

"There's All Kinds of Math:" Teachers' Domain-specific Beliefs in Response to Mathematics

Education Reform

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This paper is currently being reviewed.
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Running Head: "All Kinds of Math:" Teacher Domain-specific Beliefs

Key Words: algebra, domain-specific beliefs, mathematics beliefs, mathematics reform,
secondary education, teacher beliefs

Abstract

This article extends and refines an under-researched category of teacher mathematical beliefs called *domain-specific beliefs* that influence the way high school math teachers respond to mathematics reform. Domain-specific beliefs are beliefs associated with a specific field of mathematics (Törner, 2002). In this study, secondary mathematics teachers in an urban high school mathematics department in the United States expressed different views about the domains of algebra, geometry, and statistics/probability when faced with implementing district-mandated mathematics reform. Fine-grained analysis of the qualitative data including teacher meetings and interviews revealed two dimensions of these domain beliefs: 1) the role of abstraction within the domain and 2) the utility of the domain for future career and educational endeavors. Beliefs about the domain of algebra were particularly salient in the context of discussions about mathematics reform. The findings have important implications for research on teacher beliefs related to teaching mathematics and successful mathematics reform implementation.

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Over the past 20 years, mathematics reforms at both the state and national levels have called for changes in teaching practice to improve mathematical learning and performance (California Department of Education, 1992; 1998 NCTM, 1989, 1991, 1995, 2000; Michigan Department of Education, 1996; Vermont Department of Education, 2000). The call for change was motivated by several factors including an increased understanding of how students learn mathematics and the persistent achievement gap between African American, American Indian, and Latino/a students and their White and Asian American counterparts (NCTM, 2000; Schoenfeld, 2002; Secada, 1992; Tate, 1997; The College Board, 1999). As part of that call, researchers targeted teachers' beliefs about the nature of mathematics, teaching, learning and students as a critical lever of change (Battista, 1994; Cohen, 1990; Cooney & Shealy, 1997; Franke, Fennema & Carpenter, 1997; National Research Council, 2001; Schoenfeld, 2003). This article explores theoretical dimensions of a category of mathematics-related teacher beliefs, called *domain-specific* beliefs, that has strong implications for research and the successful implementation of mathematics reform.

Törner (2002) describes domain-specific beliefs as beliefs associated with a specific domain or field in mathematics such as calculus, stochastics, and geometry. He distinguishes this type of belief from global beliefs about the nature of mathematics, teaching, and learning – which have been the primary research focus of cognitive studies on teaching (Aguirre & Speer, 2000; Battista, 1994; Cohen, 1990; Cooney & Shealy, 1997; Lloyd, 2002) and subject-specific beliefs that focus on a particular math topic such as function (Lloyd & Wilson, 1998). Domain-specific beliefs in mathematics are also distinguished from a broader category of epistemological

beliefs characterizing different domains of knowledge such as mathematics, humanities, and social science (see Schommer, 1990; Schommer & Walker, 1995).

This article examines domain-specific beliefs of in-service secondary mathematics teachers working in an urban American high school responding to a set of district-mandated reforms aimed at mathematics content, curriculum and instruction.¹ The discussion focuses primarily on specific beliefs about algebra and to a lesser extent the domains of geometry and statistics/probability. Analysis of mathematics department meeting transcripts, teacher interviews, and member checks provided evidence of domain-specific beliefs. For example, some teachers believed algebra meant “solving equations for x ... and is the most abstract thing we do in the classroom.” And that geometry was more “visual, “concrete, and “tangible” to students. The paper demonstrates how teachers distinguished among these domains along at least two dimensions: the role of abstraction in the domain and the role of the domain’s utility for future career and educational pathways. Evidence of domain-specific beliefs often occurred when teachers responded to specific questions about the district mathematics reforms in interviews or were engaged in instructional discussions with colleagues on how to address the demands of reform and meet the needs of their students. This analysis contributes to a more comprehensive understanding of the relationship between teachers’ beliefs and mathematics reform policies.

Theoretical Framework

Defining and Assessing Teacher Beliefs

The definition of beliefs framing this analysis was informed by the work of Ernest (1989; 1991) and Thompson (1992). Beliefs are personal philosophies (often implicitly held) consisting of attitudes, values, theories and ideologies that shape practice and orient knowledge. I draw on

three aspects of the general teacher beliefs literature. First, teacher beliefs must be inferred from what teachers say and what they do, and assessed through in-depth analysis from multiple data sources (Calderhead, 1996; Pajares, 1992; Schoenfeld, 2003; Speer, 2001). Second, beliefs are organized into networks or systems, which can account for the identification of seemingly conflicting beliefs within one individual (Calderhead, 1996; Pajares, 1992; Thompson, 1992; Wilson & Cooney, 2002). Third, beliefs that are central to an individual are most likely to be explicit when there is “cognitive dissonance” (Calderhead, 1996; Pajares, 1992). A methodological assumption underlying this research was that the reform context these teachers faced would generate cognitive dissonance, and would provide opportunities to document teacher beliefs about mathematics teaching.

Beliefs Related to Mathematics Teaching in the Classroom

Research on beliefs about mathematics teaching has focused primarily on teachers’ views of the nature of mathematics (Ernest, 1989; Skemp, 1987; Thompson, 1992), student understanding of mathematics (Franke, Fennema & Carpenter, 1997; Vacc & Bright, 1999), and teaching strategies that help students learn mathematics (Cohen, 1990; Lloyd, 1999; Ma, 1999; Schoenfeld, 1988; Thompson, 1992). Most of these studies involved teachers grappling with change (Wilson & Cooney, 2002), either through professional development (Franke, Fennema & Carpenter, 1997; Hart, 2002), curriculum reform (Cohen, 1990; Lloyd, 1999; Lloyd & Wilson, 1998), or high stakes accountability assessments (Schoenfeld, 1988).

