MATH 223
FINAL EXAM STUDY GUIDE
(2017-2018)

The following questions can be used as a review for Math 223. These questions are not actual samples of questions that will appear on the final exam, but they will provide additional practice for the material that will be covered on the final exam. When solving these problems keep the following in mind: Full credit for correct answers will only be awarded if all work is shown. Exact values must be given unless an approximation is required. Credit will not be given for an approximation when an exact value can be found by techniques covered in the course. The answers, along with comments, are posted as a separate file on http://math.arizona.edu/~calc.

1. A sonic boom carpet is a region on the ground where the sonic boom is heard directly from the airplane and not as a reflection. The width of the carpet, *W*, can be expressed as a function of the air temperature on the ground directly below the airplane, *t*, and the vertical temperature gradient at the airplane's altitude, *d*. Suppose $W(t,d) = k\sqrt{\frac{t}{d}}$ for some positive constant *k*.

(a) If *d* is fixed, is the width of the carpet an increasing or decreasing function of *t*.
(b) If *t* is fixed, is the width of the carpet an increasing or decreasing function of *d*.

2. Describe the following sets of points in words, write an equation, and sketch a graph:
(a) The set of points whose distance from the line *L* is 5. The line *L* is the intersection of the plane $y = 3$ and the *xy*-plane.
(b) The set of points whose distance from the *yz*-plane is three.
(c) The set of points whose distance from the *z*-axis and the *xy*-plane are equal.

3. By setting one variable constant, find a plane that intersects $y\cos^2 x + z^2 = 3$ in a:
(a) parabola (b) sinusoidal curve (c) line(s)

4. Consider the function $f(x,y) = y - x^2$.
(a) Plot the level curves of the function for $z = -2,-1,0,1,2$.
(b) Imagine the surface whose height above any point (x,y) is given by $f(x,y)$. Suppose you are standing on the surface at the point where $x = 1, y = 2$.

(i) What is your height?
(ii) If you start to move on the surface parallel to the *y*-axis in the direction of increasing *y*, does your height increase or decrease?
(iii) Does your height increase or decrease if you start to move on the surface parallel to the *x*-axis in the direction of increasing *x*?
(iv) Are you ascending or descending when you go from (1,2) in the direction making an angle of $3\pi/4$ with the x-axis?
(v) Suppose you start to move in the direction you found in part (iv) at a rate of 3mph, at what rate is your height changing with respect to time.

5a. Describe the level curves(contour lines) for each of the following functions:

(i) $f(x,y)=\dfrac{1-x^2}{y^2}$ (ii) $g(x,y)=\sqrt{(x^2+y^2)}$ (iii) $h(x,y)=\cos y$

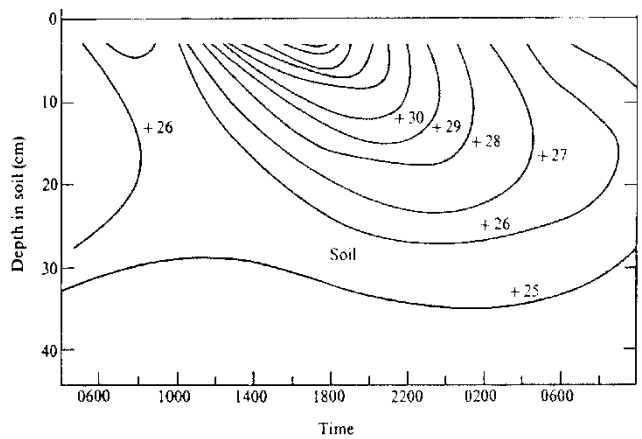
5b. Describe the level surfaces for each of the following functions:

(i) $f(x,y,z)=x-y^2-z^2$ (ii) $g(x,y,z)=e^{1-x^2-y^2-z^2}$ (iii). $f(x,y,z)=\ln(x^2+z^2)$

6. The figure at the right shows the level curves of the temperature *T* in degrees Celsius as a function of *t* hours and depth *h* in centimeters beneath the surface of the ground from midnight ($t=0$) one day to midnight ($t=24$) the next.

(a) Sketch a graph of the temperature as a function of time at 20 centimeters.

