MATH 124 AND 125
FINAL EXAM REVIEW PACKET
(Revised Fall 2011)

The following questions can be used as a review for Math 124/ 125. These questions are not actual samples of questions that will appear on the final exam, but they will provide additional practice for the material that will be covered on the final exam. When solving these problems keep the following in mind: Full credit for correct answers will only be awarded if all work is shown. Exact values must be given unless an approximation is required. Credit will not be given for an approximation when an exact value can be found by techniques covered in the course. The answers, along with comments, are posted as a separate file on http://math.arizona.edu/~calc.

1. A function $f(t)$ is continuous and differentiable, and has values given in the table below.

t	1.0	1.2	1.4	1.6	1.8
$f(t)$	0.1	0.2	0.4	0.7	1.1

Fill in the table with approximate values for the function $f'(t)$.

t	1.0	1.2	1.4	1.6	1.8
$f'(t)$					

2. Arrange the following numbers from smallest (1) to largest (5) using the graph of f shown below:

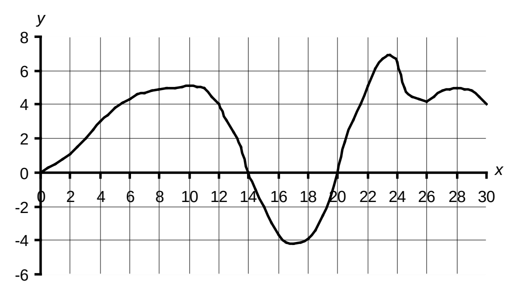
_____ $\lim_{h\to 0}\frac{f(20+h)-f(20)}{h}$

_____ The slope of f at $x=10$

_____ $f(16)$

_____ The average rate of change of f from $x=12$ to $x=24$

_____ $\left.\frac{dy}{dx}\right|_{28}$



3. Suppose $f'(x)$ is a differentiable decreasing function for all x. In each case, determine which quantity is larger:

a) $f'(a)$ or $f'(a+1)$

b) $f(a+\Delta x)$ or $f'(a)\Delta x+f(a)$

4. Let $g(2)=3$ and $g'(2)=1$. Find $g(-2)$ and $g'(-2)$ assuming

a) $g(x)$ is an even function.

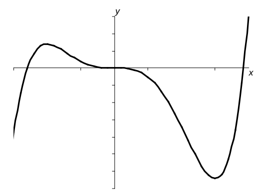
b) $g(x)$ is an odd function.

5. For a particular pain medication, the size of the dose, *D*, depends on the weight of the patient, *W*. We can write $D = f(W)$ where *D* is measured in milligrams and *W* is measured in pounds.

a) Interpret $f(150) = 125$ and $f'(150) = 3$ in terms of this pain medication.

b) Use the information in part a) to estimate $f(155)$.

6. Use the graph of $f(x)$ given below to sketch a graph of $f'(x)$.



7. Determine if the statement is true (T) or false (F). No need to make corrections.

a) _____ If $g(x)$ is continuous at $x = a$, then $g(x)$ must be differentiable at $x = a$.

b) _____ If $r''(x)$ is positive then $r'(x)$ must be increasing.

c) _____ If $t(x)$ is concave down, then $t'(x)$ must be negative.

d) _____ If $h(x)$ has a local maximum or minimum at $x = a$ then $h'(a)$ must be zero.

e) _____ $f'(a)$ is the tangent line to $f(x)$ at $x = a$.

f) _____ Instantaneous velocity can be positive, negative, or zero.

8. Sketch a graph of $f(x)$ that satisfies all of the following conditions:

i) $f(x)$ is continuous and differentiable everywhere

ii) the only solutions of $f(x) = 0$ are $x = -2$, 2, and 4

iii) the only solutions of $f'(x) = 0$ are $x = -1$ and 3

iv) the only solution of $f''(x) = 0$ is $x = 1$

9. Find the following limits for $f(x) = \dfrac{1}{1 + e^{1/x}}$.

a) $\lim_{x \to \infty} f(x)$ b) $\lim_{x \to 1} f(x)$ c) $\lim_{x \to 0^+} f(x)$ d) $\lim_{x \to 0^-} f(x)$ e) $\lim_{x \to 0} f(x)$

10. Find $\lim_{h \to 0} \dfrac{e^{2(3+h)} - e^{2(3)}}{h}$ by recognizing the limit as the definition of $f'(a)$ for some function *f* and some value *a*.

