1. Find the Taylor series about 0 for the functions below. Include at least four nonzero terms.

A. $f(x) = \arcsin x$

B. $f(x) = \ln(3x+1)$

C. $f(x) = \dfrac{\sin(x^2)}{x}$

D. $f(x) = e^{\sin x}$

2. Use an appropriate Taylor polynomial about 0 to find an approximation of $\int_0^1 e^{-x^2}\,dx$ .What can you do to find a better approximation?

3. A. Write the Taylor series about 0 for $\frac{1}{1-x}$.

B. Use the derivative of the series in part A to help you find the Taylor series about 0 for $\frac{x}{(1-x)^2}$.

C. Use your answer to part B to calculate the exact value of $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{32}+\frac{6}{64}+\cdots$.

4. A. Find the Taylor polynomial of degree 3 approximating $f(x)=\sqrt{1-x}$ near 0.

B. Use the polynomial in part A to give approximate values of $\sqrt{0.5}$ and $\sqrt{0.9}$.

C. Which approximation in part B is more accurate? Why?

5. A. Suppose all the derivatives of a function f exist at 0. If the Taylor series for f about $x = 0$ is given by $f(x) = 5x^3 - 7x^5 + 9x^7 - \cdots$ find each of the following:

A. $\boldsymbol{f}^{(4)}(0)$

B. $\boldsymbol{f}^{(5)}(0)$

C. $\boldsymbol{f}^{(7)}(0)$

6. A. If $\boldsymbol{f}(\boldsymbol{x}) = \sum_{\boldsymbol{n}=1}^{\infty} \frac{(-1)^{\boldsymbol{n}}(\boldsymbol{x}-\pi)^{\boldsymbol{n}}}{\boldsymbol{n}^2}$ find $f'''(\pi)$.

B. If $\boldsymbol{f}(\boldsymbol{x}) = \sum_{\boldsymbol{n}=1}^{\infty} \frac{(-1)^{\boldsymbol{n}}(\boldsymbol{x}-5)^{\boldsymbol{n}+1}}{\boldsymbol{n}!}$ find $\boldsymbol{f}^{(7)}(5)$.

7. Find the exact value of the following sums:

A. $5 - \frac{5(0.2)^2}{2!} + \frac{5(0.2)^4}{4!} - \frac{5(0.2)^6}{6!} + \frac{5(0.2)^8}{8!} - \cdots$

B. $(0.2)^2 - \frac{(0.2)4}{3!} + \frac{(0.2)^6}{5!} - \frac{(0.2)^8}{7!} + \cdots$

C. $5 - \frac{5(0.2)^2}{1!} + \frac{5(0.2)^4}{2!} - \frac{5(0.2)^6}{3!} + \frac{5(0.2)^8}{4!} - \cdots$

D. $\frac{5(0.2)^2}{2} + \frac{5(0.2)^4}{2} + \frac{5(0.2)^6}{2} + \frac{5(0.2)^8}{2} - \cdots$

E. $5 + 5 + \frac{5}{2!} + \frac{5}{3!} + \frac{5}{4!} + \cdots$

F. $1 - 5 + \frac{25}{2!} - \frac{125}{3!} + \frac{625}{4!} + \cdots$