(b) Sketch a graph of the temperature as a function of the depth at noon.



from S. J. Williamson, *Fundamentals of Air Pollution*, (Reading: Addison-Wesley, 1973)

7. Given the table of some values of a linear function, complete the table and find a formula for the function.

$y \backslash x$	2.5	3.0	3.5
-1.0	6		8
1.0		1	2
3.0	-6		

8. Consider the planes:

I. $3x-5y-z=2$ II. $5x=y+3$ III. $5x+3y=2$

IV. $3x+5y=2$ V. $3x+5y+z=2$ VI. $y+1=0$

List all of the planes which:

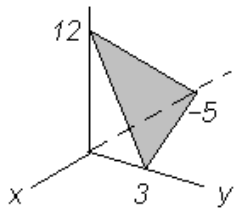
(a) Are parallel to the *z*-axis.

(b) Are parallel to $3x=5y+z+7$.

(c) Contain the point $(1,-1,6)$.

(d) Are normal to $\left(2\bar{i}+3\bar{k}\right)\times\left(3\bar{i}-\bar{k}\right)$.

9. A portion of the graph of a linear function is shown.
(a) Find an equation for the linear function.
(b) Find a vector perpendicular to the plane.
(c) Find the area of the shaded triangular region.



10. Match each of the following functions (a) – (f), given by a formula, to the corresponding tables, graphs, and/or contour diagrams (i) – (ix). There may be more than one representation or no representations for a formula.

(a) $f(x,y) = x^2 - y^2$ (b) $f(x,y) = 6 - 2x + 3y$ (c) $f(x,y) = \sqrt{1 - x^2 - y^2}$

(d) $f(x,y) = \dfrac{1}{1 + x^2 + y^2}$ (e) $f(x,y) = 6 - 2x - 3y$ (f) $f(x,y) = \sqrt{x^2 + y^2}$

(i) Table 1

y \ x	-2	-1	0	1	2
2	2.828	2.236	2.000	2.236	2.828
1	2.236	1.414	1.000	1.414	2.236
0	2.000	1.000	0.000	1.000	2.000
-1	2.236	1.414	1.000	1.414	2.236
-2	2.828	2.236	2.000	2.236	2.828

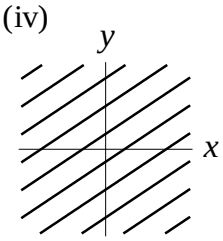
(ii) Table 2

y \ x	-2	-1	0	1	2
2	0.111	0.167	0.200	0.167	0.111
1	0.167	0.333	0.500	0.333	0.167
0	0.200	0.500	1.000	0.500	0.200
-1	0.167	0.333	0.500	0.333	0.167
-2	0.111	0.167	0.200	0.167	0.111

(iii) Table 3

y \ x	-2	-1	0	1	2
2	0.00	-3.00	-4.00	-3.00	0.00
1	3.00	0.00	-1.00	0.00	3.00
0	4.00	1.00	0.00	1.00	4.00
-1	3.00	0.00	-1.00	0.00	3.00
-2	0.00	-3.00	-4.00	-3.00	0.00

(iv)



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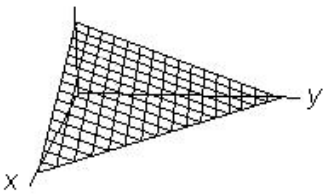

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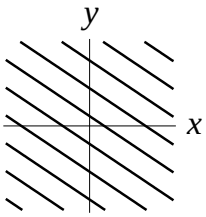

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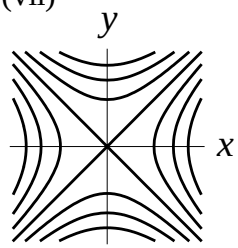

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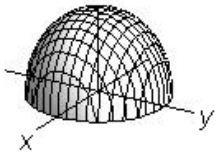

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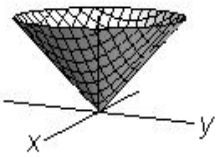


11. Let $\vec{v} = 3\vec{i} + 2\vec{j} - 2\vec{k}$ and $\vec{w} = 4\vec{i} - 3\vec{j} + \vec{k}$. Find each of the following:
(a) A vector of length 5 parallel to $\vec{w}$.
(b) A vector perpendicular to $\vec{v}$ but not perpendicular to $\vec{w}$.
(c) $\vec{v} \times \vec{w}$
(d) The angle between $\vec{v}$ and $\vec{w}$.
(e) The component of $\vec{v}$ in the direction of $\vec{w}$.
(f) A vector perpendicular(orthogonal) to both $\vec{v}$ and $\vec{w}$.

