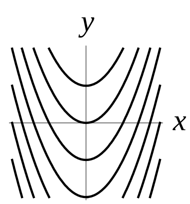
MATH 223
FINAL EXAM STUDY GUIDE ANSWERS
(2017-2018)

1. (a) increasing (b) decreasing

2. (a) $(y-3)^2 + z^2 = 25$ This is a cylinder parallel to the *x*-axis with radius 5.
(b) $x = 3$, $x = -3$. These are vertical planes parallel to the *yz*-plane.
(c) $z^2 = x^2 + y^2$. This is a cone (one opening up and one opening down) centered on the *z*-axis.

3. There are many possible answers.
(a) $x = 0$ produces the curve $y = 3 - z^2$.
(b) $y = 1$ produces the curves $z = \sqrt{3 - \cos^2 x}$ and $z = -\sqrt{3 - \cos^2 x}$.
(c) $x = \frac{\pi}{2}$ produces the curves $z = \sqrt{3}$ and $z = -\sqrt{3}$.

4. (a)



(b) (i) 1 (ii) Increase (iii) Decrease

(iv) Ascending (v) $9/\sqrt{2}$ m/hr

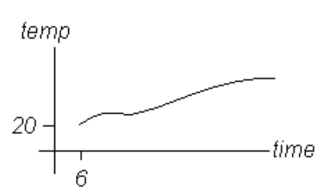
5. (a) (i) $cy^2 + x^2 = 1$. $c = 0$ gives lines, $c > 0$ gives ellipses and $c < 0$ gives hyperbolas.
(ii) $x^2 + y^2 = c^2$. Circles centered at the origin with radius *c*.
(iii) $y = \cos^{-1}(c)$. Parallel lines with $-1 \le c \le 1$.

5. (b) (i) Paraboloids centered on the *x*-axis, opening up in the positive *x* direction. $x = y^2 + z^2 + c$

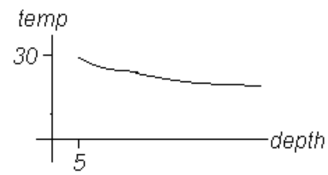
(ii) Spheres centered at the origin with radius $\sqrt{1 - \ln c}$ for $0 < c \le e$. $x^2 + y^2 + z^2 = 1 - \ln c$

(iii) Cylinders centered along the *y*-axis with radius $e^{c/2}$

6. (b) Temperature as a function of time at a depth of 20 cm.



(c) Temperature as a function of depth at noon.



7. $z = f(x, y) = 2x - 3y - 2$

y* \ *x	2.5	3.0	3.5
-1.0	6	**7**	8
1.0	**0**	1	2
3.0	-6	**-5**	**-4**

8. (a) II, III, IV, VI (b) I (c) I, III, VI (d) VI

9. (a) $z = \frac{12}{5}x - 4y + 12$ (b) There are many possible answers. $\frac{12}{5}\vec{i} - 4\vec{j} - \vec{k}$ (c) $\frac{3\sqrt{569}}{2}$

10. (a) iii, vii (b) iv (c) viii (d) ii (e) v, vi (f) i, ix

11. There are many possible answers.

(a) $\frac{5}{\sqrt{26}}\left(4\vec{i} - 3\vec{j} + \vec{k}\right)$ or $-\frac{5}{\sqrt{26}}\left(4\vec{i} - 3\vec{j} + \vec{k}\right)$ (b) $-2\vec{i} + 3\vec{j}$ (c) $-4\vec{i} - 11\vec{j} - 17\vec{k}$

(d) $\cos\theta = \frac{4}{\sqrt{442}}$, $\theta \approx 1.38$ radians (e) $\frac{4}{26}\left(4\vec{i} - 3\vec{j} + \vec{k}\right)$ (f) $-4\vec{i} - 11\vec{j} - 17\vec{k}$

12. (a) $a = -\frac{3}{5}$ (b) $a = \frac{1}{3}$ (c) $2(x-1) - (y+2) + 3(z-3) = 0$ (d) $x = 1 + 2t,\ \ y = -2 - t,\ \ z = 3 + 3t$

13. (a) $\vec{i} + 10\vec{j} - +7\vec{k}$ (There are many possible answers)