Three categories of beliefs about the nature of mathematics are: problem-solving, relational, and instrumental (Thompson, 1992). From the problem-solving view mathematics is a process of inquiry and a generation of new understandings, and is not a set of unquestioned truths to be acquired (Ernest, 1989). From a relational view mathematics is a unified body of

knowledge, emphasizing the discovery of relationships and connections, across domains and “real-world” contexts (Ernest, 1989; Skemp, 1987). From an instrumental view mathematical knowledge consists of a set of fixed procedures, rules, and facts that one applies to a specific set of narrowly defined problems/situations (Ernest, 1989; Skemp, 1987).

Törner (2002) suggests that such beliefs about the nature of mathematics are “global beliefs” about mathematics. He also identifies two other types of beliefs associated with mathematics: subject-matter beliefs and domain-specific beliefs. He describes subject-matter beliefs as analogous to subject matter knowledge (Even, 1993; Shulman, 1987). Each mathematical term, topic, or procedure can be the object of a specific belief. Cognitive studies of teaching focusing on teachers’ conceptions of function (Lloyd & Wilson, 1998; Szydlik, 2000) and proof (Mingus & Grassl, 1999; Knuth, 2002) provide examples of subject-matter beliefs.

However, Törner (2002) contends that neither global beliefs nor subject-matter beliefs adequately include all mathematics beliefs. He proposes another belief category called *domain-specific beliefs* that parallel different fields in mathematics arguing, “different fields in mathematics possess differing characteristics...” (p. 87). Törner defines *domain-specific beliefs* as beliefs associated with a specific mathematical domain such as calculus, stochastics or geometry. Based on his work with German secondary pre-service math teachers, Törner found that teachers held specific beliefs about the domain of calculus that focused on the role of logic, application, exactness, and calculation. In terms of belief structure Törner argues that, “domain-specific beliefs should be classed, hierarchically higher than, for example, notions of derivative or the term of function, although on the whole they still touch on basic views of mathematics” (p. 87). According to Törner (2002), the mathematics beliefs literature has yet to address domain-

specific and other kinds of beliefs including the nature of their relationship, evolution, and strength given particular contexts. The goal of this paper is to provide additional evidence of domain-specific beliefs within the context of mathematics reform.

A few recent studies have focused on teacher beliefs about the learning of specific mathematical content, particularly within the domain of algebra (Lloyd & Wilson, 1998; Nathan & Koedinger, 2000a, 2000b). Nathan and Koedinger (2000a) investigated secondary math teachers' (7th-12th grade) beliefs about students' algebra development. They found most teachers held a *symbolic precedence model* (SPM) about student development of algebraic reasoning. The teachers believed that facility with symbolic forms of algebraic problems were the simplest versions of algebra problems and were easier for students to solve than verbal "story" problems. These teachers believed that first students learn new procedures, concepts and laws to support conventional symbol manipulation. Later students are taught translation and modeling techniques to apply these procedures to algebra story problems as a form of problem solving. The view that reasoning with symbols precedes reasoning with text is also evident in many mathematics textbooks. The researchers argue that the textbooks may support these teacher beliefs. Interestingly, this SPM runs counter to the authors' second analysis of student problem-solving processes. The authors found that students followed a *verbal precedence model* (VPM). Observations of students' verbal competence and associated problem-solving strategies such as "guess and check" precede symbolic manipulation skills for solving algebra problems. VPM hypothesizes that reasoning with text precedes reasoning with symbols. Nathan and Koedinger (2000a) present two competing models of teacher beliefs about student development of algebraic reasoning that are specific to the domain of algebra.

Teacher Beliefs and the Logic of Mathematics Reform

Beliefs are critical to the successful implementation of mathematics reform. Cohen (1990) and Battista (1994) note that implementing reform is a tall order for teachers because many of them hold beliefs that are widely variant with the logic of mathematics reform documents. For example, recent research reports that a teacher's use of "reform-oriented" texts precludes a variety of instructional strategies and teacher beliefs that may or may not be aligned with those texts (see Lloyd, 1999; 2002; Lloyd & Wilson, 1998).

The "logic" of reform documents is a set of practices, ideas, and approaches that become bundled together to form a central organizing principle or philosophy for action (Coburn, 2001; Scott, 1995). In the context of reforms aimed at curriculum and instruction, this philosophy of action includes epistemological assumptions about student learning, conceptions of teacher and student roles, and specific instructional goals (Coburn, 2001).

Several studies of mathematics teaching focused on teachers' conceptions/beliefs about mathematics, teaching, and learning while implementing mathematics reform (Cohen, 1990; Cooney & Shealy, 1997; Lloyd, 1999;2002; Franke, Fennema & Carpenter, 1997). Using Törner's (2002) terms, these studies characterized global beliefs about the nature of mathematics, teaching, and learning, and described how these beliefs were consistent or inconsistent with the logic of mathematics reform documents (Battista, 1994). Few studies have explored domain-specific beliefs within the context of mathematics reform (Lloyd & Wilson, 1998).

Study Context and Methods

This analysis was part of a larger study exploring how teacher beliefs and department culture interact in an American high school mathematics department to affect curriculum and

instructional decisions (Aguirre, 2002). The study was conducted over two years (1998-2000) in a medium-sized urban comprehensive high school in the western region of the USA that served a socio-economically, linguistically, and culturally diverse student population of approximately 2600.²

The BVHS Mathematics Department: Teachers, Courses, Curriculum

The Buena Vista High School (BVHS) mathematics department was quite large with 17 members. Almost all (89%) of the teachers held credentials to teach secondary mathematics.³ The mathematics department faculty was very experienced with an average of 20 years of teaching. Unlike many urban high schools that suffer high faculty turnover, this department was very stable. On average, these math teachers had spent 9 years teaching together in this department. Collegial relations in the department were cooperative and congenial.⁴ In addition, organizational structures were in place that fostered frequent interaction among teachers to discuss curriculum and instruction issues. For example, teachers met in course curriculum teams or “sub-groups” (e.g. algebra, geometry) to discuss particular issues related to teaching that course.

