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Question #1
Zenia Pre-Assessment Estimation # 4

L: Omar took four point eight six days off to visit his grandmother. Andrea took four and three eighths days off for vacation. Who took more days off?

Z: Omar?

L: Omar. And how did you come to that conclusion?

Z: Because four and three eights is about four and thirty six, or something like that.

L: Four point thirty six, is that what you mean?

Z: Yes. And so you subtract, I mean compare them and Omar has more.

L: What number did you use to compare?

Z: Omar’s which is four point eight six, and Andrea which is about four point three six?

L: Good. Well, that certainly makes sense.
Question #1
Zenia Pre-Assessment Interview # 1

Z: I first multiplied the whole that was five in gallons of gas. And then I multiplied it by three which is what one gallon cost and it gave me thirteen and then I multiplied the decimals and there was point twenty five times point five, point zero five and then it give me one twenty five.

L: Okay, how did you decide where to put your decimal points?
Z: I just put it straight down.
L: Okay, so you just put it straight down, and then you had three numbers to the right of the decimal place. Right?
Z: Yes. But since we can do a dollar from one twenty five, point one twenty five, I just, I converted to a whole number so it was one point twenty five, so then I added one point twenty five plus fifteen and it gave me sixteen point twenty five.

L: And how would you normally write, maybe you can show me on the white board, how would you normally write a dollar twenty five.
L: So, in looking at your answer of point one two five, why did you decide it would be okay for you to have it be a dollar twenty five?
Z: Umm, because it had more than a hundred?
L: More than a hundred. . .
Z: I guess I thought maybe a hundred and twenty five were like pennies and so then we can make another dollar with a hundred.
L: Okay, I wasn’t sure, that was one thing that I was confused about. So you added the dollar twenty five and got sixteen twenty five.
A: Dring. Miss Berenice?
B: hi Ariel
A: Okay, I have another problem to show you. It says Omar took four point eighty six days off to visit his grandmother and Andrea took four and three eighths days off, three eights days off for vacation. Who took more days off, how many days more? Okay so what I did I wrote three point eighty six, sorry I mean four point eighty six.
B: What is that number?
A: What?
B: Four point eighty six is what?
A: It’s a decimal and those are the days that Omar took off.
B: And ok the days that Omar took off.
A: And now the days that Andrea took off was four and three eighths. So, then basically I just forgot about the eighty six and three eighths and I subtracted the two fours and it gave me zero so now I have nothing less. Basically they are in a tie. And now I go back to point eighty six and three eighths. I changed three eighths into three tenths and basically in a decimal it was point thirty. And I changed it into three ninths and it gave me thirty three.
B: How did you get the thirty three?
A: The smaller the number on the bottom, the bigger the decimal gets. The bigger it gets in a decimal.
B: How do you know that would be point three three?
A: Because like say that I have a circle and the circle I make it into nine pieces. Ohh no I have ten. I get nine pieces and then I take off three so then I’m going to have to use six more to get to nine. But now if I make it ten, sorry eight, if I have eight pieces now, I’m going to need less numbers to get to eight. So it’s going to be five. And if I make it seven I’m just going to need four and if I make it six then it’s just going to be a less number so it’s going to be easier to get to one whole. That’s how I got three point thirty three.
B: How did you get the three point thirty three?
A: Oh, I messed up right there. It’s point thirty three, sorry. And now instead of making three ninths, I make it into three eights. So as a decimal is going to be higher, three point thirty six.

B: I don’t understand how do you get the thirty six and thirty three. How do you know that would be thirty three and not thirty four or.

A: Because I’m adding by threes. The number on top, I’m adding by threes, because the number on top will be…yes. I’m just adding by threes

B: That’s because the number on top is three.

A: yeah.
(Interview)

V: Vanessa has five and one fourth pounds of grapes. She gave two and two thirds pounds of grapes to Ximena. How many pounds of grapes does Vanessa have left? So you’re going to subtract five and one fourth minus two and two thirds, but you need to multiply them because they don’t have the same denominator.

L: Ok

V: It’s going to be two and eleven twelfths. Because five minus two would be three but since the three over twelve is smaller than the eight over twelve you get a smaller whole number.

