MATH 129
FINAL EXAM REVIEW PACKET
ANSWERS

1. $\int_0^3 \left(10\pi \sin\left(\frac{\pi}{3}t\right) + 30\right) dt = 150$ people.

2. $\int_1^2 f(5-2x)dx = \frac{7}{2}$. Let $u = 5-2x$ and change the endpoints.

3. a) $\int \frac{\ln\left(z^2+1\right)}{z^2} dz = -\frac{1}{z}\ln\left(z^2+1\right) + 2\arctan z + c$. Let $u = \ln(z^2+1)$ and $v' = \frac{1}{z^2}$.

b) $\int_0^1 x \cdot g''(x)dx = 3$. Let $u = x$ and $v' = g''$ for the first integration by parts.

4. a) $\int \cos^2(3\theta+2)d\theta = \frac{1}{6}\cos(3\theta+2)\sin(3\theta+2) + \frac{1}{6}(3\theta+2) + c$. Let $u = 3\theta+2$ before using table formula # 18. If you use another approach, your answer will look different.

b) $\int \frac{2}{4t^2-9} dt = \frac{1}{6}\left(\ln\left|2t-3\right| - \ln\left|2t+3\right|\right) + c$. Let $u = 2t$ and factor the denominator before using table formula # 26. If you use another approach, your answer will look different.

c) $\int \frac{dy}{\sqrt{y^2+8y+15}} = \ln\left|(y+4) + \sqrt{y^2+8y+15}\right| + c$. Complete the square before using table formula # 29.

5. a) $\int_0^1 e^{-t^2} dt \approx \frac{1}{2}e^{-1/16} + \frac{1}{2}e^{-9/16} \approx 0.75459794$

b) Not clear because $f(t) = e^{-t^2}$ changes concavity on the given interval. When $f(t)$ is concave down, the midpoint rule provides an overestimate.

6. a) The integral converges. $\int_0^\infty \frac{1}{x^2+4} dx = \frac{\pi}{4}$. Use table formula # 24.

b) The integral converges. $\int_1^\infty \frac{1}{2^x} dx = \frac{1}{2\ln 2}$.

c) The integral diverges. $\int_0^1 \frac{e^x}{\left(e^x-1\right)^2} dx = \infty$. Let $u = e^x - 1$.

d) The integral converges. $\int_{\pi/6}^{\pi/2} \frac{\sin x}{\sqrt{\cos x}} dx = 2\sqrt{\frac{\sqrt{3}}{2}}$. Let $u = \cos x$

7. $\int_m^{\infty} e^{-\left(\frac{x-m}{s}\right)^2} dx = \frac{s\sqrt{\pi}}{2}$. Let $u = \frac{x-m}{s}$ and change the endpoints.

8. a) The integral converges. Rewrite as $\int_0^{\infty} a \cdot f(x)dx = a\int_0^{\infty} f(x)dx$.

b) The integral converges. Let $u = ax$.

c) The integral diverges. Rewrite as $\int_0^{\infty} \left(a + f(x)\right) dx = \int_0^{\infty} adx + \int_0^{\infty} f(x)dx$.

d) The integral converges. Let $u = a + x$.

9. a) The integral converges. $\int_2^{\infty} \frac{d\theta}{\sqrt{\theta^3 + 2}} < \sqrt{2}$. Use the comparison $\frac{1}{\sqrt{\theta^3 + 2}} < \frac{1}{\theta^{3/2}}$.

b) The integral converges. $\int_1^{\infty} \frac{1 + \sin^2 x}{(x+3)^3} dx < \frac{1}{16}$. Use the comparison $\frac{1 + \sin^2 x}{(x+3)^3} < \frac{2}{(x+3)^3}$.

c) The integral diverges. $\int_1^{\infty} \frac{(1 + \sin^2 x)x^2}{x^3 + 3} dx$. Use the comparison $\frac{(1 + \sin^2 x)x^2}{x^3 + 3} > \frac{x^2}{x^3 + 3}$.

10. a) $\int_0^1 3\sqrt{x}dx + \int_1^2 (6 - 3x)dx$. b) $\int_0^3 \left(\frac{6-y}{3} - \frac{y^2}{9}\right) dy$.

11. a) $\pi \cdot 12^2 \cdot 6 - \int_{-3}^{3} \pi(x^2 + 3)^2 dx = \frac{3024}{5}\pi$. b) $\int_{-3}^{3} \pi\left(12 - (x^2 + 3)\right)^2 dx = \frac{1296\pi}{5}$.

12. $\int_0^8 \pi\left(y^{1/3}\right)^2 dy = \frac{96}{5}\pi$.

