MATH 122A
FINAL EXAM STUDY GUIDE
(Fall 2017-Spring 2018)

The questions on the Math 122A final exam have a multiple choice format while the questions in this study guide are not multiple-choice in order to encourage you to solve the problems completely. These are not samples of questions that will appear on the final, but they do provide practice for the material that will be covered.

Additional practice involving multiple-choice is available as a WebAssign assignment posted in Math 122A WebAssign accounts, and again later in Math 120R and 122B Web-Assign accounts for those preparing for the Math 122A Final Exam retake.

1. Find the domain of the following functions:

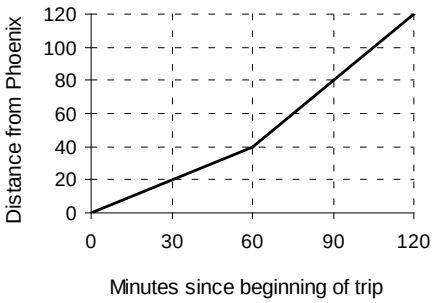
a) $g(t)=\sqrt{t^2-20}$ b) $h(y)=\dfrac{y+5}{\sqrt[3]{y-2}}$ c) $p(x)=-4x^3+2x+1$

d) $f(z)=\dfrac{z+4}{|z+4|}$ e) $w(t)=e^{\ln(t)}$

2. Use the line $Ax+By=C$ where A, B, and C are non-zero constants to answer the questions.

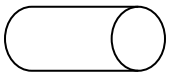
a) Find an equation of a line that is parallel to the given line and passing through the origin.
b) Find an equation of a line that is perpendicular to the given line and passing through $(2,-1)$.

3. The graph at the right illustrates a driving trip between two of Arizona's largest cities, Phoenix and Tucson. The distance between the cities is approximately 120 miles.



a) When is the driver 40 miles from Tucson?
b) What is the speed of the car over the last hour of the trip?
c) Write a piecewise function for $D(t)$, the distance from Phoenix, as a function of the number of minutes since the beginning of the trip. Use $0 \le t \le 120$.

4. Water is being pumped into a tank at a constant rate (cubic feet per minute). For each of the tanks shown below, sketch a graph of the height of the water in the tank as a function of time. Assume the tank is initially empty and will be filled.

a)


b)

c)

5. The lift L on an airplane wing at take-off is proportional to the square of the speed s of the plane and the area A of its wings.

a) Write an equation for lift.
b) If the speed is only half as much, how much larger should the area of the wings be for the lift to be the same?

6. Suppose the target heart rate when exercising, R, is a function of a person's age, A, and is given by the linear formula $R - H_0 = p(220 - A - H_0)$. Assume H_0 and p are constants. Find the slope and give a practical interpretation.

7. For certain category hurricanes, the cube of the diameter D (in miles) of the hurricane is roughly proportional to the square of the hurricane's duration t (in hours).

a) Write an equation to represent the relationship between diameter of the hurricane and duration. Solve your equation for D.
b) If the diameter of hurricane Veronica was 302 miles with a duration of 273 hours, what would be the diameter of a hurricane with a duration of 140 hours according to this model?

8. From the given data, determine if Account Balance is a function of Week.

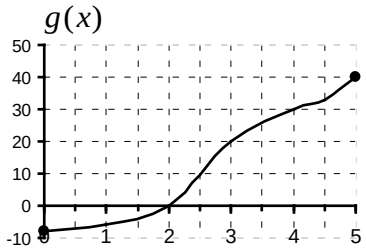
Account Balance	\$3,250	\$3,500	\$3,450	\$3,700	\$3,500	\$3,800
Week	1	2	3	4	5	6

9. Let $p(t) = \dfrac{5}{t+1}$. Find $\dfrac{p(t+h) - p(t)}{h}$ and simplify as much as possible.

10. Suppose an oil spill covers a circular area and that the radius increases according to $r(t) = 4 + \sqrt[3]{t/2}$ where t represents the number of minutes since the spill was first observed. The radius is measured in inches.

a) What was the radius of the spill when it was first observed?
b) Express the area A of the oil spill as a function of t.
c) Find the exact time when the area of the spill is 81π square inches.
d) Find and give a practical interpretation of $r(16) - r(2)$.

11. Use the graph of $g(x)$ shown below to answer the questions.



a) If $f(x) = x^3 + 2$, find $g(f(1))$.

b) Find the range of $g^{-1}(x)$.

c) Is $y = |g(x)|$ a one-to-one function?

d) Find $\dfrac{g(4) - g(2)}{4 - 2}$. What does this number represent on the graph of $g(x)$?

e) For what value(s), if any, would $y = \dfrac{1}{g(x) - 20}$ be undefined on the interval $[0, 5]$?

f) Does $g(x)$ appear to be concave up or concave down on the interval $2.5 \le x \le 3.5$?