12. Consider the vectors $\vec{u} = 2\vec{i} - \vec{j} + 3\vec{k}$ and $\vec{v} = -2a\vec{i} + a\vec{j} - \vec{k}$.
(a) For what value(s) of a are $\vec{u}$ and $\vec{v}$ perpendicular?
(b) For what value(s) of a are $\vec{u}$ and $\vec{v}$ parallel?
(c) Find an equation of the plane normal to $\vec{u}$ and containing the point $(1,-2,3)$.
(d) Find a parameterization for the line parallel to $\vec{u}$ and containing the point $(1,-2,3)$.

13a. Find a vector parallel to the line of intersection of the two planes $x + 2y - 3z = 7$ and $3x = y - z$
(b)Find parametric equations for the line in part(a)

14. Find the following:
(a) $\frac{\partial}{\partial x}\left(\ln\left(x^3 y + 3\right) - \arctan\left(x^2 + y^2\right)\right)$ (b) f_H if $f(H,T) = \frac{2H+T}{(5-H)^3}$ (c) $\frac{\partial^2}{\partial x \partial y}\left(\frac{x}{y} + \frac{y}{x}\right)$

15. Find an equation for the tangent plane to:
(a) $f(x,y) = ye^{xy}$ at $(x,y) = (1,2)$ (b) $(x-1)^2 + 4(y-2)^2 + (z-3)^2 = 17$ at $(3,3,6)$

16. A ball is thrown from ground level with initial speed v (m/sec) and at an angle of α with the horizontal. It hits the ground at a distance $s(v,\alpha) = \frac{v^2 \sin(2\alpha)}{g}$ where $g = 9.8\,\text{m/sec}^2$.
(a) Find the differential ds.
(b) What does the sign of $s_\alpha(20, \pi/3)$ tell you?
(c) Use the linearization of s about $(20, \pi/3)$ to estimate the change in α that is needed to get approximately the same distance if the initial speed changes to 19 m/sec.

17. The depth of a lake at the point (x,y) is given by $h(x,y) = 2x^2 + 3y^2$ feet. A boat is at (-1,2).
(a) If the boat sails in the direction of the point $(3,3)$, is the water getting deeper or shallower?
(b) In which direction should the boat sail for the depth to remain constant? Give your answer as a vector.
(c) If the boat moves on the curve $\vec{r}(t) = \left(t^2 - 1\right)\vec{i} + \left(t + 2\right)\vec{j}$ for t in minutes, at what rate is the depth changing when $t = 2$?

18. Calculate the following:

(a) $grad\left(\dfrac{yz^2}{1+x^2}\right)$

(b) $curl\left(\left(x^2+y^2+z^2\right)\vec{i}-\left(y+z\right)\vec{j}+\left(xz\right)\vec{k}\right)$

(c) $div\left(\left(\cos^2 x\right)\vec{i}+\left(x\sec y\right)\vec{j}+\left(e^{3z}\right)\vec{k}\right)$

(d) The maximum rate of change of $f(x,y,z)=\tan x+\sqrt[3]{z}$ at $(\pi/4,3,1)$.

(e) The directional derivative of $f(x,y,z)=e^{xy}+z$ at the point (1, 1, 0) in the direction of $2\vec{i}-3\vec{j}+4\vec{k}$.

(f) The potential function for $\vec{G}=y\vec{i}+x\vec{j}+e^z\cos(e^z)\vec{k}$.

19. Which of the following vector fields are conservative? If they are conservative find the corresponding potential function.

(a) $\vec{F}(x,y,z)=ye^{xy}\vec{i}+xe^{xy}\vec{j}+(\cos z)\vec{k}$

(b) $\vec{F}(x,y,z)=x^{-1}\vec{i}+y^{-1}\vec{j}+(z^{-1})\vec{k}$

(c) $\vec{F}(x,y)=(x+y)\vec{i}+(x-y)\vec{j}$

(d) $\vec{F}(x,y)=(2xy)\vec{i}+(-x^2+8y^3)\vec{j}$

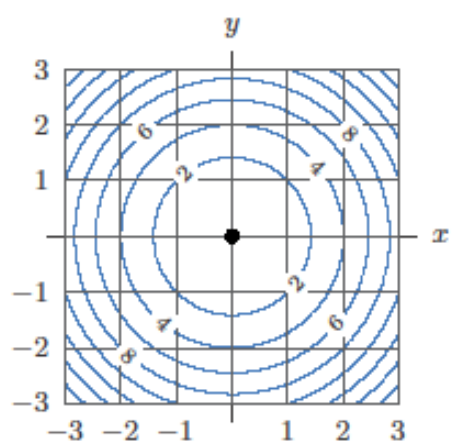
20. The contour plot for $f(x,y)$ is shown at the right. Determine if each quantity is positive, negative, or zero.