13. $\int_0^{\pi} (\sin x)^2 dx = \frac{\pi}{2}$.

14. Left hand rule:

$$10\cdot\pi\left(\frac{26}{2\pi}\right)^2+10\cdot\pi\left(\frac{22}{2\pi}\right)^2+10\cdot\pi\left(\frac{18}{2\pi}\right)^2+10\cdot\pi\left(\frac{12}{2\pi}\right)^2+10\cdot\pi\left(\frac{6}{2\pi}\right)^2=\frac{4160}{\pi}\text{ cubic inches.}$$

Right hand rule:

$$10\cdot\pi\left(\frac{22}{2\pi}\right)^2+10\cdot\pi\left(\frac{18}{2\pi}\right)^2+10\cdot\pi\left(\frac{12}{2\pi}\right)^2+10\cdot\pi\left(\frac{6}{2\pi}\right)^2+10\cdot\pi\left(\frac{2}{2\pi}\right)^2=\frac{2480}{\pi}\text{ cubic inches.}$$

Trapezoid rule: $\frac{3320}{\pi}$ cubic inches. Use the average of the left and right hand rules.

15. $\int_0^5 \left(2+0.5\cosh x\right)dx = 10+0.5\sinh 5$ grams.

16. $\int_0^{25}\left(-\frac{8}{5}h+90\right)\pi(10)^2\,dh = 175{,}000\pi$ pounds.

17. a) $\int_0^{15} 62.4(h)\pi\left(\frac{8}{15}h\right)^2 dh$ b) $\int_0^{15} 62.4(h+3)\pi\left(\frac{8}{15}h\right)^2 dh$

c) $\int_0^{10} 62.4(h)\pi\left(\frac{8}{15}h\right)^2 dh$ d) $\int_3^{15} 62.4(h+3)\pi\left(\frac{8}{15}h\right)^2 dh$

18. $500\cdot 45+\int_0^{40} 3(40-x)dx = 24{,}900$ foot-pounds.

19. $\int_0^{10}\left(\frac{240}{0.5\cdot 24\cdot 10}\right)(10-h)\left(\frac{12}{5}h\right)dh = 800$ foot-pounds.

20. a) $a_n = \frac{(-1)^n 2n}{(n+2)^2}$ b) Converges to $\frac{-3}{5}$.

21. a) $\frac{(3/64)\left(1-(1/4)^8\right)}{1-1/4} = \frac{65535}{1048576}$ b) $\frac{3/4}{1-1/4} = 1$

22. $P_n = \frac{(0.05)(200)\left(1-0.05^{n-1}\right)}{1-0.05}$ $Q_n = \frac{(200)\left(1-0.05^n\right)}{1-0.05}$ $n = 1, 2, 3, \cdots$

23. a) The series converges. $\int_2^\infty \frac{1}{x(\ln x)^2}dx = \frac{1}{\ln 2}$.

b) The series diverges. $\int_1^\infty \frac{3x^2+2x}{\sqrt{x^3+x^2+1}}\,dx = \infty$.

24. a) The series diverges. $\lim_{n\to\infty} \frac{e(n)^2}{2(n+1)^2} = \frac{e}{2} > 1$.

b) The series converges. $\lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = \frac{1}{4} < 1$

25. a) The radius of convergence is $R = 3$. The interval of convergence is $(-7,-1)$.
b) The radius of convergence is $R = \infty$. The interval of convergence is $(-\infty,\infty)$.
c) The radius of convergence is $R = 0$. The series only converges for $x = 1$.

26. a) True b) True c) Impossible to determine.

27. $P_2(x) = 3 + 2(x-2) + \frac{1}{3}(x-2)^2$

28. $c_0 < 0$, $c_1 > 0$, $c_2 > 0$.

29. a) $\frac{3}{7}$ $\lim_{x\to 2} \frac{f(x)}{h(x)} = \lim_{x\to 2}\left(\frac{\frac{3}{2}(x-2)^2+\cdots}{\frac{7}{2}(x-2)^2+\cdots}\right) = \frac{3}{7}$

b) 0 $\lim_{x\to 2} \frac{f(x)}{g(x)} = \lim_{x\to 2}\left(\frac{\frac{3}{2}(x-2)^2+\cdots}{22(x-2)+\cdots}\right) = \frac{0}{22}$

30. a) $f(3) = -1$. b) $f'(3) = \frac{1}{2}$. c) $f''(3) = -\frac{1}{6}$.

d) $\sum_{k=0}^{\infty} (-1)^{k+1} \frac{k!}{(2k)!} 3^k (x-1)^k$ Substitute $3x$ into the series, then simplify.