12. For functions $f(t) = 5t^3 - 1$ and $g(t) = \sqrt{t+1}$, find and simplify:

a) $e^{f(t)+1}$ b) $f(t) + (g(t))^2$ c) $g(t^2)$ d) $f(2t)$

13. One of the functions below is linear, one is exponential, and one is quadratic. Determine which is which and then find a formula for each.

Table A

x	$f(x)$
0	0
3	1.8
6	7.2
9	16.2
12	28.8

Table B

x	$g(x)$
0	3.5
2	31.5
4	59.5
6	87.5
8	115.5

Table C

x	$h(x)$
0	3.5
1	4.9
2	6.86
3	9.604
4	13.4456

14. Sketch a graph of $y = f(x)$ that satisfies all of the following conditions:

* domain $-2 < x < 2$
* range $-\infty < y < +\infty$
* $f(x)$ is an odd function
* $f(x)$ is not invertible

15. Let $f(x) = Ax + B$ and $g(x) = Cx + D$ where $A \neq 0$, $C \neq 0$. Find $h(x) = (g \circ f)^{-1}(x)$. What are the slope and vertical intercept of $h(x)$?

16. If the zeros of $g(x)$ are $x = -2$ and $x = 3$, what are the zeros of $y = 2g(x+1)$?

17. Let $f(x) = \frac{3}{x}$ and $g(x) = \frac{4}{x^2} - 1$. Find $f\left(g(x)\right)$ and simplify your answer.

18. Let $(2, -5)$ be a point on the graph of $y = f(x)$. Find the corresponding point on the graphs of each of these transformations.

a) $y = -f(x) + 4$ b) $y = f\left(\frac{1}{4}x\right)$ c) $y = 3f(-x)$

19. It is predicted that the population of a particular state will double every 25 years.

a) Determine the annual and monthly growth rates. Express your answers as percents.
b) Determine the continuous growth rate per year. Express your answer as a percent.

20. A typical cup of coffee contains about 100 mg of caffeine. Every hour approximately 16% of the amount of caffeine in the body is metabolized and eliminated (decays).

a) Write an equation for the amount of caffeine, *C*, in the body as a function of *t*, the number of hours since a single cup of coffee was consumed.
b) Find the time when 20% of the caffeine has been metabolized and eliminated from the body. Give both an exact answer and a decimal answer. Include units.

21. In 1997, the average tuition at four-year public universities was $2,360 per year. In 1998, that figure rose to $2,430 per year.

a) If tuition increased linearly, write a formula for the tuition as a function of years since 1997. Use your function to estimate the tuition in 2006.
b) If tuition increased exponentially, write a formula for the tuition as a function of years since 1997. Use your function to estimate the tuition in 2006.

22. Determine which of the following describe an exponential function (growth or decay).

$g(t)$: The volume of a sphere is proportional to the cube of its radius.

$f(t)$: Each week the number of organisms is reduced by 50%.

$h(t)$: The temperature increases by 1.2 degrees each year.

23. Let $f(x) = 3\log_2(8-7x) - 12$

a) Find the domain and range.
b) Use algebra to find the intercepts of $f(x)$. Simplify your answers.

24. Fish are introduced into a large lake system. The population size (in numbers of fish) can be modeled by $P(t) = 2000 - 500e^{-0.03t}$ where t is measured in months since the fish were introduced.

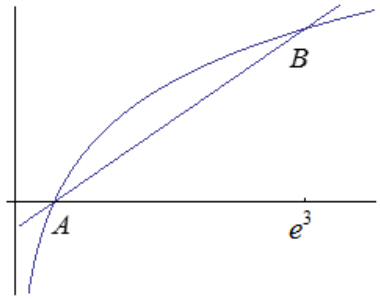
a) Find $P(3)$ and give a practical interpretation.
b) Find $P^{-1}(1500)$ and give a practical interpretation.
c) Is the population of the fish increasing or decreasing?
d) When does the population size reach 1800 fish according to the model?
e) What happens to the population size as $t \to \infty$?

25. a) Expand the following expression completely and simplify: $\log_a\left(\dfrac{\sqrt[4]{x^2 y}}{a^5}\right)$

b) Combine the following expression into a single logarithm: $\ln(x+1) - 3\ln(x) + \dfrac{2}{3}\ln(x^2+1)$

26. The graph of $y = \ln x$ along with a line passing through intersecting points A and B is shown below.

a) Find the coordinates of A and B.
b) Find the equation of the line.