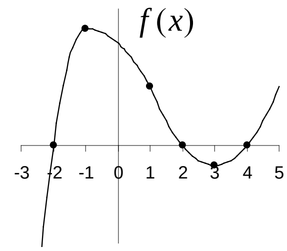
11. A particular car was purchased for $25,000 in 2004. Suppose it loses 15% of its value each year. Let $V(t)$ represent the value of the car as a function of the years since it was purchased. Find $V(t)$ and use it to find the exact value of $V'(3)$.

12. Use the graph of $f(x)$ at the right to find the value(s) of x so that

a) $f(x) = 0$

b) $f'(x) = 0$

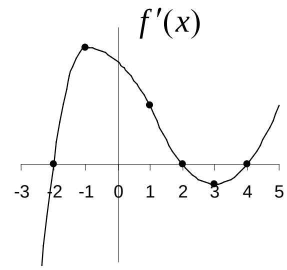
c) $f''(x) = 0$



13. Use the graph of $f'(x)$ at the right to find intervals where

a) $f(x)$ is decreasing

b) $f(x)$ is concave down



14. Let a be a positive constant. Find $\frac{dy}{dx}$ for each of the following:

a) $y = \arctan(a + x)$ b) $y = \frac{a}{\sqrt{a^2 + x^2}}$ c) $y = \sec^3(ax)$

d) $y = \frac{1}{a^x} + x^a$ e) $y = \frac{1}{a}\sinh\left(\frac{x}{a}\right)$ f) $y = e^{-ax}\ln(ax)$

15. Let $f(x)$ be a continuous function with $f(4) = 3$, $f'(4) = 5$ and $f''(4) = -9$.

a) Find the equation of the tangent line to $h(x) = 2f(x) + 7$ at $x = 4$.

b) Is $g(x) = \frac{x^2}{f(x)}$ increasing or decreasing at $x = 4$?

c) Find $k'(2)$ where $k(x) = f(x^2)$.

d) Find $m'(4)$ where $m(x) = e^{-f(x)}$.

e) Is $j(x) = \left(f(x)\right)^2$ concave up or concave down at $x = 4$?

16. If $g(x) = x^3 - 6x^2 - 12x + 5$ and $g'(x) = 3$, find x.

17. Determine where the slope of $y(\theta) = \theta + \cos^2(3\theta)$ will equal 1 on the interval $0 \le \theta \le \pi$.

18. Find the indicated derivatives:

a) $\frac{dm}{dv}$ for $m = \frac{m_o}{\sqrt{1 - \left(v^2/c^2\right)}}$ b) $\frac{d}{dz}\left(\left|z^2 - 9\right|\right)$

19. Find A and B so that $f(x)$ is continuous and differentiable on the interval $(-1,5)$.

$$f(x)=\begin{cases}\sqrt{2x+5} & -1<x\le 2\\ x^2+A(x-2)+B & 2<x<5\end{cases}$$

20. Torricelli's Theorem states that if there is a hole in a container of liquid h feet below the surface of the liquid, then the liquid will flow out at a rate given by $R(h)=\sqrt{2gh}$ where $g=32\,\text{ft}/\text{sec}^2$. Find a linear function that can be used to approximate this rate for holes that are close to 25 feet below the surface of the water.

21. For what value(s) of k will $f(x)=x^3-kx^2+kx+k$ have an inflection point at $x=5$?

22. The function $y(x)$ is defined implicitly by the equation $x^2y-2\cos(\pi x)=\ln x-y^3$

a) Find the value of the derivative of y with respect to x at the point $(1,-1)$

b) Find the equation of the tangent line to the curve at $(1,-1)$.