(a) $f_x(1,1)$ (b) $f_x(-1,1)$

(c) $f_y(-2,-2)$ (d) $f_{xy}(-2,-2)$

(e) $f_{yy}(1,1)$ (f) $f_{\vec{u}}(1,-1)$ where

$$\vec{u}=\frac{1}{2}\vec{i}-\frac{\sqrt{3}}{2}\vec{j}$$



21. Let $w(x,y)=3x\cos(\pi y)$.

(a) Find $\left.\dfrac{\partial w}{\partial u}\right|_{(1,\,1/2)}$ and $\left.\dfrac{\partial w}{\partial v}\right|_{(1,\,1/2)}$ if $x=u^2+v^2$ and $y=\dfrac{v}{u}$.

(b) Find $\left.\dfrac{dw}{dt}\right|_{t=1}$ if $x=e^{-t}$ and $y=\ln t$.

22. A steel bar with circular cross section of radius 6 cm and length 50 cm is being heated. The radius and the length increases by .005 cm for each 1 degree centigrade rise in temperature. What is the rate of change in the volume of the steel bar?

23a. Find and classify all of the critical points for $f(x,y) = 2x^3 - 3x^2 - 12x + y^3 + 3y^2 - 9y$.

(b) Find the quadratic Taylor polynomial of $f(x,y)$ near the critical point, where the point is a local minimum of $f(x,y)$.

24. Let $f(x,y) = Kx^2 + y^2 - 4xy$.
(a) Verify that the point (0, 0) is a critical point.
(b) Determine the values of *K*, if any, for which (0, 0) can be classified as the following.

(i) a saddle point (ii) a local minimum (iii) a local maximum

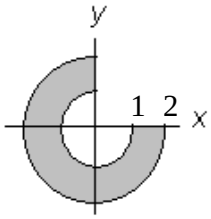
25. Find an equation for each surface:
(a) $x^2 + y^2 = 8$ in cylindrical coordinates
(b) $y = x$ in cylindrical coordinates
(c) $z = -\sqrt{x^2 + y^2}$ in spherical coordinates
(d) $z = 10$ in spherical coordinates

26. Determine (without calculation) whether the integrals are positive, negative, or zero. Let *D* be the region inside the unit circle centered at the origin, *T* be the top half of the region, *B* be the bottom half of the region, *L* be the left half of the region, and *R* be the right half of the region.
(a) $\int_T e^{-x} dA$ (b) $\int_B \cos y dA$ (c) $\int_L (x+y) dA$ (d) $\int_R y e^{-y} dA$

27. Evaluate each of the integrals:
(a) $\int_0^3 \int_0^6 \cos(3y)\sin(2x+5) dy dx$
(b) $\int_0^{\pi} \int_0^{\pi} \int_0^5 \rho^2 \sin\phi d\rho d\phi d\theta$

(c) $\int_0^9 \int_{\sqrt{y}}^3 \sqrt{1+x^3} dx dy$
(d) $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} e^{-\left(x^2+y^2+z^2\right)^{3/2}} dz dy dx$

(e) $\int_R x dA$ where *R* is shown below.



(f) $\int_{\pi/4}^{\pi/2} \int_0^{5/\sin\theta} r dr d\theta$ Hint: sketch the region first.

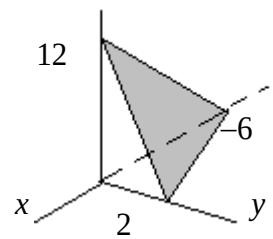
28. Set up integrals needed to find the following:

(a) The volume between the sphere $\rho = 2$ and the cone $z = r$. (Cartesian, cylindrical, and spherical)

(b) The volume between $x = 20 - y^2 - z^2$ and $x = y^2 + z^2 + 2$. (Cartesian and cylindrical)

(c) The volume of the solid in the first octant bounded from above by $x^2 + z^2 = 16$ and $y = 12$. (Cartesian in the order $dxdydz$ and Cylindrical in the order $rdydrd\theta$)

(d) The volume of the tetrahedron under the portion of the plane shown at the right, bounded by the planes $y = 0$, $x = 0$, and $z = 0$. (Cartesian)



29. A pile of dirt is approximately in the shape of an inverted cone of height 4m with base radius of 4m. The density of the dirt is proportional to the distance from the apex of the pile of dirt. Set up an integral for the mass of the dirt pile.

