MATH 122B AND 125
FINAL EXAM REVIEW PACKET ANSWERS

1.

t	1.0	1.2	1.4	1.6	1.8
$f'(t)$	1/2	3/4	5/4	7/4	2

2. __5__ $\lim_{h\to 0}\frac{f(20+h)-f(20)}{h}$

__3__ The slope of f at $x=10$

__1__ $f(16)$

__4__ The average rate of change of f from $x=12$ to $x=24$

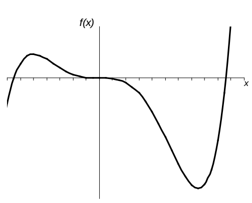
__2__ $\left.\frac{dy}{dx}\right|_{x=28}$

3. a) $f'(a)$ b) $f'(a)\Delta x+f(a)$

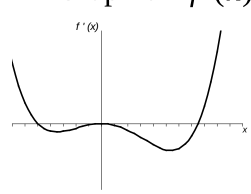
4. a) $g(-2)=3,\ \ g'(-2)=-1$ b) $g(-2)=-3,\ \ g'(-2)=1$

5. a) For $f(150)=125$: If a person weighs 150 pounds, the dose should be 125 milligrams. For $f'(150)=3$: The dose for a 151 pound person would need to be approximately 3 milligrams higher than a dose for a 150 pound person.
b) 140 milligrams.

6. Graph of $f(x)$

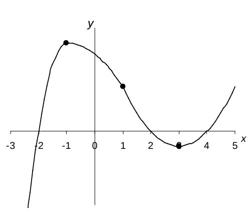


Graph of $f'(x)$



7. a) False b) True c) False d) False e) False f) True

8.



9. a) $\frac{1}{2}$ b) $\frac{1}{1+e}$ c) 0 d) 1 e) Does not exist.

10. $\lim_{h\to 0}\dfrac{e^{2(3+h)}-e^{2(3)}}{h}=f'(3)$ where $f(x)=e^{2x}$. $f'(3)=2e^{6}$.

11. $V(t)=25000(.85)^{t}$, $V'(3)=25000\ln(.85)(.85)^{3}\approx -2495.17$ dollars per year.

12. a) $x=-2$, $x=2$, $x=4$ b) $x=-1$, $x=3$ c) $x=1$

13. a) $(-\infty,-2)$, $(2,4)$ b) $(-1,3)$

14. a) $\dfrac{dy}{dx}=\dfrac{1}{1+(a+x)^{2}}$ b) $\dfrac{dy}{dx}=\dfrac{-ax}{\left(a^{2}+x^{2}\right)^{3/2}}$ c) $\dfrac{dy}{dx}=3a\sec^{3}(ax)\tan(ax)$

d) $\dfrac{dy}{dx}=-\ln a\left(a^{-x}\right)+ax^{a-1}$ e) $\dfrac{dy}{dx}=\dfrac{1}{a^{2}}\cos\left(\dfrac{x}{a}\right)$ f) $\dfrac{dy}{dx}=e^{-ax}\left(\dfrac{1}{x}-a\ln(ax)\right)$

15. a) $y=10x-27$ $h'(x)=2f'(x)$

b) Decreasing $g'(4)=-\dfrac{56}{9}$ $g'(x)=\dfrac{f(x)2x-x^{2}f'(x)}{\left(f(x)\right)^{2}}$

c) $k'(2)=20$ $k'(x)=f'(x^{2})2x$

d) $m'(4)=-5e^{-3}$ $m'(x)=e^{-f(x)}(-1)f'(x)$

e) Concave down $j''(x)=2f(x)f''(x)+2f'(x)f'(x)$

16. $x=5$, $x=-1$

17. $\theta=0, \dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{\pi}{2}, \dfrac{2\pi}{3}, \dfrac{5\pi}{6}, \pi$ $y'(\theta)=1-6\cos(3\theta)\sin(3\theta)$

18. a) $\dfrac{dm}{dv}=\dfrac{m_{o}v}{c^{2}\left(1-\left(v^{2}/c^{2}\right)\right)^{3/2}}$ b) $\dfrac{d}{dz}\left(\left|z^{2}-9\right|\right)=\begin{cases}2z & z<-3\\ -2z & -3<z<3\\ 2z & z>3\end{cases}$

The derivative is not defined at $z=3$ and $z=-3$.