L: Can you kind of show me how that works?

V: So if you would have done like five minus two that would equal two three.

L: Okay go ahead and write that down so I can kind of follow what you’re saying.

V: Put the three over twelve is smaller than the eight over twelve, the numerator is different, smaller, so the whole number is going to be smaller.

L: Oh, so that’s why you got a two. How’d you get the eleven?

V: Eleven like you add this up and get eleven.

L: Okay, why did you choose to add them?

V: I don’t remember.

L: Is there another technique you might use?

V: I think you can get them smaller.

L: Okay.

V: If they get still the same denominator you can’t get them smaller maybe something bigger than that.

L: I guess I was just not following. So if you have three twelfths minus eight twelfths. Do you want to write that here and let’s see what that looks like? Okay, so we have a subtraction.

V: That would be like negative five.

L: Do you wanna write that down?

V: I think it’s over twelve.

L: Good thinking.
V: Cause you subtract eight minus three.

L: Okay negative five over twelve and here you have a three. So, you have decided that your
answer is going to be two and a fraction. Okay, so right now we have three and a negative five
twelfths.

V: It would equal like two something.

L: Okay I’m not sure I understand, you got three and what is this?

V: Negative five twelfths. But if I you multiply twelve times three it would be forty eight over
twelve.

L: I’m not following that.

V: Twelve times three over twelve, the answer over twelve and you can subtract it from that.

L: Here you go. So you’re replacing the three with . . .

V: Five over twelfths and then you would do the same for thirty six.

L: Okay, yeah.

V: And you get thirty one.

L: Okay you have thirty one, very good.

V: Thirty one over twelve. And then it’s going to equal two and eleven twelfths.

L: Okay two, because, how’d you get two?

V: How many times does twelve go into thirty one.

L: Good that’s two and what’s two times twelve?

V: Twenty four.

L: Ok and then how many more do you need to get to thirty one?

V: Seven. Yeah. Seven.

L: So you like that answer better?

V: Yeah.
V: Yes. Vanessa has five and one fourth pounds of grapes. She gave two and two thirds pounds to Ximena. How many pounds of grapes does Vanessa have left?
B: can you repeat the numbers for me?
V: five and one fourth and two and two thirds
V: What we did was five minus two is equal to three and one fourth would be three twelfths and two thirds would be eight twelfths. So for the three . . .
B: I think that I’m lost. You said that you have five minus three.
V: No, five minus two which would equal to three. And then . . .
B: Where do you get the five?
V: Five is a whole number and then the two is a whole number for two and two thirds, and you just subtract them it’s equal to three. And then one fourth would equal to three twelfths and two thirds would be eight twelfths
V: If you subtract them you’re going to get negative five.
B: Negative five?
V: Yeah.
B: Can you read all the subtraction for me. You have three twelfths minus . . .
V: Three twelfths minus eight twelfths and then if you draw a number line you are going to go to the left and take the five from the three and you’re going to get negative five.
V: And then from the thirty six over twelve which would be the whole number, for the three you subtract that from negative five and you get thirty one over twelve. And then you’re going to see how many times does twelve fit into thirty one and it will be two and the remaining part will be seven twelfths.
B: seven twelfths. How do you get the seven?
V: Because when you see how many times the twelve fits into thirty one it’s going to be two and it would be twenty four and then you subtract thirty one minus twenty four and it would be seven.
A: Fifty times three hundred. I like to instead of doing that I like to add one point thirty-nine three hundred times. I’ll do whatever it takes just to not do that other.

B: Why?

A: It’s because for me I think that would be much simpler because I don’t really like to like where this one goes here and th . . .

B: Oh, you don’t like to do the multiplication?

A: No, I don’t really like to figure out where the decimal point goes.

B: Oh, and adding it’s easier to know where the decimal points go.

A: Yeah. So it’s sort of easier.

L: It makes more sense than memorizing the . . . all those steps, is that where you are going?

A: Yeah.

B: So you don’t like to memorize. Just try to figure out . . .

A: Figure out the hard way.