23. a) Find $\dfrac{dx}{dt}$ for $t^3+x^2-5t+9x=7$.

b) For what value(s) of t will the tangent line to the curve be horizontal?

c) For what value(s) of x will the tangent line to the curve be vertical?

24. Let $y=-2\ln(x^2+3)+8$

a) Find all the intercepts.

b) Find all the critical points and classify each as a local maximum, minimum, or neither.

c) Find all the inflection points.

25. A cable is made of an insulating material in the shape of a long, thin cylinder of radius R. It has electrical charge distributed evenly throughout it. The electrical field, E, at a distance r from the center of the cable is given below. k is a positive constant.

$$E=\begin{cases}kr & r\le R\\ \dfrac{kR^2}{r} & r>R\end{cases}$$

a) Is E continuous at $r=R$?

b) Is E differentiable at $r=R$?

c) Sketch E as a function of r.

d) Find $\dfrac{dE}{dr}$

26. Let $f(t) = -\frac{1}{t^2} + \frac{2}{t^3}$ for $t \geq 2$. Find
a) the critical point(s) and determine if it is a local maximum or minimum.
b) the inflection point(s).
c) the global maximum and minimum on the given interval.

27. Let $f(x) = x^3 - 3a^2x + 2a^4$ with constant $a > 1$. Find (answers will be in terms of a)
a) the coordinates of the local maxima and the local minima.
b) the coordinates of the inflection point(s).

28. Find the exact value of the following limits:
a) $\lim_{t\to\pi} \frac{t^2 - \pi^2}{\sin t}$ b) $\lim_{\theta\to 0} \frac{\sin(2\theta)}{\sin(7\theta)}$ c) $\lim_{x\to\infty} \arctan x$ d) $\lim_{y\to 0^+} \left(\frac{1}{e^y - 1} - \frac{1}{y} \right)$

29. Consider the family of functions $f(t) = \frac{Bt}{1 + At^2}$. Find the values of *A* and *B* so that $f(t)$ has a critical point at $(4,1)$.

30. Consider the family of functions $y(t) = at - t\ln t$ for $t > 0$.
a) Find the *t*-intercept. Your answer will be in terms of *a*.
b) Find the critical point and determine if it is a local maximum or minimum (or neither).

31. Find the values of *a*, *b*, and *k* so that the parametric equations given below trace out a circle of radius 3 centered at (0,4) $x = a + k\cos t$, $y = b + k\sin t$, $0 \leq t \leq 2\pi$.

32. Consider the lines parameterized by $\begin{Bmatrix} x = 3t - 7 \\ y = 4 - 9t \end{Bmatrix}$ and $\begin{Bmatrix} x = 5t + 6 \\ y = ct + 8 \end{Bmatrix}$
a) For what value of *c*, if any, will these two lines be parallel?
b) For what value of *c*, if any, will these two lines intersect at $(5, -32)$?

33. Suppose an object moves in the *xy* plane along a path given by parametric equations
$x = t^3 - 3t^2 + 1$, $y = t^2 - 4t - 12$, $t \geq 0$.
a) Determine the time when the object stops. Where will it stop?
b) Determine the time when the object hits the *x*-axis.
c) Find the equation of the tangent line to the curve at $t = 5$.

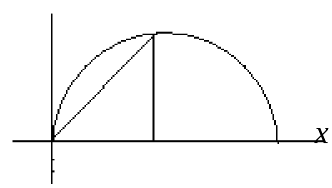
34. Wire with a total length of L inches will be used to construct the edges of a rectangular box and thus provide a framework for the box. The bottom of the box must be square. Find the maximum volume that such a box can have.

35. A closed rectangular box with a square bottom has a fixed volume V. It must be constructed from three different types of materials. The material used for the four sides costs \$1.28 per square foot; the material for the bottom costs \$3.39 per square foot, and the material for the top costs \$1.61 per square foot. Find the minimum cost for such a box in terms of V.