30. Give parametric equations for the following curves:

(a) A circle of radius 3 on the plane $y = 1$ centered at $(2,1,0)$ oriented clockwise when viewed from the origin.

(b) A line perpendicular to $z = 2x - 3y + 7$ and through the point $(1,-2,3)$.

(c) The curve $y = (x+2)^3$ oriented from $(2,64)$ to $(0,8)$.

(d) The intersection of the surfaces $z^2 = x^2 + y^2$ and $z = 6 - x^2 - y^2$.

(e) The ellipse with major diameter $2a$ along the x-axis and minor diameter $2b$ along the y axis, centered at the origin.

31. A child is sliding down a helical slide. Her position at time t seconds after the start is given in feet by $\vec{r} = (3\cos t)\vec{i} + (3\sin t)\vec{j} + (10 - t)\vec{k}$. The ground is the xy-plane.

(a) When is the child 6 feet from the ground?

(b) How fast is the child traveling at 2 seconds?

(c) At time $t = 2\pi$ seconds, the child leaves the slide tangent to the slide at that point. What is the equation of the tangent line?

32. The surface of a hill is represented by $z = 12 - x^2 - 3y^2$, where x and y are measured horizontally. A projectile is launched from the point $(1,1,7)$ and travels in a line perpendicular to the surface at that point.

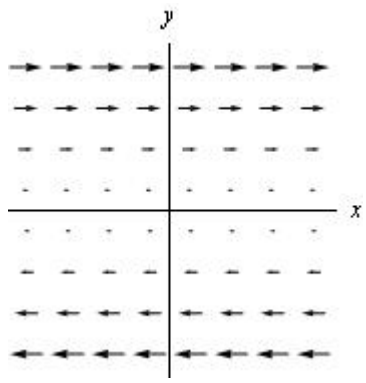
(a) Find parametric equations for the path.

(b) Does the projectile pass through the point $(1,1,8)$?

33. Match the vector field to its sketch.

(a) $x\vec{i} + y\vec{j}$ (b) $x\vec{i} - y\vec{j}$ (c) $y\vec{i} + x\vec{j}$ (d) $y\vec{i}$ (e) $\vec{i} + x\vec{j}$ (f) $x^2\vec{i} + xy\vec{j}$

(i)



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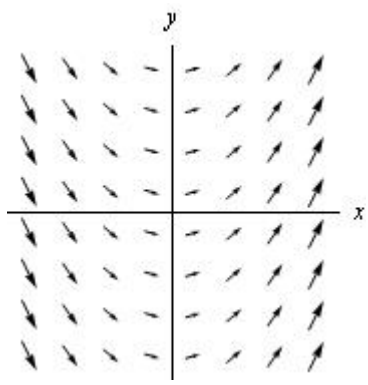

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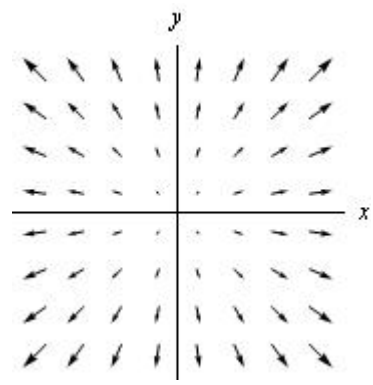

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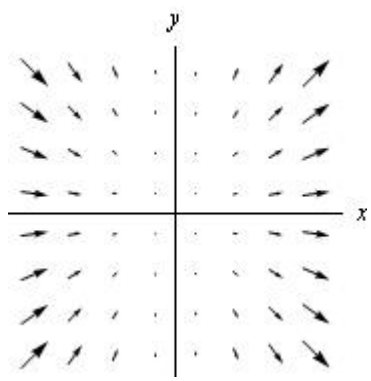