L: ((laughs)), but it makes more sense to you?

A: Yeah.
Question #2
Zenia Pre-Assessment Interview 5

L: You wanna go ahead and read it?

Z: It says compute the following problem one fourth minus one half equals. Show how you can
use a number line to find the answer. And so I had a number line with negative 2, negative one
and zero, one, and two. And then I just tried like to calculate where was about one fourth. So
divided one half and then half of a half is one fourth so about there is one half, one fourth. And
so then if I subtract one half is equivalent to two fourths. So here I have one fourth already and
so I need to find another fourth. I divided it here. It’s negative one half. And negative three
fourths, and negative two… one fourth. And so then one fourth plus one fourth is two fourths
which is equal to one half and so this is one half.

L: And why did you go that direction on the number line?

Z: Because when two fourths is bigger than one fourth and so you subtract one fourth minus a
bigger number is going to give you a negative number which goes to this side.
Question #3
Andres Pre-Assessment Estimation 1b

A: Umm, okay okay! I said that they got sixteen and that Christian’s family spent sixteen dollars and twenty five cents. So over here I said subtracted seven and twenty five cents so it gave me nine dollars and I know like if you take off the five over here, it’s just going to make it three dollars. So you add three, plus three, plus three dollars, and it’s nine. But like if you add five three times again then it will be three dollars and fifteen cents, I mean nine dollars and fifteen cents, my bad. So they wouldn’t have like enough money because they only wasted nine dollars, spend nine dollars.

L: Could they get part of a gallon. I mean could you get like three and a half gallons or two and a half gallons or . . .

A: Oh, umm.

L: So if they just had nine dollars and they had to get one gallon or two gallons or three gallons, you’re right they could only get two. But if they could get part of a gallon, do you think that would be, like if they could get two and a half gallons or two and three fourths gallons or two and a fourth, would if they put it in until that pump goes all the way to nine dollars do you think that it would be, that it would be that they would end up buying closer to two or to three gallons?

A: Oh, closer to three yeah. With like fractions and that, closer to three yeah.

L: Yeah, cause like lots of times if you’re thinking you can only buy a whole thing, but like gallons you can buy part of a gallon.

A: So, yeah I think it’s three. Closer to three.

Well that makes sense.
Question #3
Xiomara Pre-Assessment Interview 2a

X: Vanessa has five and one fourth pounds of grapes. She gave two and two thirds pounds to
Ximena. How many pounds of grapes does Vanessa have left?

X: So I put five and one fourth, minus two and two thirds. Okay, so first I did like this, five
minus two is three, then one fourth minus two thirds equals, so I know they have to have the
same denominator so I multiply times three and this one times four and that’s twelve so right
here its eight and three. Right there I got kind of lost cause its like three you can’t take off
eight so I don’t, like eight minus three so that’s six, so it’s like negative? . Negative six? But
I’m not sure if you can do that with fractions.

L: Well you can. Yeah, so you have a negative fraction there, so you have three. Do you have
any ideas about if you have three and then you have a negative six twelfths, how would you
combine those numbers do you think?

X: Well, I was thinking that three could borrow use the six could borrow some from the three,

L: That’s good thinking.

X: I’m not sure.

L: So if you did borrow that, how do you think you might do that?

X: I could do it, bars of twelve. So there is like three wholes, and you have one two three four
five six twelfths. So, six twelfths is like one half, yeah one half. There would be three and one
half. I don’t know.

L: What’s the negative, how do you think that might affect it?

X: Yeah. So, the three could, I could take out this one so it could be seven, and I could take out
this one so it would be eight. I don’t know I could reduce it. I can \{13:32\} two times two and
its twelve and that’s twenty four. I’m not sure.

L: Okay, so.

X: That would be twelve, twenty four, I’m not sure.

L: You’re not sure if that would be useful?

X: Yeah, cause I don’t like negatives.