As with most comprehensive schools, BVHS offered a variety of courses including advanced placement and remedial courses, as well as two college preparatory pathways. Almost all teachers taught a range of courses (introductory and advanced) and across curriculum pathways. The teachers considered themselves mathematics reform “pioneers” in the district with a history of *teacher-initiated* use of “reform-oriented” curricula.⁵

With an experienced, highly qualified, stable, collegial faculty with a teacher-initiated track record using innovative reform-oriented curriculum and organizational structures in place

to support teacher cooperation and decision-making, the BVHS math department appeared in a strong position to address the district-mandated reforms.

District Math Reform Context

The BVHS mathematics department faced three converging district-mandated reforms directly impacting mathematics curriculum and instruction.

1) Increased Graduation Requirements:

In 1996, the district's school board approved a substantial increase in the graduation requirements in mathematics. Starting with the class of 2001, all students were required to take and pass three years of college preparatory mathematics to receive a high school diploma. In traditional course taking patterns, this meant that students would take two years of algebra (algebra 1 and advanced algebra with trigonometry), and a year of geometry. With the Interactive Mathematics Program (IMP) pathway, this meant all students were required to take and pass Year 3 IMP.

2) Removal of all remedial mathematics courses

Also in 1996, the school board approved the removal of all "remedial" courses at the high school level such as pre-algebra or MATH A ("transition-to-college prep" course). All incoming ninth graders would be required to take algebra 1 or IMP year 1.

3) Implementation of District Content and Performance Standards

A district-wide committee that included teachers (including three from the BVHS math faculty), curriculum resource staff, and administrators developed content and performance standards. The standards included four content standards: number and operation, geometry and measurement,

function and algebra, statistics and probability; and two performance standards: problem solving/mathematics reasoning, and mathematical communication. The standards were consistent with the new graduation requirements.

These reforms addressed equity, accountability, and content quality. The first two reforms directly responded to the district's mathematics achievement gap for African American and Latino/a students. These students were underrepresented in more advanced mathematics courses and over-represented in the remedial courses at the high school level. Eliminating the remedial courses and mandating more mathematics for all students as a general education requirement, was intended to remove a primary gatekeeper to post-secondary education while simultaneously increasing the stakes and thus responsibility on students and teachers to learn. A guiding assumption of the reforms was that all students were capable of meeting these new requirements. In addition, following the national and state reform and accountability movements, the content and performance standards explicitly delineated the district's learning expectations for mathematics. The intention was to clearly communicate to students, parents, teachers, and administrators what students at each grade level should know and be able to do. A district-wide assessment program aligned with the standards was in development at the time of the study. Thus the district put in place a set of progressive reforms that increased standards and accountability to learn mathematics.⁶

Methods

The methodological framework used to collect and analyze data about teacher beliefs was informed by grounded theory (Strauss, 1987) and a naturalistic paradigm (Moschkovich & Brenner, 2000). The guiding principles of grounded theory propose that theorizing grows from the data rather than from a pre-existing framework used to confirm/disconfirm a theory (Strauss,

1987). Furthermore, in keeping with a naturalistic paradigm, themes that emerged from the data informed the design of further data collection (Moschkovich & Brenner, 2000).

There were several data sources including participant observation fieldnotes of all department faculty meetings, two semi-structured teacher interviews, and a mid-project “work-in-progress” session (Yin, 1994). These multiple data sources were used to generate categories of beliefs, confirm emergent categories, and triangulate across data sources.

There are several ways to document and describe teacher beliefs. Each method brings tradeoffs in relation to the descriptive and explanatory power of these methods (Leder & Forgasz, 2002; Schoenfeld, 2003). As Calderhead (1996) notes qualitative approaches to investigate teacher beliefs tend to utilize at least two data sources: teacher interviews and observations of instructional practice. Instructional practice usually refers to in class teaching. Writing in a review of studies on teachers’ epistemological world views (EWV), Schoenfeld (2003) argued that if researchers want to link EWV to practice, then researchers have to look directly at their practices. However, it is a misinterpretation of Schoenfeld’s axiom to narrow teacher practice to their classroom actions. A fundamental issue to consider is when are teachers’ beliefs consequential? And, what actions would indicate whether the teachers are or are not acting in ways consistent with their professed beliefs?

Here, observations of “instructional practice” refer not to classroom teaching (which was not a source of data), but to professional interactions among teachers during meetings because these interactions are seen as important aspects of teachers’ work (Little, 1990). Horn (2002; 2004) studied the impact of teachers’ epistemological world views on curriculum construction and student placement. Her study comparing two high school math departments demonstrated that the views teachers held about students and curriculum impacted their decisions to re-

construct their curriculum and, for one department, eliminate curricular tracking. Horn's research offers triangulation between teacher beliefs (about students, about curriculum) and teacher actions (the construction of curricular opportunities for students). In that way, consistent with Schoenfeld's argument, her study does look directly at teacher practices – without stepping inside the classroom.⁷

This analysis uses a view of teacher communities informed by situated cognition theory (Lave & Wenger, 1991; Wenger, 1998). From this perspective, mathematics departments are “communities of practice” where teachers jointly engage in the enterprise of teaching through the presence of meaningful relationships, shared understandings, negotiation of meanings, common language, and structures of participation (Horn, 2002; 2004; Lave & Wenger, 1991; Little, in press; Wenger, 1998). Using this view, “instructional practice” includes activities that occur *outside the classroom*. Building on the social-cultural frame, organizational theorists identify meetings as settings that in part define an organization's work and relationships, and within which sense-making and decisions occur (Schwartzman, 1989; Weick, 1995).