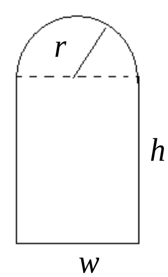
36. The speed of a wave traveling in deep water is given by $V(w) = k\sqrt{\frac{w}{c} + \frac{c}{w}}$ where w is the wavelength of the wave. Assume c and k are positive constants. Find the wavelength that minimizes the speed of the wave.

37. An electric current, I, in amps, is given by $I = \cos(\omega t) + \sqrt{3}\sin(\omega t)$ where $\omega \neq 0$ is a constant. Find the maximum and minimum values of I. For what values of t will these occur if $0 \le t \le 2\pi$.

38. The hypotenuse of the right triangle shown at the right is the line segment from the origin to a point on the graph of $y = \sqrt{4-(x-2)^2}$. Find the coordinates on the graph that will maximize the area of the right triangle.



39. A stained glass window will be created as shown at the right. The cost of the semi-circular region will be \$10 per square foot and the cost of the rectangular region will be \$8 per square foot. Due to construction constraints, the outside perimeter must be 50 feet. Find the maximum total cost of the window. What are the dimensions of the window?

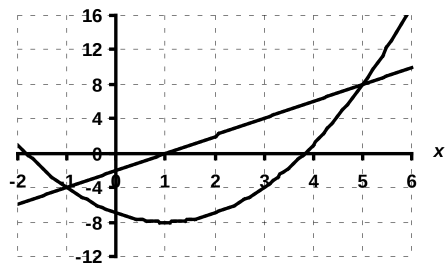


40. For $b > 0$, the line $b\left(b^2+1\right)y = b - x$ forms a triangle in the first quadrant with the x-axis and the y-axis.

a) Find the value(s) of b so that the area of the triangle is exactly $1/5$.

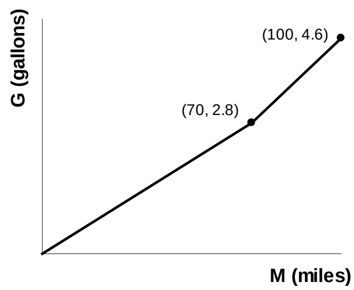
b) Find the value of b that maximizes the area of the triangle.

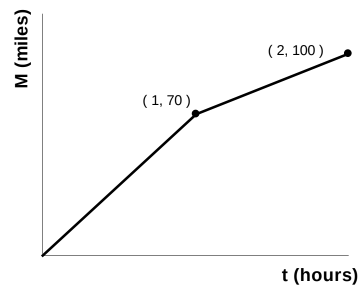
41. The graph of the function $f(x)$ and its derivative $f'(x)$ are given at the right.



a) Determine which graph is $f(x)$ and which graph is $f'(x)$.
b) Use the graphs to find the values of *x* that maximize and minimize the function $g(x) = f(x)e^{-x}$.

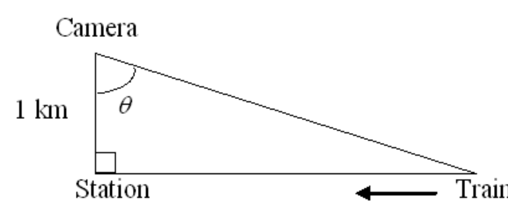
42. The graph below on the left shows the number of gallons, *G*, of gasoline used on a trip of *M* miles. The graph below on the right shows distance traveled, *M*, as a function of time *t*, in hours since the start of the trip. You can assume the segments of the graphs are linear.



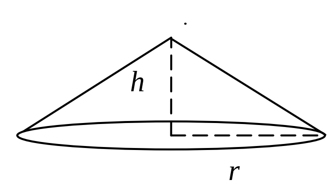


a) What is the gas consumption in miles per gallon during the first 70 miles of the trip? During the next 30 miles?
b) If $G = f(M)$ and $M = h(t)$, what does $k(t) = f(h(t))$ represent? Find $k(0.5)$.
c) Find $k'(0.5)$ and $k'(1.5)$. What do these quantities tell us?