L: ((laughs)). Okay. Any other ideas about how you might combine that three and that negative
six twelfths?
Question #3
Zenia Post-Assessment Interview 2b

Z: Ariel drove his bike thirty nine and one sixth meters and Nick drove his bike twenty eight and five ninths meters. How many more meters did Ariel drive than Nick?
Z: But then I have to find a common denominator to subtract and so I multiplied two hundred and thirty five over six by nine over nine.
L: Why did you choose nine?
Z: So that it would have a common denominator?
L: Okay, so you got the nine from where?
Z: Over here.
L: Okay.
Z: And I got two thousand one hundred and fifteen. And then I need to multiply this to get the same common denominator.
L: And where did you get the six over six?
Z: From the other denominator. And, then I need to subtract this and this. And then I got five hundred and seventy three. Over fifty four and then I need to I can convert this to a mixed number.
L: Oh great.
Z: And so I need to divide five hundred seventy three into fifty four. Yes.

so this is equal to ten and six over ten.
Question #4
Andres Pre-Assessment-Estimation 5

1  L: Okay, your problem is to take one fourth minus one half.
2  A: That’s one fourth.
3  L: Okay, one fourth minus one half is one fourth. And how did you get that?
4  A: Because one fourth plus one fourth is one half so, yeah.
5  A: I’m changing my mind again. I think it’s minus one fourth.
6  L: Okay.
7  A: I think it’s minus one fourth because, once again you have the two quarters and a quarter over here.
8  L: Okay.
9  A: And one of the quarters you subtract minus this quarter and it just gives you nothing. And it’s like you have twenty five cents here and you have minus zero here and you subtract it and it’s going to give you like minus twenty five. And so then twenty five as a fraction is one fourth and so it’s minus one fourth.
Question #4
Andres Post-Assessment Interview 4b

A: It says one sixth minus two thirds. And that one is kind of difficult. So right here was one half
one fourth, and like three fourths cause those are the most simpler. And right here it will
probably be like one sixth. Sorry, nevermind. Right here is going to be one sixth and right here
one eighth and stuff.

L: Good, yeah.

A: So now I label one sixth and I gotta label two thirds, so right here will be like one third. And
this one will be like two thirds, two thirds right here. So what I did was I labeled, I went like
that two thirds and one sixth. Since it was one sixth minus two thirds, what I did was . . . Let’s
say that this part is five inches. I did the same here and so minus five. Let’s say this whole
chart is like fifteen inches. So right here I have five inches, minus five inches. And it gave me
like… It was just five. So what it gave me was one. Minus one.

L: From one sixth if you go the same distance in the other direction you think you’re going to get
all the way to negative one?

A: Oh, no sorry! It’s cause I did it wrong. I think I’ll probably go like minus two thirds
something like that.

L: Okay, so if you go the same distance in the other direction you get about…?

A: two thirds. Oh, minus two thirds.

L: Ok. You want to write that down? So we don’t forget that. So my question is, if you start at
one sixth and you go this far. If you go this far in the other direction, are you going to be at
negative two thirds? Cause you’re starting at one sixth.

A: Yeah, it’s cause like let’s say this is one sixth and this is just two thirds. What I did was I just
went like that.

L: And that makes sense

A: Right here is like I said is minus two thirds. It’s probably like…

L: Since this, if you’re going two thirds, that is your distance here. And you’re at one sixth and
you’re going the same distance, two thirds here. If you went that same distance and started at
zero, where would you end up?

A: I’ll probably end up at two thirds.
L: Right, yeah. So I think it’s a really, really good strategy, but what about that one sixth?
A: I guess it will have to be. Oh wait! It’s like one third
B: Do you know another way to use those fractions? In a way that you don’t have to work with big numbers?

Z: You could use decimals?

B: Decimals? What else? Can you still use fractions?

Z: Yes.

B: In a different way?

Z: Um, maybe I can just subtract 39 minus 28 and I get 11, and 1/6 minus 5/9 is gonna give you a negative number. And so a negative number is subtracted from the whole number.

B: Do you want to try it that way? Can you say aloud what you are writing just to know?

Z: I subtracted 39 minus 28 and it gave me 11 and now I am going to multiply 1/6 times 3 over 3 and its gonna give me 3/18. Then I am going to multiply 5/9 times 2 over 2 and it gives me 10/18. And so then I need to subtract 3/18 minus 10/18 and it gives me negative 7/18. Then I need to subtract negative 7/18 minus 11. Maybe if I just subtract 11 minus 7/18 equals . . . 10 and . . . 11/18ths

B: 10 and 11/18ths? How did you get that?