Because teachers' participation in meetings is a part of instructional practice, meetings are a crucial setting to examine teacher beliefs. In the case of BVHS math department, the faculty meetings were the structured forums in which teachers came together to discuss issues related to curriculum and instruction. I audio-taped and transcribed each meeting and recorded meeting minutes. The data set included 28 meetings including 14 department meetings, 12 curriculum subgroup meetings (i.e. algebra, geometry, IMP), and 2 special meetings focused on parents and honor course articulation. Participants' verbal contributions during meetings were one source of data on teacher beliefs.

Another source of data were semi-structured teacher interviews conducted twice over the course of the project. The average length of each interview was 70 minutes. Fourteen teachers were interviewed twice, 2 teachers interviewed once, and 1 teacher was not interviewed. The interview questions probed teachers' perspectives on mathematics, teaching, learning, students, and reform initiatives. All interviews (except two) were audio-taped and transcribed. The interviews were used as a primary source for evidence of teacher beliefs. Appendix A provides examples of some interview questions that explicitly probed beliefs about mathematics, learning, teaching, and reform.

A "member check" provided a third source of data (Miles & Huberman, 1994). I conducted a "Work-In-Progress" (WIP) session designed to share emerging findings with the study participants. Fourteen out of seventeen teachers attended. This enabled me to check the accuracy of my initial interpretations, ask the participants clarifying questions, and provide the department with information for future planning.

Analysis utilized a modified constant comparative method (Strauss, 1987). I coded both first round interviews and the meeting transcripts using codes for beliefs derived from the literature based on teacher beliefs.⁸ These included beliefs about the nature of mathematics, student learning of mathematics, and teaching of mathematics. For evidence of beliefs in the meeting data, I focused on teachers' public descriptions of their teaching, public statements of philosophy, and places in the discussion where there were disagreements, "provocative" topics on the table for discussion, or a decision required. The themes and questions about beliefs that emerged from the initial stages of analysis of the meetings and first-round interviews were summarized and shared with the participants in the WIP session.

Initial coding of the data provided evidence of domain-specific beliefs, particularly about algebra. To further explore this type of belief, I incorporated questions about math content into the WIP session and the second interview protocol. Selective coding of the data was instituted after the second round of interviews refining previous categories and yielding other emergent categories of beliefs (Strauss, 1987). The categories were cross-referenced within and across data sources for triangulation purposes. This analysis produced belief profiles that summarized different beliefs for each teacher.

Teachers' Domain-specific Beliefs

Analysis showed that almost half (8/17) of the BVHS mathematics teachers held domain-specific beliefs as evidenced by their contributions to instructional discussions and interview responses. The domain-specific beliefs that surfaced from the analysis centered on three mathematical domains: algebra, geometry, and probability/statistics. The examples of domain-specific beliefs primarily focus on the domain of algebra. Beliefs about geometry and probability/statistics are indirectly illustrated through teacher comparisons with the domain of algebra.

To illustrate the prevalence of domain-specific beliefs I present a two-part analysis. First, I present excerpts from two different teacher meetings. The excerpts provide two instances that show the importance of domain-specific beliefs when teachers consider student mathematics competency while implementing mathematics reform. The first excerpt is from a geometry meeting in which teachers discussed curriculum and instruction issues. The second excerpt concerns teachers discussing alignment of current curricula with the district's content and performance standards. The analysis uses specific assertions as evidence of domain-specific

beliefs. However, in many cases what was said in the meetings provided insufficient evidence alone to make claims about domain specific beliefs. A second analysis of the interviews and WIP session data provides a more comprehensive understanding of domain-specific beliefs. The combination analysis illuminates the following findings: 1) domain-specific beliefs consists of at least two dimensions: the role of abstraction in the domain and the role of the domain's utility for future career and education choices and 2) Domain-specific beliefs about algebra are crucial to teachers' responses to mathematics reform.

Domain-Specific Beliefs Analysis 1: Teacher Meetings

Meeting Example 1: Geometry Meeting

In this exchange, Pamela brings up her concern about students' algebra skills. Using a specific example from her practice, she described how her students understand a mathematical idea, but cannot manipulate equations needed to solve the problem. Characterizing her concern with students' algebra skills as "weak," Pamela solicited her colleagues' insights.⁹

PAMELA: You know what I find out that they, um, they know which ones and they know how to do it. But the thing is, I find out that their algebra skills are very weak.

CURTIS: Very weak.

PAMELA: You know like setting up equations and finding the r [radius variable] or find the cubic?

JOSCELYN: yeah.

PAMELA: You know they have a big problem right there.

JOSCELYN: yeah.

PAMELA: That's the algebra part. (chuckles)

JOSCELYN: right.

PAMELA: When I say they have a hard time, it is not just like you have to get to the answer or the formula. Right? And then part of it is. They don't get that part right. And that is the algebra part. Do you feel that?

JOSCELYN: Yeah. There are some kids who are very weak in algebra, just because they took algebra doesn't mean they know anything about algebra. And I think that is a universal problem throughout the ages.

PAMELA: oh okay (softly)

JOSCELYN: I mean I think there are kids who really don't get algebra.

Joscelyn's response confirmed Pamela's observation that some students' competence in algebra is weak. However, Joscelyn took this observation a step further and generalized this situation as a "universal problem." Joscelyn's assertion suggests an innate quality to student competency in the domain of algebra. Some kids can learn algebra and some kids cannot. Moreover, experience with algebra through a class does not necessarily indicate competency. What is unclear from this exchange is why Joscelyn thinks some kids "really don't get algebra." Is there something unique about algebra that makes it difficult for some students to learn, suggesting domain-specific beliefs? Joscelyn's assertion remained unexamined by the teachers.

Meeting Example 2: Aligning Curriculum with New District Standards

This excerpt is taken from a meeting involving most of the algebra 1 and geometry teachers. The teachers were discussing the "algebra and function" content standard of the district standards. During the discussion, a concern is raised about student competency in algebra. The responses point to domain-specific beliefs about student capacities to learn algebra.