19. $A=-\dfrac{11}{3}$, $B=-1$ $f'(x)=\begin{cases}\dfrac{1}{\sqrt{2x+5}} & -1<x\le 2\\ 2x-\dfrac{11}{3} & 2<x<5\end{cases}$

20. $f(h)=\frac{4}{5}h+20$

21. $k=15$

22. a) $\frac{3}{4}$ $\quad 2xy+x^2\frac{dy}{dx}+2\pi\sin(\pi x)=\frac{1}{x}-3y^2\frac{dy}{dx}$ $\qquad$ b) $y=\frac{3}{4}x-\frac{7}{4}$

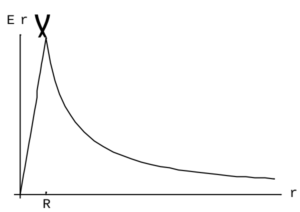
23. a) $\frac{dx}{dt}=\frac{5-3t^2}{2x+9}$ $\qquad$ b) $t=\pm\sqrt{\frac{5}{3}}$ $\qquad$ c) $x=-\frac{9}{2}$

24. a) $\left(\pm\sqrt{e^4-3},0\right)$ $\quad \left(0,-2\ln(3)+8\right)$ $\qquad$ b) $x=0$ is a local maximum $\quad y'=\frac{-4x}{x^2+3}$

c) $x=\pm\sqrt{3}$ are inflection points $\qquad y''=\frac{4x^2-12}{\left(x^2+3\right)^2}$

25. a) Yes. $E(R)=\lim_{r\to R^+}E(r)=kR$ $\qquad$ b) No. The slope is k for $r<R$, but the slope approaches $-k$ as r approaches R from the right.

c)



d)

$$\frac{dE}{dr}=\begin{cases} k & r<R \\ \frac{-kR^2}{r^2} & r>R \end{cases}$$

26. a) $t=3$ is a local minimum, $(3,-1/27)$
b) $t=4$ is the inflection point, $(4,-1/32)$
c) $t=2$ is the global maximum, $t=3$ is the global minimum

27. a) $\left(-a,2a^4+2a^3\right)$ is the local maximum, $\left(a,2a^4-2a^3\right)$ is the local minimum.
b) $\left(0,2a^4\right)$ is the inflection point.

28. a) $\lim_{t\to\pi}\frac{t^2-\pi^2}{\sin t}=-2\pi$ $\qquad$ Use L'Hopital's rule.
b) $\lim_{\theta\to 0}\frac{\sin(2\theta)}{\sin(7\theta)}=\frac{2}{7}$ $\qquad$ Use L'Hopital's rule.
c) $\lim_{x\to\infty}\arctan x=\pi/2$

d) $\lim_{y\to 0^+}\left(\frac{1}{e^y-1}-\frac{1}{y}\right)=-\frac{1}{2}$ Get common denominator then use L'Hopital's rule twice.

29. $A=1/16$, $B=1/2$ Use $f(4)=1$ and $f'(4)=0$ to find *A* and *B*.

30. a) $\left(e^a,0\right)$ b) $t=e^{a-1}$ is a local maximum.

34. The maximum volume is $\frac{L^3}{1728}$ cubic inches. The dimensions are $\frac{L}{12}\times\frac{L}{12}\times\frac{L}{12}$.

$V(x)=x\cdot x\cdot\left(\frac{L-8x}{4}\right)$

35. The minimum cost is $(9.6)V^{2/3}$ dollars. The dimensions are $(0.8)V^{1/3}$ by $(0.8)V^{1/3}$ by $\frac{V^{1/3}}{0.64}$.

