

## EXPLORING LATINO STUDENTS' UNDERSTANDING OF MEASUREMENT ON NAEP ITEMS

Cynthia Anhalt  
The University of Arizona  
canhalt@math.arizona.edu

Anthony Fernandes  
The University of Arizona  
azmafam@math.arizona.edu

Marta Civil  
The University of Arizona  
civil@math.arizona.edu

*This study explores Latino students' thinking when solving NAEP measurement problems via task-based interviews. Our study is grounded on a combination of sociocultural and cognitive perspectives in which multiple resources were considered when students represented their mathematical ideas. We discuss themes that emerged from the data, some challenges that students experienced, and the richness in their thinking across three items.*

This exploratory study seeks to complement large scale studies that have been done on the NAEP (e.g. Abedi & Lang, 2001, Lubienski, 2003) to shed light on Latino students' thinking as they solve measurement problems from NAEP. Lubienski (2003) reported that in the 2000 NAEP, the greatest gap between white and black students and between white and Hispanic students occurred in the measurement strand. Our effort was to gain a better understanding of how working-class Latino students thought about some of these NAEP measurement items. The goals of this study were (a) to uncover challenges that selected NAEP measurement items raise for a small group of Latino students (b) to document their interpretations of the problems; (c) to understand their reasoning and communication of their solutions; and (d) to investigate the role that language plays in their thinking process of procedures and concepts, especially if the students' first language is not English.

### Theoretical Perspectives

This study is part of a larger research agenda that looks at the interplay of mathematics, language and culture among Latino students. Our perspective is essentially a combination of a sociocultural perspective and a cognitive perspective (Brenner, 1998; Civil, 2006; Cobb & Yackel, 1996). On one hand, we used task-based interviews, which are typical of cognitively-based studies; on the other hand, our analysis of these interviews is guided by a "situated-sociocultural view of mathematics cognition, language, and bilingual learners" (Moschkovich, 2002, p. 196). Further, the work of Bielenberg and Fillmore (2005) has informed us of critical language features needed for discourse development, such as academic English vocabulary, common academic English structures, and such language functions as explaining, defending, and discussing.

By utilizing task-based interviews, researchers are able to describe and assess mathematical thinking as well as the interplay of contextual and social factors, observed behaviors, and inferred cognitions. The use of language is central to our study in that we are interested in the students' use of language, both everyday and academic language, as they interpret the tasks and explain their thinking; in some cases, our students are bilingual, but more proficient in one of their two languages. Because language structure of assessment items can present added cognitive demands for the students, especially ELL students (Campbell, Adams, & Davis, 2007), it is pertinent to take into consideration the students' multiple resources for their communication of their reasoning in solving the problems. Moschkovich (2002) points out that communication is multifaceted involving gestures, expressions, and objects as resources to simultaneously

communicate mathematical ideas, and they are especially crucial for students who are less proficient in English. Thus, in our approach for this study we focus on the students' use of various resources, including non-verbal and linguistic resources.

### **Modes of Inquiry**

We conducted and videotaped task-based interviews with twenty-eight Latino students in grades 4 through 8. The students were attending elementary and middle schools in predominantly working class / low income neighborhoods. The first phase included 11 students working on various measurement items taken from the NAEP; the second phase had 17 students working on these three problems: (1) "if both the square and the triangle above have the same perimeter, what is the length of each side of the square?" (Lengths of the triangle are given as 4, 7 and 9); (2) a problem in which the student is given cutouts of a triangle and a square and is asked to compare their areas; and (3) a problem asking students to find the area of a trapezoid ABCD (made of a rectangle and a right triangle). The area of rectangle BCDE is given; and the lengths ED and AE are given.

The students first solved the problems independently and then they were asked to explain their thinking in their solutions. After they finished their explanations, we asked probing questions based on their responses. In some cases, the students' interactions with the researcher prompted them to revise their initial solution. The video tapes of the students were first analyzed by problem. We paid attention to the students' thinking that was observable as they worked on the problems independently. We then looked across the different problems for common themes. In the next section we address three of the themes: (1) linguistic demands; (2) reliance on visual cues; and (3) area and perimeter confusion.