- SARA: My theory is that it is ingrained maturated problem. I think it is maturation.
- ALEX: [That is what I thought too].
- ELLEN: [It is not working.] The kids are=
- SARA: =It is brain development and maturation=
- ALEX: =But it is still an individual thing.=
- JOSCELYN: =Absolutely, it is an individual thing.=
- SARA: =Um hum=
- JOSCELYN: And it doesn't mean they are stupid.
- SARA: No. (in agreement)
- ELLEN: [CPM 4 they are not remembering from one month away from trig. They lose it.]
- JOSCELYN: [I thought so, but I have some students who just can't, can't do algebra.]
- PAMELA: Oh (to ELLEN)
- JOSCELYN: They can do all the other parts of math beautifully, but algebra. And so it's (.) it's a problem area for (.) Some of the kids see it and do fine. And some of the kids are just really totally on a level that they don't get.

Sara's and Alex's comments are examples of beliefs about algebra and a view of learning in this domain as having a unique trajectory, an individual "maturation" process. It is important to point out that they do not suggest students are incapable of learning algebra. In addition, Joscelyn agrees with Alex that learning is an "individual thing." Her comments suggest a belief that students' capacities to learn mathematics are not only developmental, but also domain-specific. She asserts her view, grounded in her experience, that some students can demonstrate

competency (learning) in other domains of mathematics, but not in algebra. She does not consider a lack of competency in algebra as an indicator of low intelligence. Her comments suggest that she believes there is something special about the domain of algebra and the learning in that domain.

These teachers were in the midst of aligning their curricula to the new district-mandated standards. While the public comments made by Joscelyn and Sara suggest domain-specific beliefs about algebra, further evidence is needed to characterize these domain-specific beliefs.

The analysis presented in the next section focuses on domain-specific beliefs documented during individual teacher interviews and the member check "Work-In-Progress" (WIP) session. This analysis in conjunction with the analysis of the two examples above provides a fuller picture of domain beliefs held by some of these teachers that contributed to how they responded to the district-mandated reforms.

Domain-Specific Beliefs Analysis 2: Abstraction and Utility Dimensions

Teachers described algebra as less accessible to students than other domains such as geometry and statistics in two ways, the role abstraction plays in learning the domain and the domain's perceived utility for educational and career choices.

Role of Abstraction

Teachers described the role of abstraction as defining particular domains of mathematics. And, they linked abstraction to students' capacities to learn the domain. In example 2, Sara proposed that student difficulty with learning involved cognitive development and "maturation." In her first interview, Sara also proposed that "maturation" is related to student capacity to

abstract. Sara was discussing her concerns about the graduation requirement and the elimination of the remedial courses. She felt the reforms pushed students who were not “mature enough” to take more mathematics. When I asked her to clarify what she meant by “maturity” she answered:

Mature mathematically and emotionally, I think, run hand in hand as mathematics is abstract. And, things have to connect. And, if you are not able to abstract enough, at some level, you are not going to be dealing with it very well. Some kids freak when I give them some algebra. And they have to do some algebra, because they are okay with the geometry but they are not okay with the abstraction of the algebra and how you apply it.
[Sara, Int.1.2, p. 5]

Sara’s description suggests a difference in “abstraction” among different domains. She describes the “abstraction of algebra” as a stumbling block for students. Whereas, in geometry students are “okay.” Sara elaborated on her beliefs about geometry and algebra during the WIP session.

In geometry we have a little more options. Especially with the course geometry, there is a lot more options, and some kids really come to the fore. And, um, because we move into the, you know, more concrete and manipulate things and what have you. Now, in the IMP, you have to put the equation in vertex form and read off from the equation and I mean... I'm helping some kids in Walt’s class and he's expecting them to produce the vertex form of the equation. They are stymied. [Sara, WIP, p. 7]

Working with the symbolic representations of equations and “reading off” information from those representations is different than what students do in geometry. While Sara did not provide an example of why geometry was “more concrete,” her characterization of the abstraction associated with algebra is clearly linked to symbolic notation, which Sara found very difficult for students to understand or use. Sara made a distinction between the domains of algebra and geometry on the basis of students' capacities to learn those domains and the level of abstraction each domain demands. In addition, she explicitly connected this distinction to her concerns about the district's reforms requiring formal study of two years of algebra for all students.

Joscelyn provides further examples of the role of abstraction. In both meeting excerpts analyzed above, Joscelyn asserted that she had some students who “really don't get algebra” or could “do all the other parts of math beautifully, but algebra.” She further elaborated on the distinction between domains and linked learning to abstraction in both the WIP session and interviews. To illustrate, here is part of Joscelyn's response to the query I posed during a WIP session, “why is algebra a department concern?”

It seems to me, as a math teacher, that algebra, meaning solving equations for x , y , or whatever...that to me seems to be the hardest part, solving equations. And that seems to me the most abstract thing that we do in the math classroom. And I think that's what's the difficulty is the abstraction level. Because when you are working with triangles or you are working with circles or you are doing things that are visual and even making a graph, I think the students are good at that kind of thing, because it is tangible relatively speaking. But then when you get into solving very complex algebraic equations that is kind of like going up a level somewhat. You are leaving the land of the visual, and you

are entering the land of the abstraction or theoretical. And there are a lot of kids that aren't ready for that journey. [Joscelyn, WIP, p. 6-7]

Joscelyn associated student learning difficulties with the level of abstraction involved in two domains of mathematics, algebra and geometry. She was explicit about viewing algebra as “solving equations,” which she believed is the “most abstract thing we do in the mathematics classroom.” She characterized solving equations as “abstract” and qualitatively more difficult than “working with triangles...or circles” or “even making a graph” all of which are geometric representations or competencies.

Both Sara and Joscelyn described geometry as a domain that all students could learn and algebra a domain only some students could learn. They believed that students experience difficulties when required to formalize or codify mathematical relationships into symbolic notation. For these two teachers abstraction is an important dimension distinguishing algebra from geometry.