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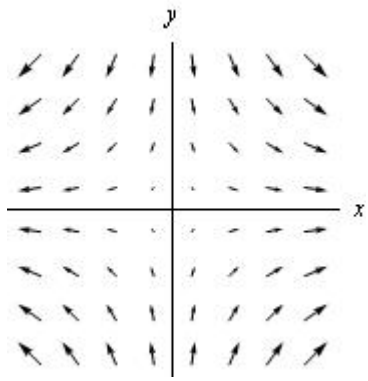

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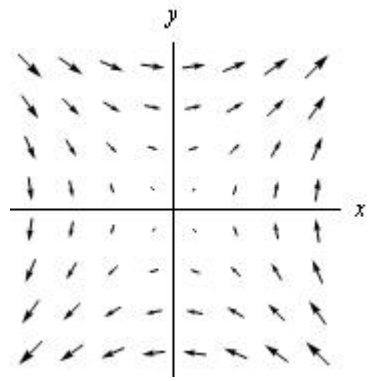
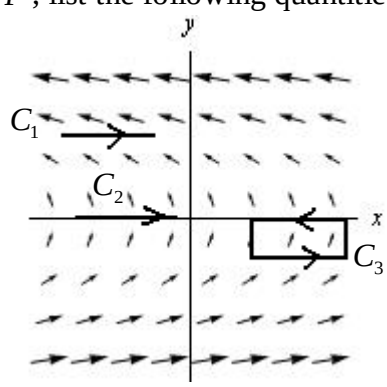


34. Given the plot of the vector field, $\vec{F}$, list the following quantities in increasing order.

(i) $\int_{C_1} \vec{F} \cdot d\vec{r}$

(ii) $\int_{C_2} \vec{F} \cdot d\vec{r}$

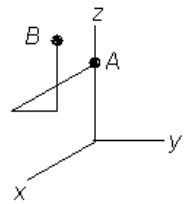
(iii) $\int_{C_3} \vec{F} \cdot d\vec{r}$



35. Evaluate $\int_C \vec{F}\cdot d\vec{r}$:

(a) $\vec{F}=(x+z)\vec{i}+z\vec{j}+y\vec{k}$. C is the line from $(2,4,4)$ to $(1,5,2)$.

(b) $\vec{F}=x^2\vec{i}+z\sin(yz)\vec{j}+y\sin(yz)\vec{k}$. C is the curve from $A(0,0,1)$ to $B(3,1,2)$ as shown below.



(c) $\vec{F}=y\vec{i}-x\vec{j}+z\vec{k}$. C is the circle of radius 3 centered on the z-axis in the plane $z=4$ oriented clockwise when viewed from above.

(d) $\vec{F}=4x^3\vec{i}+(x+y)\vec{j}$. C is the curve $y=\sin(2x)$ from $(0,0)$ to $(\pi/2,0)$.

(e) $\vec{F}=\left(-y^3+\sin(x^2)\right)\vec{i}+\left(x^3-\ln(y^2+1)\right)\vec{j}$. C is the circle of radius 5 centered at $(0,0)$ in the xy-plane oriented counterclockwise.

(f) $\vec{F}=y\vec{i}+x\vec{j}$. C is the parameterized path $\vec{r}(t)=(2(3-t)+\sin(\pi t/3))\vec{i}+((3-t)^2+\log(t^2+1))\vec{j}$, $0\le t\le 3$.

36. Calculate the flux of $\vec{F}$ through the surface, S, given below:

(a) $\vec{F}=3\vec{i}+4\vec{j}+(z-x)\vec{k}$. S is a square of side 2 on the plane $z=x$ oriented upward.

(b) $\vec{F}=-5\vec{i}+z\vec{j}-y\vec{k}$. S is $x=\sqrt{y^2+z^2}$ for $0\le x\le 8$, oriented in the negative x-direction.

(c) $\vec{F}=2x\vec{i}-(z^3-y)\vec{j}+(x^5+7z)\vec{k}$. S is the closed cylinder centered on the y-axis with radius 3, length 5, oriented outward.

(d) $\vec{F}=x\vec{i}+y\vec{j}+z\vec{k}$. S is the part of the surface $z=25-(x^2+y^2)$ above the disk of radius 5 centered at the origin, oriented upward.

37. (a) Evaluate $\int_C grad(x^2yz^3)\cdot d\vec{r}$ where C is the square of side 2 centered at $(1,1)$ in the xy-plane, oriented counterclockwise.

(b) Evaluate $\int_S curl(x^2\vec{i}-(y+z)\vec{j}+xz\vec{k})\cdot d\vec{A}$ where S is the cube of side 4 centered at $(2,1,3)$, oriented outward.