27. Use $f(x) = 9^x$ and $g(x) = \log_3(x)$ to answer the following questions.

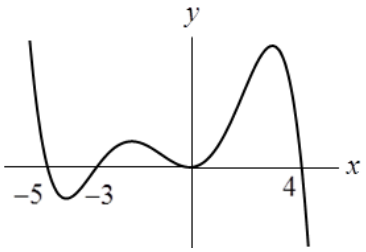
a) Find and simplify $g(f(x))$.
b) Find and simplify $f(g(x))$.
c) Find the asymptotes of $f(x)$ and $g(x)$.

28. Answer true or false:

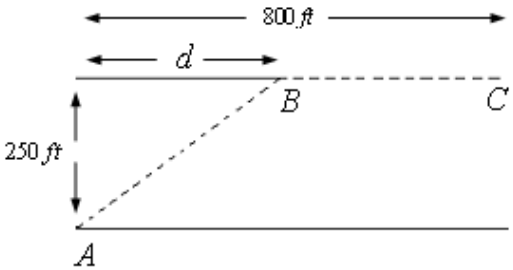
a) _____ The domain of all polynomials is $(-\infty, \infty)$.
b) _____ If $g(x)$ passes the vertical line test, it is a one-to-one function.
c) _____ An even degree polynomial must have at least one maximum or minimum.
d) _____ All rational functions have vertical asymptotes.
e) _____ If $y = f(x)$ is an odd function, then $y = |f(x)|$ is an even function.

29. Answer the following questions about the polynomial graphed at the right.

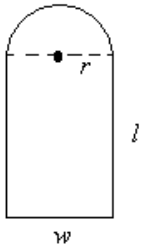
a) What is the smallest possible degree?
b) Is the leading coefficient positive or negative?
c) Write a possible equation for this polynomial.



30. A cable must be laid from point A to a point C across a river. The plan is to go from point A to point B under water and then continue from point B to point C on land. The cost of cable laid under water is \$74 per foot while the cable laid on land is \$48 per foot. Write an equation for the total cost of laying the cable in terms of *d*.



31. Consider a window consisting of a rectangle topped by a semicircle. If the total area of the window is 50 square feet, express the total <u>outer</u> perimeter in terms of r only.



32. If the hypotenuse of a right triangle is four times its base, *b*, express the area, *A*, of the triangle as a function of *b*.

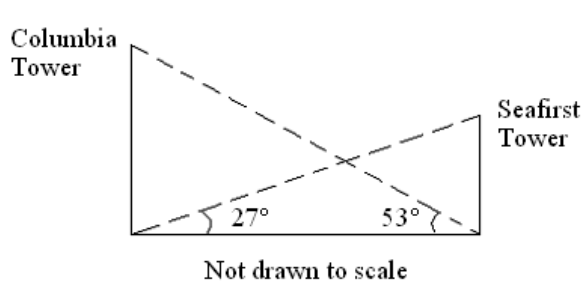
33. Find all intercepts, asymptotes, and holes (if any) for $f(x)=\dfrac{3x^2+14x-5}{(x+5)(5-4x)}$.

34. In each case, find the value(s) of *k* so that the following is true for $p(t)=\dfrac{2t^2+k}{3t+1}$.

a) $p(1)=5$ b) $p(3)=0$ c) The graph of $p(t)$ has no zeros.

35. Each time a person's heart beats, their blood pressure increases and then decreases as the heart rests between beats. A certain person's blood pressure is modeled by the function $b(t)=A+B\sin(Ct)$ where $b(t)$ is measured in mmHg and t is measured in minutes. Find values for *A*, *B*, and *C* if the person's average blood pressure is 115 mmHg, the range in blood pressure is 50 mmHg, and one cycle is completed every 1/80 of a minute.

36. The Columbia Tower in Seattle is 954 feet tall. The Seafirst Tower is *T* feet tall and stands *d* feet away from the Columbia Tower. Find the height of the Seafirst Tower. Give your answer to 2 decimal places.

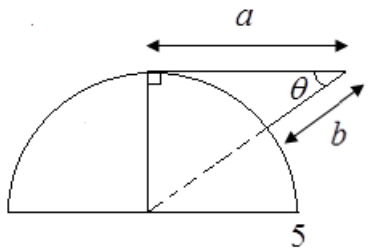


37. If $\csc(\theta)=\dfrac{3}{x}$, express $\tan(\theta)$ in terms of *x*.

38. Let $f(\alpha)=\cos(\alpha)$ and $g(\alpha)=\dfrac{\pi}{4}\sin(\alpha)$. Find the exact value of $f\left(g\left(\dfrac{3\pi}{2}\right)\right)$.