$C(x)=5x^2+\frac{5.12V}{x}$

36. The wavelength is $w=c$ $V'(w)=\frac{k}{2}\left(\frac{w}{c}+\frac{c}{w}\right)^{-1/2}\left(\frac{1}{c}-\frac{c}{w^2}\right)$

37. The maximum value of *I* is 2, occurs at $t=\frac{\pi}{3\omega}$.

The minimum of value of *I* is -2, occurs at $t=\frac{4\pi}{3\omega}$

38. The coordinates that maximize the area of the triangle are $\left(3,\sqrt{3}\right)$

$A(x)=\frac{1}{2}x\sqrt{4-(x-2)^2}$

39. $C(r)=400r-16r^2-3\pi r^2$ The total cost will be maximized when $r=\frac{200}{16+3\pi}$ (approximately 7.87 feet) and $h=\frac{200-25\pi}{16+3\pi}$ (approximately 4.78 feet) . Maximum total cost is approximately \$1573.27

40. a) $b=\frac{1}{2}$, $b=2$ b) $b=1$ $A(b)=\frac{b}{2\left(b^2+1\right)}$

41. a) $f'(x)$ is the graph that looks linear.
b) $x=-1$ is a local minimum. $x=5$ is a local maximum.
$g'(x)=-f(x)e^{-x}+e^{-x}f'(x)$, $g'(x)=0$ when $f(x)=f'(x)$.

42. a) 25 miles per gallon during the first 70 miles.
$50/3 \approx 16.67$ miles per gallon in the next 30 miles.
b) The total gallons used t hours into the trip. $k(0.5) = 1.4$ gallons.
c) $k'(0.5) = 2.8$, $k'(1.5) = 1.8$ gallons per hour. Gas consumption is better in the first 70 miles, but gas is being consumed more quickly.

43. The camera is rotating at $1/24$ radians per minute. $x = \tan\theta$, $\dfrac{d\theta}{dt} = -2.5$

44. The height is growing at $\dfrac{5}{432\pi}$ feet per minute. $V = \dfrac{1}{3}\pi(3h)^2 h$.

45. The resistance is decreasing at 10 ohms per minute. $\dfrac{dV}{dt} = I\dfrac{dR}{dt} + R\dfrac{dI}{dt}$

46. a) Population is increasing when $0 < P < L$ and decreasing when $P > L$. When $P = L$ the, population remains constant.
b) $\dfrac{d^2P}{dt^2} = k^2 P(L-P)(L-2P)$ $P = \dfrac{L}{2}, \quad P = L, \quad P = 0$

47. $\dfrac{dM}{dt} = K\left(1 - \dfrac{1}{1+r}\right)\dfrac{dr}{dt}$

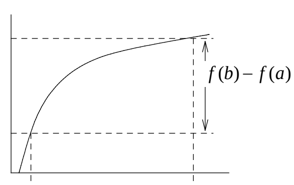
48. The volume is decreasing at a rate of $180\pi \text{ cm}^3$ per hour. The surface area is decreasing at a rate of $24\pi \text{ cm}^2$ per hour. Note: the formula for the surface area of a sphere would be given in the problem. $S = 4\pi r^2$.

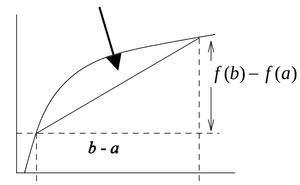
49. a) The lower estimate is 0.28. The upper estimate is 0.48.
b) Using the Fundamental Theorem of Calculus, $\int_{1.2}^{1.6} f'(t)dt = f(1.6) - f(1.2) = 0.5$.

50. __1__ $\sum_{k=1}^{10} g(t_k)\Delta t$ __3__ $\sum_{k=0}^{9} g(t_k)\Delta t$ __2__ $\lim_{n\to\infty}\sum_{k=1}^{n} g(t_k)\Delta t$

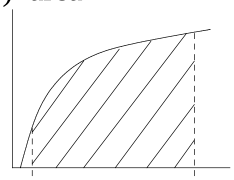
51. a) Object B b) Objects A and D c) Objects A and D d) Object C