(c) Evaluate $\int_c xdx+zdy-ydz$ where C is the circle of radius a in the yz plane centered at the origin, oriented clockwise when viewed from the positive x-axis.

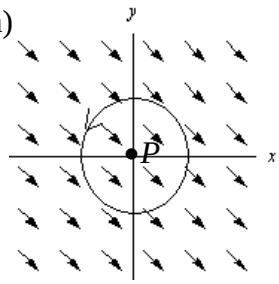
38. Consider the flux of the vector field $\vec{H}=\dfrac{\vec{r}}{\|\vec{r}\|^p}$ for $p\ge 0$ out of the sphere of radius 5 centered at the origin. For what value of p is the flux a maximum? What is that maximum value?

39. Let S_1 be the ellipsoid $4x^2 + y^2 + 4z^2 = 64$ and let S_2 be the sphere $x^2 + y^2 + z^2 = 1$, both oriented outward. Let $\vec{F} = \dfrac{\vec{r}}{\|\vec{r}\|^3}$, $\vec{r} \neq \vec{0}$.

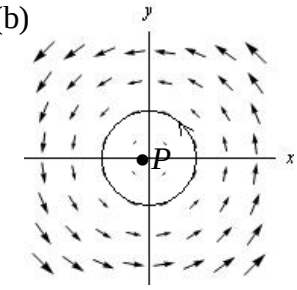
(a) Find div $\vec{F}$.
(b) Calculate the flux out of the sphere.
(c) Using the answers to part (a) and (b), find the flux out of the ellipsoid.

40. The vector fields below have the form $\vec{F} = F_1\vec{i} + F_2\vec{j}$. Assume F_1 and F_2 depend only on *x* and *y*. For each vector field, circle the best answers.

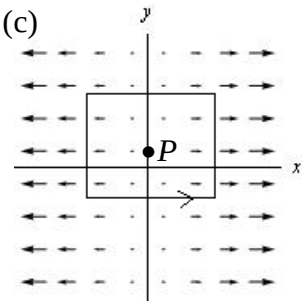
(a)



(b)



(c)



(i) $\int_C \vec{F} \cdot d\vec{r}$ is positive negative zero

(ii) $div\vec{F}(P)$ is positive negative zero

(iii) $curl\vec{F}$ at P has positive $\vec{k}$ component negative $\vec{k}$ component zero $\vec{k}$ component

(iv) $\vec{F}$ could be a gradient field could not be a gradient field

41. Let $\vec{F} = (75x - x^3)\vec{i} - y^3\vec{j} - z^3\vec{k}$ and let S_1, S_5, and S_6 be spheres of radius 1, 5, and 6 respectively, centered at the origin.
(a) Where is $div\vec{F} = 0$?
(b) Without computing the flux, order the flux out of the spheres from smallest to largest.

42. In the region between the circles C_1 : $x^2 + y^2 = 4$ and C_2 : $x^2 + y^2 = 25$ in the *xy*-plane, the vector field $\vec{F}$ has $curl\vec{F} = 3\vec{k}$. If C_1 and C_2 are both oriented counterclockwise when viewed from above, find the value of $\int_{C_2} \vec{F} \cdot d\vec{r} - \int_{C_1} \vec{F} \cdot d\vec{r}$.

43. A particle travels along the helix C given by $x = \cos t,\ y = \sin t,\ z = t,$ for $0 \le t \le 4\pi$ and is subject to a force $\vec{F} = -y\vec{i} + x\vec{j} + 5\vec{k}$. Find the total work done by the force for $0 \le t \le 4\pi$.

44. Determine if each of the following quantities is a vector (V), a scalar (S), or is not defined (ND). Assume that $\vec{u}$ and $\vec{v}$ are 3-D vectors, $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, S is a smooth surface, C is a smooth curve, $\vec{G}$ is a differentiable 3-D vector field, and f is a differentiable scalar function of x, y, and z.

(a) $(curl\vec{G})\times\vec{r}$ (b) $div(\vec{G}\times\vec{r})$ (c) $f_{\vec{u}}(a,b,c)$ (d) $(div\vec{G})\vec{r}$ (e) $\dfrac{\vec{u}\cdot\vec{v}}{\|\vec{r}\|}$

(f) $curl(f\vec{G})$ (g) $\int_C (curl\vec{G})\cdot d\vec{r}$ (h) $\int_S (div\vec{G})\cdot d\vec{A}$ (i) $grad\vec{G}$

45. True or False?

(a) If all of the contours of a function $g(x,y)$ are parallel lines, then the function must be linear.

