MATH 124 AND 125
FINAL EXAM REVIEW PACKET

The following questions can be used as a review for Math 124/ 125. These questions are not actual samples of questions that will appear on the final exam, but they will provide additional practice for the material that will be covered on the final exam. When solving these problems keep the following in mind: Full credit for correct answers will only be awarded if all work is shown. Exact values must be given unless an approximation is required. Credit will not be given for an approximation when an exact value can be found by techniques covered in the course. The answers, along with comments, are posted as a separate file on http://math.arizona.edu/~calc.

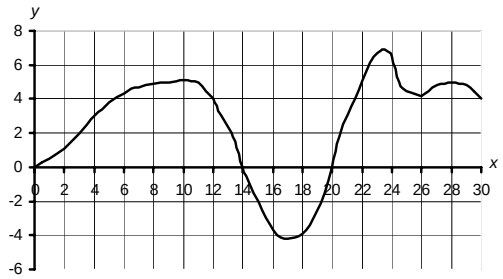
1. A function $f(t)$ is continuous and differentiable, and has values given in the table below.

t	1.0	1.2	1.4	1.6	1.8
$f(t)$	0.1	0.2	0.4	0.7	1.1

Fill in the table with approximate values for the function $f'(t)$.

t	1.0	1.2	1.4	1.6	1.8
$f'(t)$					

2. Arrange the following numbers from smallest (1) to largest (4) using the graph of *f* shown below:



_____ $\lim_{h\to 0}\dfrac{f(20+h)-f(20)}{h}$

_____ $f(16)$

_____ The slope of *f* at $x=10$

_____ The average rate of change of f from $x=12$ to $x=24$

3. Suppose $g(2)=3$ and $g'(2)=1$. Find $g(-2)$ and $g'(-2)$ assuming

a) $g(x)$ is an even function.

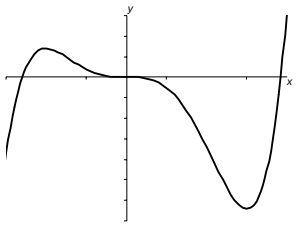
b) $g(x)$ is an odd function.

4. For a particular pain medication, the size of the dose, *D*, depends on the weight of the patient, *W*. We can write $D=f(W)$ where *D* is measured in milligrams and *W* is measured in pounds.

a) Interpret $f(150)=125$ and $f'(150)=3$ in terms of this pain medication.

b) Use the information in part a) to estimate $f(155)$.

5. Use the graph of $f(x)$ given below to sketch a graph of $f'(x)$.



6. Determine if the statement is true (T) or false (F). No need to make corrections.

a) _____ If $g(x)$ is continuous at $x = a$, then $g(x)$ must be differentiable at $x = a$.

b) _____ If $r''(x)$ is positive then $r'(x)$ must be increasing.

c) _____ If $t(x)$ is concave down, then $t'(x)$ must be negative.

d) _____ If $h(x)$ has a local maximum or minimum at $x = a$ then $h'(a)$ must be zero.

7. Sketch a graph of $f(x)$ that satisfies the following conditions:

i) $f(x)$ is continuous and differentiable everywhere

ii) the only solutions of $f(x) = 0$ are $x = -2$, 2, and 4

iii) the only solutions of $f'(x) = 0$ are $x = -1$ and 3

iv) the only solution of $f''(x) = 0$ is $x = 1$

8. Find the following limits for $f(x) = \dfrac{1}{1 + e^{1/x}}$.

a) $\lim\limits_{x \to \infty} f(x)$ b) $\lim\limits_{x \to 1} f(x)$ c) $\lim\limits_{x \to 0^+} f(x)$ d) $\lim\limits_{x \to 0^-} f(x)$ e) $\lim\limits_{x \to 0} f(x)$

9. A particular car was purchased for $25,000 in 2004. Suppose it loses 15% of its value each year. Let $V(t)$ represent the value of the car as a function of the years since it was purchased. Find $V(t)$ and use it to find $V'(3)$.