52. a) length b) slope of the line

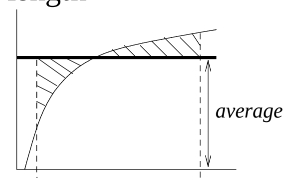




c) area



d) length



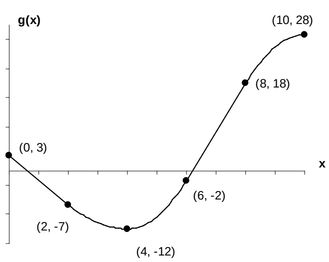
53. a) $\frac{b}{3}x^3 + x + c$ b) $b\ln|x| + \frac{1}{2}x^2 + c$ c) $\frac{1}{2}\ln\left|b + x^2\right| + c$ d) $\frac{1}{b}\arctan(bx) + c$

54. a) $\int_0^4 x(4-x)dx = \frac{32}{3}$ b) $\int_0^4 \left((x+2) - \left(x^2 - 3x + 2\right)\right)dx = \frac{32}{3}$

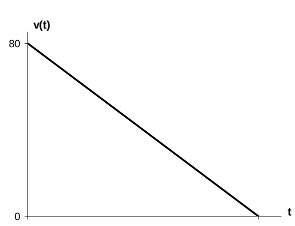
55. 2400 people will be added to the city. $\int_0^4 \left(3\sqrt{t} + 2\right)dt = 24$

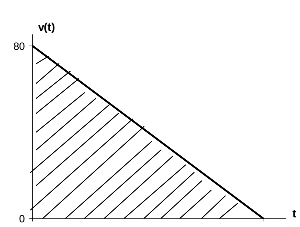
56. $r(t) = \begin{cases} 75 & 0 \le t < 2 \\ -3(t-2)^2 + 75 & 2 \le t \le 7 \end{cases}$ $\int_0^2 75dt + \int_2^7 \left(-3(t-2)^2 + 75\right)dt = 400$ gallons

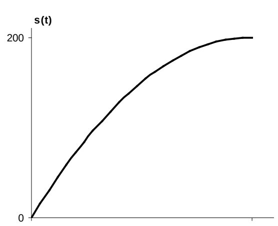
57.



58. a) $V(t) = -16t + 80$ b) 200 feet c) $s(t) = -8t^2 + 80t$







59. a) $F(0)=\int_0^0 e^{-t^2}dt=0$

b) $F'(x)=\dfrac{d}{dx}\int_0^x e^{-t^2}dt=e^{-x^2}$

c) Increasing.

d) Concave down. $F''(x)=-2xe^{-x^2}$

60. $\dfrac{1}{\pi/4-0}\int_0^{\pi/4}\dfrac{3}{\cos^2 x}dx=\dfrac{12}{\pi}$

61. a) $\int_{-\pi}^{\pi}\ln\left(5+4\cos x\right)dx=4\pi\ln 2$ $f(x)$ is an even function.

b) $\int_0^{\pi/2}\ln\left(5+4\cos(2x)\right)dx=\pi\ln 2$ Let $u=2x$ and change the endpoints.

62. a) True b) True c) False d) True e) False

63. a) $17/6$ b) 20 c) $37/2$ d) M

64. a) $g(C)>g(D)$ b) $g'(B)>g'(C)$ c) $g''(A)>g''(B)$

65. a) $F'(t)=e^{2t}(2t+11)$

b) Using the Fundamental Theorem $\int_{\ln(3)}^{2}e^{2t}(2t+11)dt=e^{2t}(t+5)\Big|_{\ln(3)}^{2}=7e^4-9\ln(3)-45$

66. a) Positive – look at signed area b) Negative – look at y value

c) Positive – look at slope

67. a) Approximately -775 algae per hour $\dfrac{\int_0^{2.5}r(t)dt}{2.5-0}$

b) The population decreased by approximately 1800 algae. $\int_0^3 r(t)dt$

68. $K=\dfrac{7}{18}$ $\int_0^9 K\sqrt{x}dx=7$

69. a) $\int\sin(3\theta)\cos^4(3\theta)d\theta=-\dfrac{1}{3}\int w^4dw=-\dfrac{1}{15}\cos^5(3\theta)+C$

b) $\int\dfrac{1}{5\tan(4v)}dv=\dfrac{1}{5}\int\dfrac{\cos(4v)}{\sin(4v)}dv=\dfrac{1}{5}\cdot\dfrac{1}{4}\int\dfrac{1}{w}dw=\dfrac{1}{20}\ln\left|\sin(4v)\right|+C$

c) $\int_0^1 te^{-t^2}\,dt = -\frac{1}{2}\int_0^{-1} e^w\,dw = \frac{1}{2}\int_{-1}^{0} e^w\,dw = \frac{1}{2}e^w\Big|_{-1}^{0} = \frac{1}{2}\left(1-e^{-1}\right)$