(b) $x(t) = 1 + t$, $y(t) = 3 + 10t$, $z(t) = 7t$ (There are many possible answers)

14. (a) $\frac{\partial z}{\partial x} = \frac{3x^2 y}{x^3 y + 3} - \frac{2x}{1 + \left(x^2 + y^2\right)^2}$ (b) $f_H = \frac{10 + 4H + 3T}{\left(5 - H\right)^4}$ (c) $\frac{\partial^2 z}{\partial x \partial y} = -\frac{1}{y^2} - \frac{1}{x^2}$

15. (a) $z=4e^2(x-1)+3e^2(y-2)+2e^2$ (b) $4(x-3)+8(y-3)+6(z-6)=0$

16. (a) $ds=\dfrac{v\sin(2\alpha)}{5}dv+\dfrac{v^2\cos(2\alpha)}{5}d\alpha$
(b) The distance s decreases if the angle α increases and the initial speed v remains constant.
(c) $\Delta\alpha\approx -0.0886$. The angle decreases by about 0.089 radians.

17. (a) The water is getting shallower. $h_{\vec{u}}(-1,2)=-\dfrac{4}{\sqrt{17}}$
(b) There are many possible answers. $3\vec{i}+\vec{j}$
(c) 72 ft/min

18. (a) $grad\left(\dfrac{yz^2}{1+x^2}\right)=-\dfrac{2xyz^2}{\left(1+x^2\right)^2}\vec{i}+\dfrac{z^2}{1+x^2}\vec{j}+\dfrac{2yz}{1+x^2}\vec{k}$
(b) $curl\left(\left(x^2+y^2+z^2\right)\vec{i}-\left(y+z\right)\vec{j}+\left(xz\right)\vec{k}\right)=\vec{i}+z\vec{j}-2y\vec{k}$
(c) $div\left(\left(\cos^2 x\right)\vec{i}+\left(x\sec y\right)\vec{j}+\left(e^{3z}\right)\vec{k}\right)=-2\cos x\sin x+x\sec y\tan y+3e^{3z}$
(d) $\dfrac{\sqrt{37}}{3}$
(e) $\dfrac{4-e}{\sqrt{29}}$
(f) $g(x,y,z)=xy+\sin e^z+c$

19. (a) Yes, $f(x,y)=e^{xy}+\sin z+C$ (b) Yes, $f(x,y)=\ln C\left|xyz\right|$ where C is a positive constant.
(c) Yes, $f(x,y)=\dfrac{x^2}{2}+yx-\dfrac{y^2}{2}+C$ (d) No, Curl $\vec{F}=-4x\neq 0$

20. (a) positive (b) negative (c) negative (d) zero (e) positive (f) positive

21. (a) $\left.\dfrac{\partial w}{\partial u}\right|_{(1,\ 1/2)}=\dfrac{15}{8}\pi$ and $\left.\dfrac{\partial w}{\partial v}\right|_{(1,\ 1/2)}=-\dfrac{15}{4}\pi$
(b) $\left.\dfrac{dw}{dt}\right|_{t=1}=-\dfrac{3}{e}$

22. 3.18π cubic cm/ C^o

23. (a) $(-1,-3)$ local maximum, $(2,1)$ local minimum, $(-1,1)$ and $(2,-3)$ saddle points

(b) $f(x,y) \approx 9(x-2)^2 + 6(y-1)^2 - 25$

24. (b) $K < 4$, saddle point, $K > 4$ local minimum, no values of K for local maximum.

25. (a) $r = \sqrt{8}$ (b) $\theta = \dfrac{\pi}{4}$ (c) $\phi = \dfrac{3\pi}{4}$ (d) $\rho = \dfrac{10}{\cos\phi}$

26. (a) positive (b) positive (c) negative (d) negative

27. (a) $\dfrac{1}{6}\sin 18(\cos 5 - \cos 11)$

(b) $\dfrac{2}{3}\pi(5)^3$ volume of a half sphere

(c) $\dfrac{2}{9}\left(28^{3/2} - 1\right)$ change the order

(d) $\dfrac{\pi}{2}\left(\dfrac{1}{3} - \dfrac{1}{3}e^{-8}\right)$ convert to spherical