10. Let a be a positive constant. Find $\dfrac{dy}{dx}$ for each of the following:

a) $y = \arctan(a + x)$ b) $y = \dfrac{a}{\sqrt{a^2 + x^2}}$ c) $y = \cos^3(ax)$ d) $y = \dfrac{1}{a^x} + x^a$

11. Let $f(x)$ be a function so that $f(4) = 3$ and $f'(4) = 5$. Find the following:

a) $h'(4)$ where $h(x) = 2f(x) + 7$

b) $g'(4)$ where $g(x) = \dfrac{x^2}{f(x)}$

c) $k'(2)$ where $k(x) = f(x^2)$

d) $m'(4)$ where $m(x) = e^{-f(x)}$

12. If $g(x) = x^3 - 6x^2 - 12x + 5$ and $g'(x) = 3$, find x.

13. Find the indicated derivatives:

a) $\dfrac{dm}{dv}$ for $m = \dfrac{m_o}{\sqrt{1-\left(v^2/c^2\right)}}$

b) $g'(x)$ for $g(x) = \left|x^2 - 9\right|$

14. Torricelli's Theorem states that if there is a hole in a container of liquid h feet below the surface of the liquid, then the liquid will flow out at a rate given by $R(h) = \sqrt{2gh}$ where $g = 32\,\text{ft}/\text{sec}^2$. Find a linear function that can be used to approximate this rate for holes that are close to 25 feet below the surface of the water.

15. For what value(s) of k will $f(x) = x^3 - kx^2 + kx + k$ have an inflection point at $x = 5$?

16. The function $y(x)$ is defined implicitly by the equation $x^2 y - 2\cos(\pi x) = \ln x - y^3$

a) Find the value of the derivative of y with respect to x at the point $(1, -1)$

b) Find the equation of the tangent line to the curve at $(1, -1)$

17. A cable is made of an insulating material in the shape of a long, thin cylinder of radius R. It has electrical charge distributed evenly throughout it. The electrical field, E, at a distance r from the center of the cable is given below. k is a positive constant.

$$E = \begin{cases} kr & r \le R \\ \dfrac{kR^2}{r} & r > R \end{cases}$$

a) Is E continuous at $r = R$?

b) Is E differentiable at $r = R$?

c) Sketch E as a function of r.

d) Find $\dfrac{dE}{dr}$

18. Let $f(t) = -\dfrac{1}{t^2} + \dfrac{2}{t^3}$ for $t \ge 2$. Find

a) the critical point(s) and determine if it is a local maximum or minimum.

b) the inflection point(s).

c) the global maximum and minimum on the given interval.

19. Let $f(x) = x^3 - 3a^2x + 2a^4$ with constant $a > 1$. Find (answers will be in terms of a)

a) the coordinates of the local maxima and the local minima.

b) the coordinates of the inflection point(s).

20. Find the exact value of the following limits:

a) $\lim_{t\to\pi}\dfrac{t^2-\pi^2}{\sin t}$ b) $\lim_{\theta\to 0}\dfrac{\sin(2\theta)}{\sin(7\theta)}$ c) $\lim_{x\to\infty}\arctan x$

21. Consider the family of functions $f(t)=\dfrac{Bt}{1+At^2}$. Find the values of *A* and *B* so that $f(t)$ has a critical point at $(4,1)$.

22. Find the values of *a*, *b*, and *k* so that the parametric equations given below trace out a circle of radius 3 centered at (0,4) $x=a+k\cos t$, $y=b+k\sin t$, $0\le t\le 2\pi$.

23. Consider the lines parameterized by $\left\{\begin{matrix}x=3t-7\\y=4-9t\end{matrix}\right\}$ and $\left\{\begin{matrix}x=5t+6\\y=ct+8\end{matrix}\right\}$

a) For what value of *c*, if any, will these two lines be parallel?
b) For what value of *c*, if any, will these two lines intersect at $(5,-32)$?

24. Suppose an object moves in the *xy* plane along a path given by parametric equations $x=t^3-3t^2+1$, $y=t^2-4t-12$, $t\ge 0$.

a) Determine the time when the object stops. Where will it stop?
b) Determine the time when the object hits the *x*-axis.

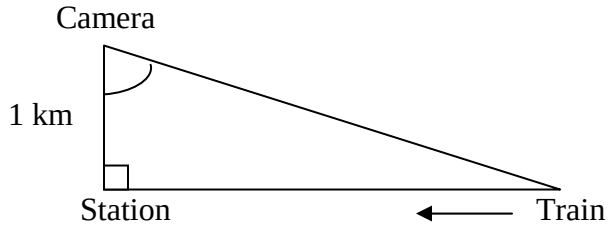
25. Wire with a total length of *L* inches will be used to construct the edges of a rectangular box and thus provide a framework for the box. The bottom of the box must be square. Find the maximum volume that such a box can have.