70. a) $\int_5^{15} g\left(\frac{x}{5}\right)dx = 5\int_1^3 g(w)\,dw = 60$

b) $\int_{-1/3}^{1/3} g\left(2-3t\right)dt = -\frac{1}{3}\int_3^1 g(w)\,dw = 4$

71. a) $s(t) = 2t^{3/2} + 5t - 2^{5/2}$

b) $\theta(x) = \arctan(x) + \frac{3\pi}{4}$

72. The thickness of the ice is decreasing by $\frac{1}{12\pi}$ cubic inches per minute.

$V = \pi\left(4+T\right)^2 \cdot 6 - \pi(4)^2 \cdot 6 \qquad \frac{dV}{dt} = 12\pi(4+T)\frac{dT}{dt}$

73. a) people per dollars, negative, this is the rate at which the number of people purchasing the skateboard changes as the price of the skateboard increases (the number of people who will not buy the skateboard if its price was increased from \$30 to \$31).

b) people, positive, this is the number of people willing to have the skateboard if the skateboard was free.

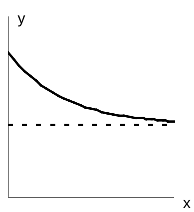
c) people, negative, this is the change in the number of people purchasing the skateboard if the price changed from \$32 to \$40.

74. $\frac{2}{3\sqrt{5}}$ feet per second

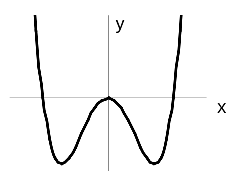
$D^2 = x^2 + (6-x)^2$

$D\frac{dD}{dt} = x\frac{dx}{dt} - (6-x)\frac{dx}{dt}$

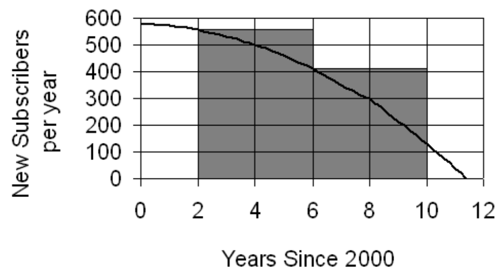
75. a)



b)



76. a) $550(4)+400(4)=3800$ new subscribers



b) 2, 3, 1

c) negative

d) $r(8)-r(4)=-200$

77. $\dfrac{\pi}{10\sqrt{g}}\left(x-100\right)+\dfrac{20\pi}{\sqrt{g}}$

78. $t=0$, $t=\dfrac{5}{2}$

79. $x=-4+2\sqrt{6}$ $\qquad \dfrac{-1}{(x+4)^2}=\dfrac{1/6-1/4}{2}$

80. $\int_0^1 (8-f(x))dx+\int_1^3 (8-g(x))dx$

81. A. $\lim\limits_{x\to 0}\int_0^x \sin(2t)dt=0$ and $\lim\limits_{x\to 0}\int_0^x \tan t\,dt=0$

B. $\lim\limits_{x\to 0}\dfrac{\int_0^x \sin(2t)dt}{\int_0^x \tan(t)dt}=\lim\limits_{x\to 0}\dfrac{\frac{d}{dx}\int_0^x \sin(2t)dt}{\frac{d}{dx}\int_0^x \tan(t)dt}=\lim\limits_{x\to 0}\dfrac{\sin(2x)}{\tan(x)}=\lim\limits_{x\to 0}\dfrac{2\cos(2x)}{\sec^2(x)}=2$ Yes.