields and Galois theory. Some exposure to algebraic geometry in the form of étale cohomology and de Rham cohomology will be useful for motivation but is not required. We will mainly follow the notes of Brinon and Conrad from the Clay Mathematics Summer School on Galois representations (Hawaii, 2009).