39. Substitute $x=3\sec\theta$ into the expression $\dfrac{\sqrt{x^2-9}}{x}$ and simplify as much as possible. Assume $0<\theta<\pi/2$.

40. Use the figure below to answer each question. The vertical side of the right triangle cuts the semicircle of radius 5 exactly in half.



a) Which of the following can be used to express θ as a function of a only.

(A) $\theta(a) = \arccos\left(\frac{a}{5+a}\right)$ (B) $\theta(a) = \arcsin\left(\frac{5}{5+a}\right)$ (C) $\theta(a) = \arctan\left(\frac{5}{a}\right)$

(D) $\theta(a) = \operatorname{arccot}\left(\frac{5}{a}\right)$ (E) Not enough information

b) Which of the following can be used to express b as a function of θ only.

(A) $b(\theta) = 5\sec\theta + 5$ (B) $b(\theta) = 5\csc\theta - 5$ (C) $b(\theta) = 5\cot\theta - 5$

(D) $b(\theta) = 5 - 5\sin\theta$ (E) Not enough information

41. Suppose $\sin\theta = A$ for $\frac{\pi}{2} < \theta < \pi$.

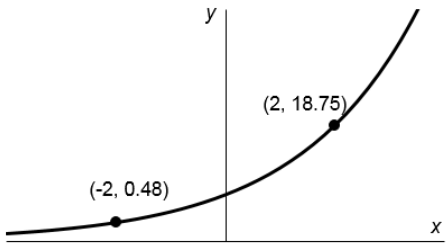
a) What are the possible values of A?
b) Find $\sin^3\theta$ and $\cot\theta$ in terms of A:

42. On a day when the sun passes directly overhead at noon, a six foot tall man casts a shadow of length $L(t) = 6\left|\cot\left(\frac{\pi}{12}t\right)\right|$ where L is in feet and t is the number of hours since 6 a.m.

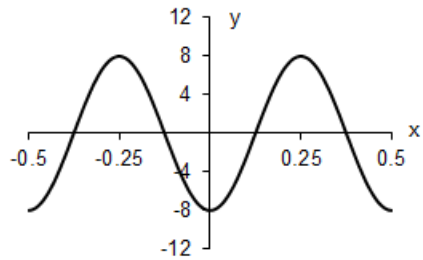
a) Find exact values for the lengths of the shadow at 8:00 a.m., noon, and 2:00 p.m.
b) Use your calculator to help you sketch an accurate graph of $L(t)$ for $0 < t < 12$.
c) Determine the values of t at which the length of the shadow equals the man's height.

43. Find a possible formula for the functions graphed below.

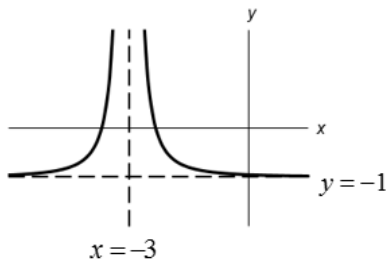
a) Use the general exponential form.



b) Use a periodic function.



c)



44. In each problem solve for the indicated variable on the given interval. Do not use your calculator.

a) For t: $\cos^2(t)\sin(t)+\sin(t)=0$ $\quad 0 \le t \le 2\pi$

b) For x: $\dfrac{\cos\left(\frac{x}{2}\right)}{2-\sin\left(\frac{x}{2}\right)}=0$ $\quad 0 \le x \le \pi$

c) For α: $\tan(3\alpha)=-1$ $\quad 0 \le \alpha \le \pi$

45. In each case, determine if the expression is defined. If the expression is defined, simplify it as much as possible.

a) $\arcsin\left(-\frac{1}{2}\right)$

b) $\cos^{-1}(2\pi)$

c) $\tan\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$

d) $\arccos\left(\cos\left(\frac{5\pi}{4}\right)\right)$

e) $\tan\left(\arcsin\left(\frac{20}{29}\right)\right)$

46. Let $f(x)=\begin{cases} 2+\cos\left(\dfrac{\pi}{4}x\right) & x\le 2 \\ \dfrac{12-5x}{x+2} & x>2 \end{cases}$

a) Find $f(0)$.
b) Find x such that $f(x)=0$.
c) Which of the following statements is true? (select all that apply)

____ $f(x)$ is continuous at $x=2$.
____ $f(x)$ has a jump (break) at $x=2$.
____ $f(x)$ has a hole at $x=2$.
____ $f(x)$ has a vertical asymptote at $x=2$.