43. A camera is focused on a train as the train moves along a track towards a station as shown at the right. The train travels at a constant speed of $10\,\text{km/hr}$. How fast is the camera rotating (in radians/min) when the train is 2 km from the camera?



44. Sand is poured into a pile from above. It forms a right circular cone with a base radius that is always 3 times the height of the cone. If the sand is being poured at a rate of 15 ft^3 per minute, how fast is the height of the pile growing when the pile is 12 ft high?



45. A voltage, *V* volts, applied to a resistor of *R* ohms produces an electrical current of *I* amps where $V = I \cdot R$. As the current flows, the resistor heats up and its resistance falls. If 100 volts is applied to a resistor of 1000 ohms, the current is initially 0.1 amps but increases by 0.001 amps per minute. At what rate is the resistance changing if the voltage remains constant?

46. The rate of change of a population depends on the current population, P, and is given by $\frac{dP}{dt} = kP(L-P)$ for some positive constants k and L.

a) For what nonnegative values of P is the population increasing? Decreasing? For what values of P does the population remain constant?

b) Find $\frac{d^2P}{dt^2}$ as a function of P. For what value of P will $\frac{d^2P}{dt^2} = 0$?

47. The mass of a circular oil slick of radius r is $M = K\left(r - \ln(1+r)\right)$, where K is a positive constant. What is the relationship between the rate of change of the radius with respect to time and the rate of change of the mass with respect to time?

48. A spherical snowball is melting. Its radius is decreasing at 0.2 cm per hour when the radius is 15 cm. How fast is the volume decreasing at that time? How fast is the surface area decreasing at that time?

49. A function $f(t)$ is continuous and differentiable, and has values given in the table below. The values in the table are representative of the properties of the function.

t	1.0	1.2	1.4	1.6	1.8
$f(t)$	0.1	0.2	0.4	0.7	1.1

a) Find upper and lower estimates for $\int_1^{1.8} f(t)dt$ using $n = 4$.

b) Find $\int_{1.2}^{1.6} f'(t)dt$.

50. A function $g(t)$ is positive and decreasing everywhere. Arrange the following numbers from smallest (1) to largest (3).

_____ $\sum_{k=1}^{10} g(t_k)\Delta t$ _____ $\sum_{k=0}^{9} g(t_k)\Delta t$ _____ $\lim_{n\to\infty} \sum_{k=1}^{n} g(t_k)\Delta t$

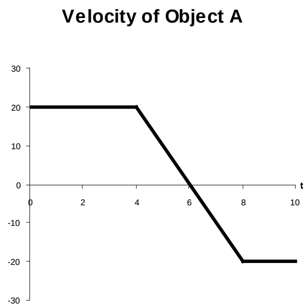
51. Several objects are moving in a straight line from time $t = 0$ to time $t = 10$ seconds. The following are graphs of the velocities of these objects (in cm/sec).

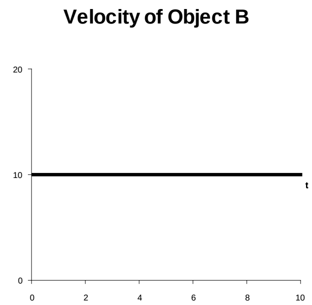
a) Which object(s) is farthest from the original position at the end of 10 seconds?

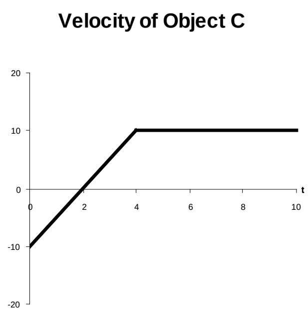
b) Which object(s) is closest to its original position at the end of 10 seconds?

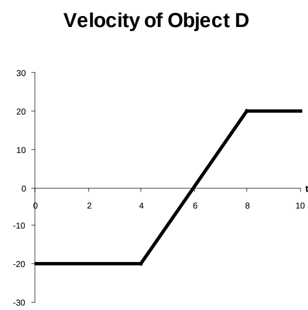
c) Which object(s) has traveled the greatest total distance during these 10 seconds?

d) Which object(s) has traveled the least distance during these 10 seconds?