Distinctions were also made between the domain of algebra and probability/statistics. Probability/statistics was a domain that, although “complex,” all students could learn because it was perceived to be less abstract than algebra. In her second interview, Joscelyn discussed this perspective.

They can understand standard deviation. They get the concept. I mean to me that is a very complex thing. Um, the fact that you've got a central value and you've got two deviations and when you got two deviations on either side you got almost all the data. I mean they really get that. They get the spread of the data. But you give those same kids

an equation to solve and they can't do it. So there are, I'm, as I'm teaching IMP, particularly, I'm understanding that there are all kinds of math. And there are lots of parts of math that the kids, ALL kids, get. And I'm finding that the algebra is really the hardest things for them to learn, for an awful lot of kids to learn.” [Joscelyn, Int. 2, p. 6]

For Joscelyn, learning mathematics was not dependent upon the degree of complexity but on the capacity to formalize mathematical relationships through symbolic notation, manipulation, and representation (e.g. solving equations). Joscelyn believed that if one could abstract, one could learn algebra. And, alternatively, if one could not abstract, one could still learn geometry and statistics/probability. These examples show that the role of “abstraction” is a defining characteristic of beliefs about algebra.

It is more difficult to infer specific characterizations of domain-specific beliefs about geometry and probability/statistics from Sara’s and Joscelyn’s comments because they did not provide as much detail on those domains. However, we can infer from their responses that they did have beliefs associated with those two domains. They described geometry as “concrete,” “visible,” and “tangible.” Joscelyn described the concept of standard deviation as both complex and understandable to students. They characterized the role of abstraction by symbolic representation and notation as distinguishing algebra from those two domains. In addition, these teachers’ beliefs about specific domains and student learning in those domains were connected to their perspectives on mathematics reform. As these teachers contemplated all students taking more mathematics, they differentiated among domains of mathematics by using abstraction, and making algebra the domain of most concern.

Utility Dimension of Domain-specific Beliefs

A second dimension of teachers' domain-specific beliefs was the *utility* of a domain for future career and educational trajectories of students. Martin (2000) found that students hold beliefs about the *instrumental* importance and value of mathematics knowledge.¹⁰ "Instrumental importance" relates to how individuals situate and give meaning to mathematics knowledge in everyday life experiences, socioeconomic attainment, and education goals. In the case of domain-specific beliefs, the utility dimension describes how individuals situate and give meaning to mathematical knowledge of a particular domain in relation to everyday life, educational goals, and socio-economic attainment.

Several teachers described the utility of a domain when discussing their views on the district's new graduation requirement. About 70% of the teachers in the department disagreed with the increase in graduation requirements to three years of college preparatory math. Teachers expressed several domain-specific beliefs, particularly about algebra, when articulating their views about the graduation requirement. For instance, taking advanced mathematics, in this case advanced algebra (which included trigonometry), was only necessary for specific career choices.

You don't need trigonometry to be a lawyer. You don't need trigonometry to be a school superintendent. You know, I could understand if you are going on to college to, to pursue something that you would need a lot more math like in business world, or the engineering world, or the sciences. [Alex, Int. 1, p. 12]

Because not every kid is going to be an engineer, scientist, doctor. And, that the people who are liberal arts major don't need the algebra. They don't need the advanced algebra. I think that the kids who are not going to college should be given a, some occupational approach classes to help them succeed.

[Walt, Int. 2, p. 5]

We keep teaching algebraic skills which are not very well addressed to practical matters. We say it is true. It's not. I mean, there's, you know, an awful lot of algebra that is, is really pointed towards calculus. And calculus has a practical aspect to it. But, it surely doesn't, you know, it's great for a certain engineering skill, but we're not talking about all engineering skills. [Elliot, Int. 2, p. 11]

These three teachers associated advanced studies in mathematics, particularly in the domain of algebra with “professional” careers such as business, science, and engineering. Instead of algebra, students who were not pursuing professional careers or a college education needed to study other kinds of mathematics with a more practical or occupational emphasis.

Joscelyn also linked career choice with advanced mathematics, particularly algebra.

I would venture to say that all of us agree that it [increase in graduation requirement] is a bad decision. And, um we fear what is going to happen next year because we just don't THINK that all the kids are going to pass three years of college prep math. And, then what happens? Do they not graduate? I mean that's a horrible consequence. And we REALLY don't believe that they need three years of college prep math. There are an

awful lot of wonderful jobs out there that do not require three years of college prep math. And, um, we just think it's foolhardy to ask the kids to go through three years of college prep math when they don't need it. And as I said to you before, some of the parts of advanced algebra are so, um::, so far beyond some students.

[Joscelyn, Int. 2, p.15]

Joscelyn believed that the future welfare of her students (finding a job, graduating from high school) might be compromised by the new graduation requirements. Her statement reaffirming (“as I said to you before...”) that some parts of advanced algebra are “far beyond some students” implies domain-specific beliefs about algebra. She referred back to an earlier statement in which she indicated that some students “really have trouble with manipulating symbols and translating situations into an equation [Joscelyn, Int. 2, p. 7].” By requiring more algebra of students, there was a potential for unnecessarily diverting some students from “wonderful jobs” requiring less mathematics or into academic failure. Although it is unclear what she meant by “wonderful jobs,” it is clear that those jobs did not require advanced study in algebra.