31. $\frac{1}{12}$ Integrate term by term. The result is recognizable as the series for $\frac{x}{1-x}$.

32. a) *ii* b) *iv* c) *i* d) *iii*

33. a) $\sin 1$ b) $\ln(1.5)$

34. a) $\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots$

b) $-\frac{1}{11}$ Use a) to find the series for $\frac{\sin x}{x}$, consider the term containing x^{11}.

35. a) $x\ln(1+2x) = 2x^2 - \frac{4x^3}{2} + \frac{8x^4}{3} - \frac{16x^5}{4} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} x^{n+2}}{n+1}$

b) $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

36. $\frac{1}{(a+r)^2} = \frac{1}{a^2}\left(1+\frac{r}{a}\right)^{-2} = \frac{1}{a^2}\left(1-2\left(\frac{r}{a}\right)+3\left(\frac{r}{a}\right)^2-4\left(\frac{r}{a}\right)^3+\cdots\right)$

37. a) *ii* b) *iii* c) *i* d) *iv*

38.

x	y	Δx	Δy
0.0	3.0	0.1	$\left(10(0)^2+3\right)(0.1)=0.3$
0.1	3.3	0.1	$\left(10(0.1)^2+3.3\right)(0.1)=0.34$
0.2	3.64	0.1	$\left(10(0.2)^2+3.64\right)(0.1)=0.404$
0.3	4.044	0.1	$\left(10(0.3)^2+4.044\right)(0.1)=0.4944$
0.4	4.5384		

39. $y(x) = 2\tan\left(x^2 + \frac{\pi}{4}\right)$

40. a) $\dfrac{dQ}{dt} = -\alpha Q$ where $\alpha > 0$. b) $Q(t) = Ae^{-\alpha t}$. c) $t = \dfrac{7\ln(90)}{\ln(9/5)} \approx 53.59$ hours.

41. a) *ii* b) *iii* c) *i* d) *iv* e) *vi* f) *v*

42. a) *i* b) *ii* c) *iii* d) *iv*

43. $\dfrac{dL}{dt} = 4 - 0.6L$, $L(t) = \dfrac{Ae^{-0.6t} + 4}{0.6}$

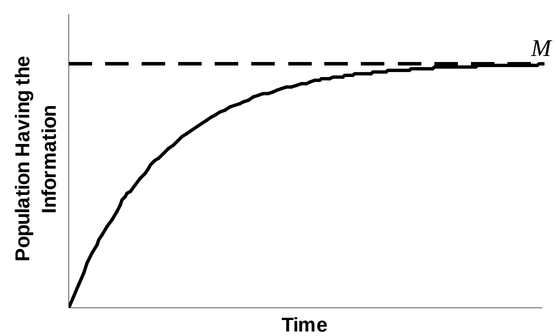
Equilibrium solution is $L = \dfrac{20}{3}$ grams per square centimeter.

44. a) $r(t) = \dfrac{1}{15}t + 60$ b) $\dfrac{dH}{dt} = k\left(H - \left(\dfrac{1}{15}t + 60\right)\right)$ $H(0) = 180$ where $k < 0$

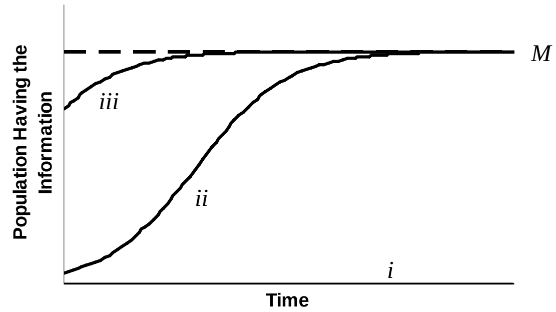
45. $\dfrac{dA}{dt} = 0.06\sqrt{A}$, $A(t) = (0.03t + c)^2$

46. a) $\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i = e^{-(\pi/6)i}$ b) $2e^{\frac{-\pi}{4}i} = \sqrt{2} - i\sqrt{2}$ c) $e^{(3+4i)t} = e^{3t}\cos(4t) + i \cdot e^{3t}\sin(4t)$

47. a) $\dfrac{dP}{dt} = k(M - P)$ where $k > 0$.



b) $\frac{dP}{dt} = kP(M - P)$ where $k > 0$.



48. $\frac{-y\sqrt{5-y^2}}{2} + \frac{5}{2}\arcsin\left(\frac{y}{\sqrt{5}}\right) + c$

49. $-\frac{2}{t} + 3\ln|t| - 2\ln|t+1| + c$