26. What are the dimensions of the largest rectangle that can be inscribed under the graph of $y=25-x^2$ so that one side is on the x-axis?

27. A closed rectangular box with a square bottom has a fixed volume *V*. It must be constructed from three different types of materials. The material used for the four sides costs $1.28 per square foot; the material for the bottom costs $3.39 per square foot, and the material for the top costs $1.61 per square foot. Find the minimum cost for such a box in terms of *V*.

28. The speed of a wave traveling in deep water is given by $V(w)=k\sqrt{\dfrac{w}{c}+\dfrac{c}{w}}$ where *w* is the wavelength of the wave. Assume *c* and *k* are positive constants. Find the wavelength that minimizes the speed of the wave.

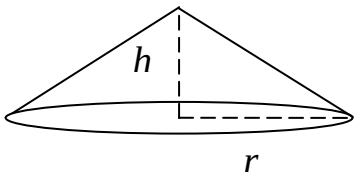
29. A camera is focused on a train as the train moves along a track towards a station as shown below. The train travels at a constant speed of 10 km/hr. How fast is the camera rotating (in radians/min) when the train is 2 km from the camera?



30. Sand is poured into a pile from above. It forms a right circular cone with a base radius that is always 3 times the height of the cone. If the sand is being poured at a rate of 15 ft^3 per minute, how fast is the height of the pile growing when the pile is 12 ft high?

The volume of a right circular cone is given by $V = \frac{1}{3}\pi r^2 h$

.



31. Let *b* be a positive constant. Evaluate the following:

a) $\int \left(bx^2 + 1\right) dx$ b) $\int \frac{b + x^2}{x} dx$ c) $\int \frac{x}{b + x^2} dx$ d) $\int \frac{dx}{1 + \left(bx\right)^2}$

32. Find the areas of the regions. Include a sketch of the regions.

a) The region bounded between $y = x(4 - x)$ and the *x*-axis.

b) The region bounded between $y = x + 2$ and $y = x^2 - 3x + 2$

33. A function $f(t)$ is continuous and differentiable, and has values given in the table below. The values in the table are representative of the properties of the function. Find upper and lower estimates for $\int_1^{1.8} f(t)dt$.

t	1.0	1.2	1.4	1.6	1.8
$f(t)$	0.1	0.2	0.4	0.7	1.1

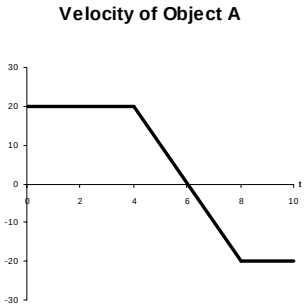
34. Several objects are moving in a straight line from time $t = 0$ to time $t = 10$ seconds. The following are graphs of the velocities of these objects (in cm/sec).

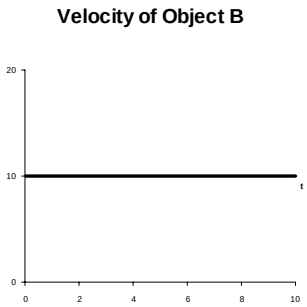
a) Which object(s) is farthest from the original position at the end of 10 seconds?

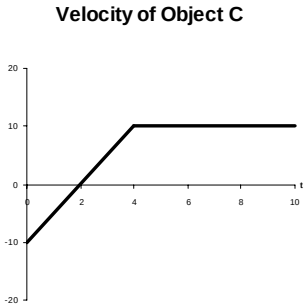
b) Which object(s) is closest to its original position at the end of 10 seconds?

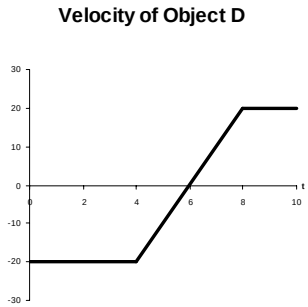
c) Which object(s) has traveled the greatest total distance during these 10 seconds?

d) Which object(s) has traveled the least distance during these 10 seconds?