47. Find the value of C that makes $q(t)$ continuous on $(-\infty,\infty)$.

$$q(t)=\begin{cases} Ct^2+2 & t\le -2 \\ -2t+8 & t>-2 \end{cases}$$

48. Determine if each function is continuous on the given interval.

a) $g(\theta)=\dfrac{e^{\sin\theta}}{\cos\theta}$ on $\left[-\dfrac{\pi}{4},\dfrac{\pi}{4}\right]$

b) $f(x)=3x-x^{-1}$ on $[-1,2]$

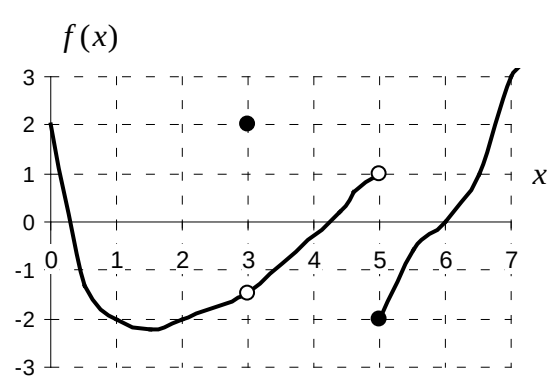
49. Find the exact values of the following limits:

a) $\displaystyle\lim_{z\to 4}\frac{28z-7z^2}{z^2-2z-8}$

b) $\displaystyle\lim_{t\to\infty}\frac{3t^2+5t^3}{7t^3+t-1}$

50. Use the graph of $f(x)$ at the right to find the following:

a) $f(5)$
b) $f(3)$
c) $\displaystyle\lim_{x\to 5^-} f(x)$
d) $\displaystyle\lim_{x\to 3} f(x)$



51. Suppose f and g are functions such that $\lim_{x\to 1} f(x) = 8$ and $\lim_{x\to 1} g(x) = 2$. Find the following:

a) $\lim_{x\to 1}\left(2f(x)+9\right)$ b) $\lim_{x\to 1}\dfrac{g(x)}{\left(f(x)\right)^2}$ c) $\lim_{x\to 1}\cos(g(x))$

52. Find the following limits for $f(x) = \dfrac{1}{1+e^{1/x}}$.

a) $\lim_{x\to\infty} f(x)$ b) $\lim_{x\to 1} f(x)$ c) $\lim_{x\to 0^+} f(x)$ d) $\lim_{x\to 0^-} f(x)$ e) $\lim_{x\to 0} f(x)$

53. Which of the following describes the end behavior of $p(x) = a(x-2)^2(x+9)^4$ for $a > 0$?

(A) $p(x)\to -\infty$ as $x\to -\infty$
$p(x)\to -\infty$ as $x\to \infty$

(B) $p(x)\to \infty$ as $x\to -\infty$
$p(x)\to \infty$ as $x\to \infty$

(C) $p(x)\to -\infty$ as $x\to -\infty$
$p(x)\to \infty$ as $x\to \infty$

(D) $p(x)\to \infty$ as $x\to -\infty$
$p(x)\to -\infty$ as $x\to \infty$

54. Factor completely (simplify the factors):

a) $-30z^3(1-6z)^4 + 3z^2(1-6z)^5$ b) $9 + 6e^x + e^{2x}$

55. Simplify as much as possible:

a) $\dfrac{b^n 5^{n+1}}{5^n b^{n+1}}$ b) $\dfrac{4-m^2}{3m-6}$ for $m \neq 2$ c) $\ln(5e^y)$

d) $10^{5\log x}$ e) $\dfrac{4(y+2)^{1/2} - 2y(y+2)^{-1/2}}{y+2}$

56. Use algebra to solve for the indicated variable:

a) For u: $3^{4u+1} = e^{1-2u}$

b) For p: $\sqrt{p^2+4} = 9$

c) For x: $\dfrac{5x^2 - 9x - 2}{x-7} = 0$

d) For t: $7te^{5t} - 3t^2e^{5t} = 0$

e) For h_2: $A = \dfrac{1}{2}b(h_1 + h_2)$

f) For w: $\log(w-3) + \log(w) = \log(28)$

g) For x: $y = \dfrac{4+3x}{5+2x}$

h) For z: $5z^3 - z^4 \geq 0$