While some teachers dismissed the idea that students should be required to take three years of college preparatory mathematics, other teachers agreed students needed to study mathematics throughout high school. However, their choice of coursework for certain students reflected domain-specific beliefs. For example, the department chair, Ellen, suggested the following:

I don't mind three years of math, but three years of college preparatory, A-F Berkeley requirement math, I think is a little bit unrealistic...I think taking a third

year of advanced algebra for our students is stretching it a bit. I can see kids taking a stat/prob third year math or maybe a computer-based geometry graphics or integrated or something you could create for a third year math. A very rich course I think you could create that wouldn't necessarily have to be advanced algebra. I don't know if they've been given that choice at all. I mean I think three years of math is fine. I mean I can see four years of math. I wouldn't mind that. But, not what this has to be. [Ellen, Int. 1, p. 26]

Ellen believed that all students could learn mathematics and should study advanced levels of mathematics. However, she offered two particular third-year courses as alternatives to algebra-based courses. Her statement suggests she held domain-specific beliefs about algebra, geometry, and statistics/probability. Ellen believed it was important to nurture students' mathematical strengths by providing advanced mathematics courses built on domain-specific strengths. Ellen tied the utility of a domain to student education pathways. Ellen's statement suggests that she believed students should focus on different domains (i.e. geometry, statistics/probability, or algebra). Implicit in her statement is a connection between eligibility to top universities and mathematics course content. Since a third year college preparatory course in algebra is a pre-requisite for admission to the University of California (UC), by suggesting that advanced study in the domain of algebra was not for all students she implicitly questioned whether the UC system is for all students, a sentiment expressed by other colleagues. This belief directly conflicts with one of the underlying rationales for the district-mandated reform increasing the graduation requirement.

Teachers connected the utility of particular mathematical domains to specific career and educational trajectories. Many described how the reforms' emphasis on the extended study of mathematics, particularly algebra, was seriously problematic and unnecessary. Some teachers saw algebra as necessary only for students pursuing careers in business, engineering or science or those pursuing a post-secondary education at a top four-year university. The advanced study of mathematics proposed by the district reforms severely limited career options for students. Furthermore, while a few teachers endorsed advanced study of mathematics, they also suggested that not all students should study algebra. Instead, they proposed that some students should study geometry or statistics. The examples illustrate how the perceived roles of abstraction and the utility of different domains for future careers and educational pathways of students are important dimensions of domain-specific beliefs that affect how teachers respond to mathematics reform.

Summary and Discussion

This article documents and describes teacher beliefs about specific mathematical domains that inform how high school mathematics teachers respond to reform. Building on the work of Törner (2002), this analysis revealed two dimensions that teachers used to distinguish algebra from geometry and probability/statistics: the role of abstraction in the domain and the utility of the domain for future career and educational trajectories. Teachers believed algebra was the least accessible domain because it demands abstraction. Teachers characterized abstraction in terms of symbolic notation, manipulation, and representation. They also believed extended study of algebra was not necessary for all students. In contrast, they described geometry and statistics/probability as domains all students could learn because of the decreased role of abstraction and the increased utility of these domains.

Steen (1990) regards abstraction as a “deep idea that nourishes the growing branches of mathematics” (p. 3). He characterizes abstraction in several ways including symbols, logic, equivalence, similarity, and recursion. In contrast, these teachers focused on only one aspect of abstraction characterized by Steen (1990), namely symbol manipulation in the domain of algebra. Because abstraction is such a fundamental idea across the field of mathematics, future research should explore teachers’ understandings of abstraction within and across domains.

The findings about the role of abstraction in the domain of algebra contradict the symbolic precedence model (SPM) described by Nathan and Koedinger (2000a, 2000b). The BVHS teachers’ emphases on symbolic representation and manipulation in algebra as the “hardest thing we do in a math classroom” (Joscelyn, Member Check, p. 6-7) stands in contrast to the beliefs expressed by the high school mathematics teachers surveyed by Nathan and Koedinger (2000a) who viewed symbolic forms of algebraic problems easier for students to solve than verbal “story” problems. As Nathan and Koedinger suggest, there may be a connection between teacher beliefs and the curriculum they use. In that study, teachers used traditional textbooks that emphasized a symbolic precedence model (SPM). In contrast, BVHS mathematics teachers utilized “reform-oriented” curriculum texts that emphasized a verbal precedence model (VPM). Future work should investigate the interaction and influence of curriculum on the development of domain-specific teacher beliefs.

Domain-specific beliefs complicate successful implementation of mathematics reforms and accountability policies that call for increased access to the study of mathematics, particularly algebra (NCTM, 2000; No Child Left Behind, 2002). The domain-specific beliefs discussed here challenge two key components at the heart of both the district-mandated and more recent national standards-based mathematics reforms: *content and equity*.

The findings suggest that algebra has a higher status than other domains of mathematics such as geometry and statistics/probability. The privileged status of algebra is partly predicated on the role of abstraction as characterized by symbolic manipulation and representation. This view stands in sharp contrast to the logic of reform standards where symbol manipulation is viewed as procedural rather than conceptual knowledge. These teachers' narrow view of abstraction in algebra stands in contrast to the field's broader notion of algebra as understanding patterns and generalization, and a view of abstraction as pervasive in all domains of mathematics (Steen, 1990). This finding raises interesting questions about how content is perceived by teachers. What are the roots of the higher status ascribed to algebra? Have teachers developed a view of algebra and abstraction through their own mathematics education? Is this view grounded in their experiences with students? How might this view be different/similar to mathematicians' views of particular domains?

The beliefs documented here also challenge the equity component of mathematics reform stating all students should demonstrate competency in higher-level mathematics. Instead of creating opportunities to pursue particular careers or post-secondary education, several BVHS teachers believed that requiring students to take more mathematics actually narrowed student choice and opportunities to learn. Teachers tied the differential status of algebra to particular career and educational trajectories that have serious implications for the courses offered at the high school level and students mathematical preparation for the future. They also believed that the demands of these reforms may increase academic failure, a consequence that provoked fear and concern.

Ironically, it may have been the recent mathematics reforms and the teachers' use of reform-oriented curriculum that contributed to the development of these domain-specific beliefs

by raising teachers' awareness about different mathematics domains and perhaps increasing opportunities for students to pursue advanced study of mathematics. In the case of Joscelyn, the reform-oriented curriculum IMP was a major influence in helping her see different strengths students bring to different mathematical domains. It could be argued that some progress has been made with the successful implementation of standards-based mathematics reform because teachers no longer believe some students cannot learn mathematics at all. Thus if more students gain competency in domains such as probability /statistics and geometry, this is progress.