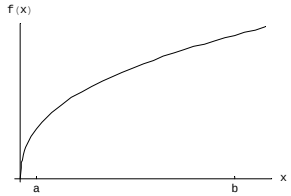




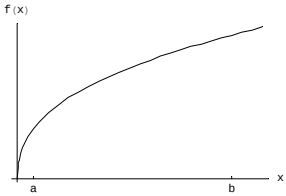


35. Illustrate the following on the graph of $f(x)$ given below. Assume $F'(x) = f(x)$.

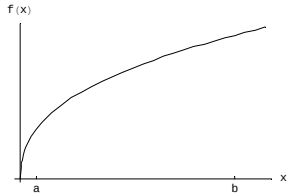
a) $f(b) - f(a)$



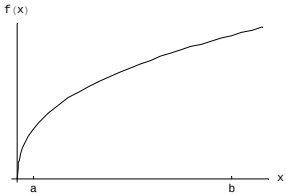
b) $\dfrac{f(b) - f(a)}{b - a}$



c) $F(b) - F(a)$

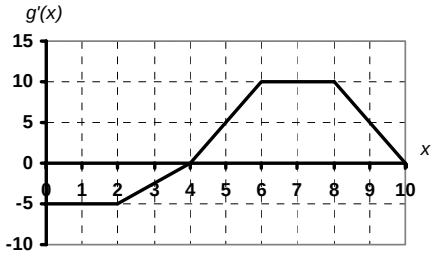


d) $\dfrac{F(b) - F(a)}{b - a}$



36. Water is pumped into a large tank at the rate of $r(t) = 3\sqrt{t} + 2$ liters per hour. Find the amount of water that enters the tank in the first four hours $(0 \le t \le 4)$.

37. Use the graph of $g'(x)$ given below to sketch a graph of $g(x)$ so that $g(0) = 3$.



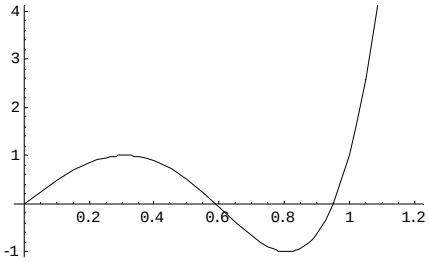
38. A car going 80 $ft/\sec$ brakes to a stop in five seconds. Assume the deceleration is constant.
a) Find an equation for $v(t)$, the velocity function. Sketch $v(t)$.
b) Find the total distance traveled from the time the brakes were applied until the car came to a stop. Illustrate this quantity on the graph of $v(t)$ in part a).
c) Find an equation for $s(t)$, the position function. Sketch $s(t)$.

39. Consider the function $F(x) = \int_0^x e^{-t^2} dt$.
a) Find $F(0)$ b) Find $F'(x)$ c) Is $F(x)$ increasing or decreasing for $x \ge 0$?
d) Is $F(x)$ concave up or concave down for $x \ge 0$?

40. The average value of f from *a* to *b* is defined as $\frac{1}{b-a}\int_a^b f(x)dx$. Find the average value of $f(x) = \frac{3}{\cos^2 x}$ over the interval $0 \le x \le \frac{\pi}{4}$.

41. According to a book of mathematical tables, $\int_0^{\pi} \ln(5 + 4\cos x)dx = 2\pi \ln 2$.
a) Find $\int_{-\pi}^{\pi} \ln(5 + 4\cos x)dx$ b) Find $\int_0^{\pi/2} \ln(5 + 4\cos(2x))dx$

42. Use the graph of $f(x)$ below to answer the following. Circle True or False.



a) $\int_0^{0.1} f(x)dx \le \int_0^{0.2} f(x)dx$ True False

b) $\int_0^{0.4} f(x)dx \le \int_0^{0.5} f(x)dx$ True False

c) $\int_0^{0.1} f(x)dx \le \int_0^{0.1} \left(f(x)\right)^2 dx$ True False

d) $\int_0^{1} f(x)dx \ge 0$ True False

e) $\int_0^{1} \left|f(x)\right| dx \ge 1$ True False

43. The graph of the function $f(x)$ and its derivative $f'(x)$ are given below.

a) Determine which graph is $f(x)$ and which graph is $f'(x)$.

b) Use the graphs to find the values of *x* that maximize and minimize the function $g(x) = f(x)e^{-x}$. Hint: Find $g'(x)$ and then use the graphs to determine where $g'(x) = 0$.