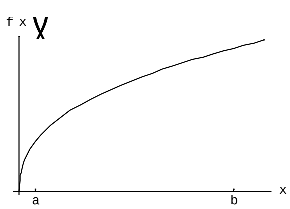




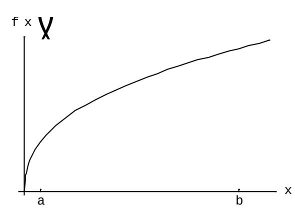


52. Illustrate the following on the graph of $f(x)$ given below. Assume $F'(x) = f(x)$.

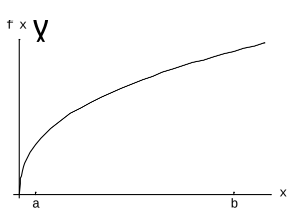
a) $f(b) - f(a)$



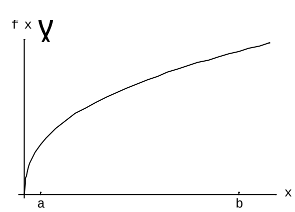
b) $\dfrac{f(b) - f(a)}{b - a}$



c) $F(b) - F(a)$



d) $\dfrac{F(b) - F(a)}{b - a}$



53. Let b be a positive constant. Evaluate the following:

a) $\int (bx^2+1)\,dx$ b) $\int \frac{b+x^2}{x}\,dx$ c) $\int \frac{x}{b+x^2}\,dx$ d) $\int \frac{dx}{1+(bx)^2}$

54. Find the exact areas of the regions. Include a sketch of the regions.

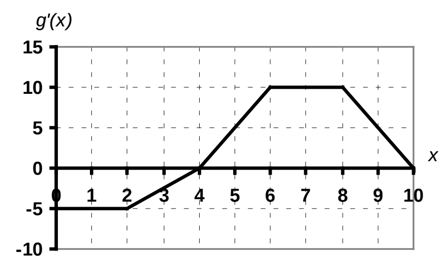
a) The region bounded between $y = x(4-x)$ and the x-axis.

b) The region bounded between $y = x+2$ and $y = x^2-3x+2$.

55. It is predicted that the population of a particular city will grow at the rate of $p(t) = 3\sqrt{t}+2$ (measured in hundreds of people per year). How many people will be added to the city in the first four years according to this model?

56. At time $t = 0$ water is pumped into a tank at a constant rate of 75 gallons per hour. After 2 hours, the rate decreases until the flow of water is zero according to $r(t) = -3(t-2)^2+75$, gallons per hour. Find the total gallons of water pumped into the tank.

57. Use the graph of $g'(x)$ given at the right to sketch a graph of $g(x)$ so that $g(0) = 3$.



58. A car going 80 $ft/\sec$ brakes to a stop in five seconds. Assume the deceleration is constant.

a) Find an equation for $v(t)$, the velocity function. Sketch the graph of $v(t)$.

b) Find the total distance traveled from the time the brakes were applied until the car came to a stop. Illustrate this quantity on the graph of $v(t)$ in part a).

c) Find an equation for $s(t)$, the position function. Sketch the graph of $s(t)$.

59. Consider the function $F(x) = \int_0^x e^{-t^2}\,dt$.

a) Find $F(0)$ b) Find $F'(x)$ c) Is $F(x)$ increasing or decreasing for $x \geq 0$?

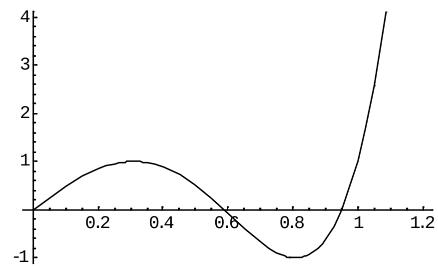
d) Is $F(x)$ concave up or concave down for $x \geq 0$?

60. The average value of f from a to b is defined as $\frac{1}{b-a}\int_a^b f(x)dx$. Find the average value of $f(x) = \frac{3}{\cos^2 x}$ over the interval $0 \leq x \leq \frac{\pi}{4}$.