The belief that not all students can or should have to learn algebra continues to be debated within the mathematics community and the broader U.S. society (Chazan, 1996; Chazan, 2000; Moses, 1994; 2000; NCTM Dialogues, 2000; Noddings, 1994; 2000). For example, civil rights leader and mathematics educator, Robert Moses, argues mathematics literacy is a civil rights issue – fundamental for citizenship, and critical for economic access and educational advancement (Moses, 1994; 2000). He contends that algebra is the “floor” for mathematics literacy. In contrast, Daniel Chazan (1996) challenged the mathematics education community to think critically about the slogan “algebra for all” as he raised questions about the quality of traditional algebra curriculum, lack of teaching for understanding, and the perceived needs of “lower-track” students and their teachers. Chazan’s perspective echoes the sentiments of some of the BVHS teachers when he stated, “I believe it is wrongheaded to force students to take a class that almost half the students will fail... I think it is not fair to hold out college as the only avenue for successful adulthood (p.475).” The connection to failure and utility of mathematics is evident in both sides of the debate.

Furthermore, the NCTM Mathematics Education Dialogue (2000) document specifically raised the question to the mathematics education community about whether all students should

study algebra. Responses included what should be taught in algebra courses, why it should be taught, and whether all students might benefit from learning algebra. The utility of algebra was a prevalent theme in many responses. Clearly, the utility dimension of domain-specific beliefs is important and generates questions about what makes the domain of algebra so special and controversial. Why do we not have similar debates about geometry or statistics/probability? The findings discussed here suggest that domain beliefs are a key factor in focusing the debate so much on algebra.

Domain-specific beliefs provide an additional arena for teacher education. In support of teachers deepening their understanding of the content, connecting mathematical domains, and understanding how students learn different areas of mathematics teacher education/professional development can also address domain-specific beliefs. In addition, the awareness of differential status placed on particular domains may help teacher educators address issues of equity and content within their professional development initiatives.

This analysis demonstrates the power of domain-specific teacher beliefs and their influence on how teachers respond to mathematics reform. The findings suggest that teachers distinguish among domains and privilege algebra. Two dimensions of domain-specific beliefs warrant further exploration in future research on domain-specific beliefs: the role of abstraction in the domain and the utility of the domain for future career and education pathways. Beliefs about algebra, in particular, are critical and problematic for current mathematics reforms that call for advanced study and increased performance in mathematics. Teachers' domain-specific beliefs need to be central concerns if mathematics reforms about content and equity are to succeed.

APPENDIX A: SAMPLE INTERVIEW QUESTIONS

- What view of mathematics would you want your students to have when they leave your course? Why is this view important?
- What do you think are the **biggest reasons** that students **don't learn** mathematics as well as you, as their teacher, would like them to?
 - Are the reasons the same for students in your advanced courses?
- What **factors** make it possible for students to **learn math** well?
 - Are the reasons the same for students in your advanced courses?
- What are your thoughts about the increase in graduation requirements to 3 years college preparatory math?
- What does this requirement do that is good for students? How is it problematic?
- How does the decision to increase graduation requirements to three years of college prep math affect your work as an individual teacher?

Author's Notes

This work was supported by Center for Mathematics Education of Latinos/as: National Science Foundation, Grant No. ESI-0424983 and the Spencer Foundation. The author would like thank the following colleagues for their helpful comments on earlier drafts of this paper: Judit Moschkovich, Richard Kitchen, Sylvia Celedon-Pattichis, Judy Scott, and Betty Achinstein.

¹ American governance of public schools is primarily at the local state and district level. Governance structures and policy regulations vary by state and district. Often the appointed district superintendent and the publicly elected school board mandate district policies. Public schools must comply with district and state policies, rules and regulations.

² The school's student population comprised of 81% non-White students (e.g. Asian American, African American, Latino, Pacific Islander). Approximately 47% of the students spoke another language other than English at home.

³ Two teachers held "supplementary" or "multiple" credential that enabled them to teach mathematics courses offered at the middle school level which includes algebra and geometry.

⁴ For a full account of collegial relations and department culture see (Aguirre, 2002).

⁵ For six years the department utilized the College Preparatory Mathematics program - CPM (Sallee, Kysh, Kasamatis, & Hoey, 1998) and Interactive Mathematics Program -IMP (Fendel, Resek, & Alper, 1998). Both curricula were developed from NCTM reform documents (1989, 1991). The National Science Foundation funded their development. Furthermore, the teachers made explicit distinctions between reform-oriented curriculum programs and those they deemed "traditional." During the larger study, the department rejected an opportunity to choose a more "traditional" curriculum for their math courses because such an action would usurp their efforts to change instructional practices and improve student learning.

⁶ At the time of the larger study, the state graduation requirements included a two-year minimum mathematics course requirement, one of which had to include the study of algebra topics. The national reform documents are recommendations only. There are no national graduation requirements. In addition, there are no federal-level sanctions if schools do not implement these recommendations.

⁷ The larger study's research questions focused on the interaction and influence of teacher beliefs and department culture norms. Thus formal observation of teacher meetings was a critical data source (See Aguirre, 2002). Other studies utilize data sources other than observations of classroom practice to make claims about teacher beliefs (e.g. Nathan & Koedinger, 2000a; 2000b; Hart, 2002). The researcher recognizes that future studies on domain-specific beliefs should include classroom practice as another data source.

⁸ Strauss (1987) refers to this aspect of coding as "sociologically constructed codes" and is part of the open coding approach of grounded theory.

⁹ All names are pseudonyms

¹⁰ I choose the term utility to avoid confusion between Martin's characterization of "instrumental importance" with the word "instrumental" in the teacher beliefs literature which characterizes a specific view of the nature of mathematics as a set of useful but unrelated collection of facts, rules and skills (Ernest, 1989; Thompson, 1992)

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