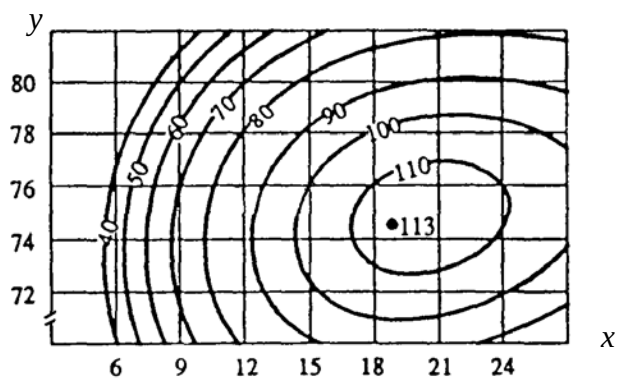
(b) If $curl\vec{F}$ is parallel to the x-axis for all x, y, and z and if C is a circle in the xy-plane, then the circulation of $\vec{F}$ around C must be zero.

(c) If f is a differentiable function, then $f_{\vec{u}}(a,b) \geq -\|\nabla f(a,b)\|$.

(d) If $\vec{F}$ is a divergence free vector field defined everywhere and S is a closed surface oriented inward, then $\int_S \vec{F}\cdot d\vec{A} = 0$.

(e) If $\vec{G}$ is a curl free vector field defined everywhere and C is a simple closed path, then $\int_C \vec{G}\cdot d\vec{r} = 0$.

46. Use the portion of the contour diagram of $f(x,y)$ shown below to estimate the following:



(a) $gradf(15,78)$ (b) $f_{\vec{u}}(15,76)$ in the direction $-\vec{i}+\vec{j}$

(c) A critical point of $f(x,y)$. (d) $\int_C \nabla f\cdot d\vec{r}$ where C is the path from $(15,76)$ to $(24,76)$.

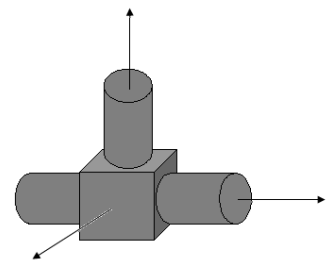
(e) $\int_R f(x,y)dA$ where R is the rectangle $9 \leq x \leq 15$, $76 \leq y \leq 80$.

47. Calculate $\int_C (-y^3\vec{i} + x^3\vec{j} + e^z\vec{k})\cdot d\vec{r}$ where C is $x^2 + y^2 = 16,\ z = 8,$ oriented counterclockwise when viewed from above.

48. Suppose S is the surface obtained by taking the union of the upper hemisphere of radius 2 centered at (0, 0, 4) and an open cylinder of radius 2 centered around the *z*-axis, with $0 \le z \le 4$. If the orientation of S is away from the origin, evaluate the integral $\int_S (curl(y\vec{i} - x\vec{j} + xy\vec{k})) \cdot d\vec{A}$.

49. Suppose *W* is a solid consisting of cube and three solid cylinders of height 6 and radius 2. (See picture below.) The cube is centered at the origin and has side 5. One cylinder is centered on the z-axis and the other two cylinders are centered on the y-axis. Let *S* be the whole surface of *W* except for the circular disk of the cylinder centered on the z-axis with outward orientation.

Find $\int_S \vec{F} \circ d\vec{A}$, given that $\vec{F} = y^2\vec{i} - xz^2\vec{j} - 3z\vec{k}$.



50. Consider the line L_1 parameterized by $\vec{r}_1(t) = (2-t)\vec{i} + (3+2t)\vec{j} + (4-5t)\vec{k}$ and the line L_2 parameterized by $\vec{r}_2(t) = (5-8t)\vec{i} + (-3+t)\vec{j} + (6+2t)\vec{k}$. Decide whether the following statements are True or False.

(a) The lines L_1 and L_2 are perpendicular to each other.
(b) The line L_1 is normal to the plane $8x - y - 2z = 0$.
(c) The line L_2 is parallel to the plane $x = 2y - 5z$.
(d) The line L_2 is parallel to the vector $-16i + 2\vec{j}$.
(e) The plane $x + 4z - 29 = 0$ contains the line L_2.