61. According to a book of mathematical tables, $\int_0^{\pi} \ln(5+4\cos x)dx = 2\pi \ln 2$.

a) Find $\int_{-\pi}^{\pi} \ln(5+4\cos x)dx$ b) Find $\int_0^{\pi/2} \ln(5+4\cos(2x))dx$

62. Use the graph of $f(x)$ below to answer the following. Circle True or False.



a) $\int_0^{0.1} f(x)dx \le \int_0^{0.2} f(x)dx$ True False

b) $\int_0^{0.4} f(x)dx \le \int_0^{0.5} f(x)dx$ True False

c) $\int_0^{0.1} f(x)dx \le \int_0^{0.1} \left(f(x)\right)^2 dx$ True False

d) $\int_0^{1} f(x)dx \ge 0$ True False

e) $\int_0^{1} |f(x)|dx \ge 1$ True False

63. Assume all functions below are continuous everywhere.

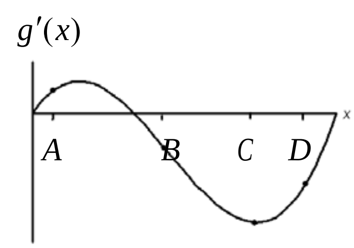
a) If $\int_2^5 6f(x)dx = 17$, find $\int_2^5 f(x)dx$.

b) If $g(x)$ is an odd function and $\int_{-2}^{3} g(x)dx = 20$, find $\int_2^3 g(x)dx$.

c) If $h(x)$ is an even function and $\int_{-2}^{2} (h(x)-3)\,dx = 25$, find $\int_0^2 h(x)dx$.

d) If $\int_a^b f(t)dt = M$, find $\int_{a+5}^{b+5} f(t-5)dt$.

64. Use the graph of $g'(x)$ at the right to determine which sign is appropriate:



a) $g(C)$ < = > $g(D)$

b) $g'(B)$ < = > $g'(C)$

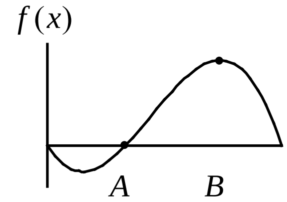
c) $g''(A)$ < = > $g''(B)$

65. Let $F(t) = e^{2t}(t+5)$

a) Find $F'(t)$.

b) Find the exact value of $\int_{\ln(3)}^{2} e^{2t}(2t+11)dt$

66. Let $g(x)=\int_0^x f(t)dt$. In each case explain what graphical feature of $f(x)$ you used to determine the answer.

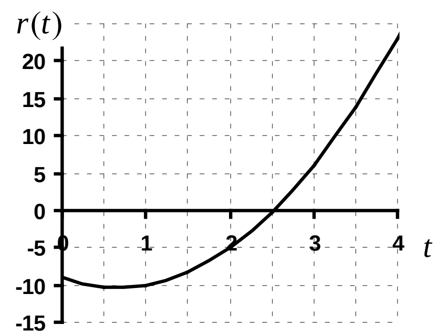


a) What is the sign of $g(B)$?

b) What is the sign of $g'\left(\frac{A}{2}\right)$?

c) What is the sign of $g''(A)$?

67. The graph at the right shows the rate, $r(t)$, in hundreds of algae per hour, at which a population of algae is growing, where t is in hours.



a) Estimate the average value of the rate over the first 2.5 hours.
b) Estimate the total change in the population over the first 3 hours.

68. Find the value of K so that the total area bounded by $f(x)=K\sqrt{x}$ and the x-axis over the interval $[0,9]$ is 7.