Z: Well, I just kind of, um, took away 10 from 11 and then just put them as a whole number and so 1 is 18 over 18.

B: Uh huh.

Z: And then I subtracted 7 and it gave me 11/18th and I just put it with the whole number.

B: Uh, huh. And then your answer is?

Z: 10 and 11/18ths.

B: Okay. Thank-you.

Z: Bye.

B: Bye-bye.

L: I think I’m going to tear up, that was so good.

B: Yeah.
Question 5
Andres Classroom Explanation

A: So the pattern is like subtracting one zero. Like when you have 21 times 100 its going to be
like wait oh yeah, that’s gotta be 2 thousand and well, you just get that.
R: Okay
A: And here you just take like 21 times 10 and here you just subtract, like take off a zero. And
here it is just 21 times 1 and you get another zero and you just keep going like that. And here
you keep adding another zero but here the number actually it makes it smaller
R: Is it getting smaller?
A: Yeah.
R: The answers are getting smaller? What about the movement of the decimal point? Anything
going on with the decimal?
A: Right here you don’t have the decimal point and right here you add it. Because here is the
one and here you keep moving it like that forward.
R: If I had to ask now by how much are they getting smaller by each time, could you tell me
how much it is getting smaller by? You said, what was the word you used, you said, its
getting smaller, you said, it’s, what’d you say, the numbers are getting. . .
A: Yeah, smaller by zero
R: Okay, smaller by zero. So what’s that, what’s another way to say that? It’s getting smaller
by zero. That’s a really nice way to put it, A. I like that idea. Give him a hand by the way,
that was brilliant.
Question #5
Scaffolding. Quiz Bits and Pieces III

In this clip, students are taking a quiz for Bits and Pieces III. The teacher is engaged in scaffolding to assist students in understanding the questions by addressing issues of language and background knowledge. These interactions enable the teacher to have a deep understanding of student interpretation of the questions, resulting in a more accurate assessment of student mathematical knowledge.

Without the scaffolding student responses may be more representative of language and background issues thereby obscuring their mathematical knowledge.
Question 6
Xiomara Post-Assessment 5

X: It has one sixth minus two thirds. Um, I have to have an equivalent denominator so I multiply. . .
B: Equivalent denominator?
X: Um, yes, denominator, the same
B: Good, you want to have the same denominator?
X: Yeah, so I multiply times two and I multiply the numerator by 2 too.
B: What is the fraction?
X: Two over three
B: Okay.
X: Um, so then I have 4 over 6 and it gives me negative, negative 3 over 6.
B: So when you, you start with 1/6 minus 2/3 then you change to what?
X: 1/6 minus 4/6.
B: And you’ve got?
X: Three negative, three negative over six.
B: Okay.
X: And so I twist it, so I multiply times 2, times 2 and I got 6/12ths and that’s when I noticed that I could reduce it into one half because 6 is a half of 12 and 3 is a half of 6 so…
B: Um, I, can I ask you something? When you say that you have the 3/6 and you say that you reduce it and you get 6/12 then you say that you reduce it and you get ½ and both times you were reducing the fraction?
X: I don’t get it.
B: So, you said that you have 3/6, then you said that you reduce it to 6/12?
X: Uh huh.
B: Yeah? But, then you also say you have 3/6 and you reduce it to ½. Is that right?
X: Weeelll.
B: In both cases, were you reducing the fraction?
X: No. Because, when I got half, its just I knew it automatically because its like 3 is a half of 6 so I know its like.

B: Oh, okay.

L: Can you explain to us what you mean when you say “reduce” the fraction? What does that mean?

X: Um, when you do something to the fraction to get another fraction that’s equivalent . .

L: Okay

X: to it.

B: Do something—what do you do?

X: You can divide to make it smaller and or you could multiply to make it bigger.

B: Oh okay, that’s what you mean when you say reduce it. Okay