(e) $-\dfrac{7}{3}$

(f) $\dfrac{25}{2}$ area of a triangle

28. (a) $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} dzdydx$

$$\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} rdzdrd\theta$$

$$\int_0^{2\pi} \int_0^{\pi/4} \int_0^2 \rho^2 \sin\phi d\rho d\phi d\theta$$

(b) $\displaystyle\int_{-3}^{3} \int_{-\sqrt{9-z^2}}^{\sqrt{9-z^2}} \int_{y^2+z^2+2}^{20-y^2-z^2} dxdydz$ $\displaystyle\int_0^{2\pi} \int_0^{3} \int_{r^2+2}^{20-r^2} rdxdrd\theta$

(c) $\int_0^4 \int_0^{12} \int_0^{\sqrt{16-z^2}} dxdydz \qquad \int_0^{\pi/2} \int_0^4 \int_0^{12} rdydrd\theta$

(d) $\int_{-6}^0 \int_0^{1/3x+2} \int_0^{2x-6y+12} dzdydx$

29. $\int_0^{2\pi} \int_0^4 \int_0^{4-r} k(4-z)rdzdrd\theta$

30. There are many possible answers.
(a) $x = 3\cos t + 2,\ \ y = 1,\ \ z = -3\sin t \qquad 0 \le t \le 2\pi$
(b) $x = 1 + 2t,\ \ y = -2 - 3t,\ \ z = 3 - t \qquad -\infty < t < \infty$
(c) $x = -t,\ \ y = \left(-t + 2\right)^3 \qquad -2 \le t \le 0$
(d) $x = 2\cos t,\ \ y = 2\sin t,\ \ z = 2 \qquad 0 \le t \le 2\pi$
(e) $x = a\cos t,\ \ y = b\sin t \qquad 0 \le t \le 2\pi$

31. (a) $t = 4$ seconds (b) $\sqrt{10}$ feet per second
(c) There are many possible answers. $x = 3,\ \ y = 3(t - 2\pi),\ \ z = 10 - 2\pi - (t - 2\pi) \qquad t \ge 2\pi$

32. (a) There are many possible answers. $x = 1 + 2t,\ \ y = 1 + 6t,\ \ z = 7 + t \qquad t \ge 0$ (b) No

33. (a) iii (b) v (c) vi (d) i (e) ii (f) iv

34. $\int_{C_1} \vec{F} \cdot d\vec{r} < \int_{C_2} \vec{F} \cdot d\vec{r} < \int_{C_3} \vec{F} \cdot d\vec{r}$

35. (a) $-\frac{21}{2}$ (b) $10 - \cos 2$ (c) 18π (d) $\frac{\pi^4}{16} - 1$ (e) $\frac{1875\pi}{2}$ (f) -54

36. (a) $\frac{-12}{\sqrt{2}}$ (b) 320π (c) 450π (d) $\frac{1875\pi}{2}$

37. (a) 0 (b) 0 (c) $2\pi a^2$

38. $p = 0,\ \ flux = 500\pi$

39. (a) 0 (b) 4π (c) 4π

40. (a) (i) 0 (ii) 0 (iii) zero $\vec{k}$ component (iv) could be a gradient field
(b) (i) positive (ii) 0 (iii) positive $\vec{k}$ component (iv) could not be a gradient field
(c) (i) 0 (ii) positive (iii) zero $\vec{k}$ component (iv) could be a gradient field

41. (a) On a sphere of radius 5. (b) $\int_{S_6} \vec{F}\cdot d\vec{A} < \int_{S_5} \vec{F}\cdot d\vec{A} < \int_{S_1} \vec{F}\cdot d\vec{A}$

42. $75\pi - 12\pi = 63\pi$

43. 24π

44. (a) V (b) S (c) S (d) V (e) S (f) V (g) S (h) ND (i) ND

45. (a) false (b) true (c) true (d) true (e) true

46. There are many possible answers.
(a) $\frac{10}{3}\vec{i} - \frac{10}{2}\vec{j}$ (b) $-\frac{8}{\sqrt{2}}$ (c) $(18.5, 74.5)$ (d) 10 (e) $6(60+80+50+70) = 1560$

47. 384π

48. -8π

49. $-114\pi - 375$

50. (a) True (b) False (c) True (d) False (e) True