69. Evaluate the following:

a) $\int \sin(3\theta)\cos^4(3\theta)d\theta$ b) $\int \frac{1}{5\tan(4v)}dv$ c) $\int_0^1 te^{-t^2}dt$

70. Suppose $\int_1^3 g(t)dt=12$.

a) Find $\int_5^{15} g\left(\frac{x}{5}\right)dx$ b) Find $\int_{-1/3}^{1/3} g(2-3t)\,dt$

71. Find the solution to the initial value problems.

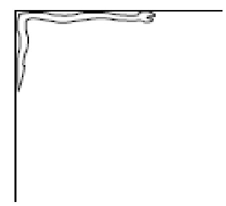
a) $\frac{ds}{dt}=3\sqrt{t}+5, \quad s(2)=10$ b) $\frac{d\theta}{dx}=\frac{1}{x^2+1}, \quad \theta(1)=\pi$

72. A metal cylindrical can with radius 4 inches and height 6 inches is coated with a uniform layer of ice on its side (but not the top or bottom). If the ice melts at 5 cubic inches per minute, how fast is the thickness of the ice decreasing when it is 1 inch thick?

73. A company has produced a new model of skateboard that is marketed toward inexperienced skateboarders who prefer not to spend a lot of money on their first skateboard. Let $D(p)$ represent the number of people who are willing to buy this type of skateboard when the price is p dollars. For each of the following expressions, give the units, the sign, and a practical interpretation.

a) $D'(30)$ b) $\lim_{p\to 0^+} D(p)$ c) $\int_{32}^{40} D'(p)dp$

74. A 6 foot long snake is crawling along the corner of a room. It is moving at a constant 2/3 foot per second while staying tucked snugly against the corner of the room. At the moment that the snake's head is 4 feet from the corner of the room, how fast is the distance between the head and the tail changing?

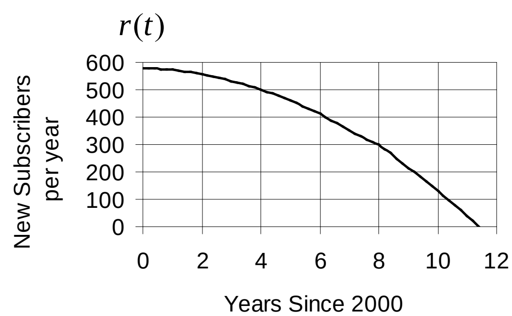


75. For each of the following descriptions, sketch a graph of a function satisfying the given conditions.

a) A function $f(x)$, whose slope is increasing for all x, such that $\lim_{x\to\infty} f(x) = 1$.

b) A function $g(x)$ that has a global minimum and a local maximum but not global maximum.

76. An internet provider modeled their rate of new subscribers to an old internet service package by $r(t)$ as shown at the right.



a) Use the Left Hand Sum with $n = 2$ to estimate the total number of new subscribers between 2002 and 2010. Include an illustration of this estimate.

b) Rank the following from smallest (1) to largest (3):

_____ $\int_0^5 r(t)dt$

_____ Left Hand Sum with $n = 20$.

_____ Right hand Sum with $n = 20$.

c) What is the sign of $r'(3)$?

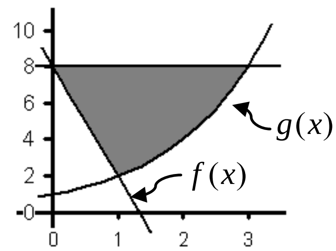
d) Estimate $\int_4^8 r'(t)dt$.

77. Find the local linearization of $2\pi\sqrt{\frac{x}{g}}$ near $x = 100$.

78. Determine the inflection points of $h(t)$ if $h''(t) = -2t(t+3)^2(2t-5)e^t$.

79. Consider the function $f(x) = \dfrac{1}{x+4}$. Find a number c in the interval $(0,2)$ so that the instantaneous rate of change of f is identical to the average rate of change of f over $[0,2]$. Why are we guaranteed to find such a c?

80. Set up the integral(s) needed to find the area of the shaded region.



81. A. Find $\lim_{x\to 0}\int_0^x \sin(2t)dt$ and $\lim_{x\to 0}\int_0^x \tan t dt$.

B. Find $\lim_{x\to 0}\dfrac{\int_0^x \sin(2t)dt}{\int_0^x \tan(t)dt}$. Does L'Hopital's Rule apply?