## **Results**

### ***Linguistic Demands***

Linguistic complexity can stem from the written form of the problem, especially for English learners. For example, in the perimeter problem (1), the problem reads, "If both the square and the triangle above have the same perimeter, what is the length of each side of the square?" One of the fourth grade students interpreted the "if" statement as "they do not have the same perimeter." When probed, she said "but they do not because it says **IF** (emphasis added)." This child was interpreting the "if" statement as a negation statement, therefore, the square and the triangle could not possibly have the same perimeter. Her facial expression indicated that she was faced with conflict. By the student's interpretation of the written language used in this problem, it is impossible to assess her mathematical understanding of two shapes having the same perimeter.

We found that the students who were successful in solving these problems were able to work at a more abstract level with the measurements given and had moved beyond the concrete level of counting. For example, a successful 6<sup>th</sup> grade student simultaneously represented his manipulations of the concrete shapes in problem (2) with rectangles and with the equation  $2P=2N$  ("P" represented the triangle and "N" represented the square). He reasoned that if  $2P=2N$ , then half of each rectangle is the triangle P and the square N, therefore,  $P=N$  and the areas were equal. Successful students were able to explain their reasoning with more linguistic precision than those who struggled, who tended to use lots of pronouns with no clear referents.

### ***Reliance on Visual Cues***

Some students relied on the drawings as their key source of information, yet these were not drawn to scale. A few students requested a ruler for finding the lengths of the sides for some of the figures. Regarding the first problem (finding the length of one side of a square with the same perimeter as a triangle), most students were able to arrive at the correct solution, but those that did not arrive at the correct solution relied on approximating the side of the square visually to be 4 since it looked like one of the sides of the triangle that was labeled 4. In this same problem on perimeter, some students who were incorrect relied on the use of grids to find the perimeter, and understandably, this strategy can be tied to reform curriculum that emphasizes meaning making, but in this example, we found that such drawings by the students interfered with their reasoning. In fact, this reliance on the grid deserves further attention, as we found it to be not always helpful and part of the confusion with area and perimeter that we address next (also, see Kamii & Kysh, 2006, for their insights on counting squares on a grid to find area of shapes).

### ***Confusion of Area and Perimeter***

We consistently found that when the problem presented both square units for area and linear units for lengths of sides of a figure, that students confused the concepts and procedures for finding the area or perimeter. In the first problem on perimeter, most students had no trouble with finding the solution since the measurements were given in linear units. But multi-step problem (3), involving both linear and square units, was more challenging. The 8<sup>th</sup> and 6<sup>th</sup> grade students who were able to solve it, effectively combined their knowledge of the formulas with the given information to solve the problem.

### ***Closing Thoughts***

We found that some of the challenges that students faced were related to their interpretation of the problem due to the linguistic demands, their confusion of area and perimeter, the lack of problem-solving strategies used, their reliance on concrete and visual approaches, and their representations of the mathematics. However, we found that some students were resourceful in their use of tools and problem-solving strategies and in drawing on previous knowledge, but could not always clearly express their thinking about the problems. Our findings show that most students were able to participate meaningfully in mathematical discourse at some level about their solutions and convey their thinking through various resources.

### ***Acknowledgment***

This research was supported by the National Science Foundation under Grant No. ESI-0424983.

### ***References***

- Abedi, J., and Lord, C. (2001). The language factor in mathematics tests. *Applied Measurement in Education*, 14, 219-234.
- Bielenberg, B., & Fillmore, L. W. (2005). The English they need for the test. *Educational Leadership*, 45-49.
- Brenner, M. E. (1998). Adding cognition to the formula for culturally relevant instruction in mathematics. *Anthropology & Education Quarterly*, 29, 214-244.
- Campbell, A., Adams, V., & Davis, G. (2007). Cognitive demands and second-language learners: A framework for analyzing mathematics instructional contexts. *Mathematical Thinking and Learning*, 9(1), 3-30.

- Civil, M. (2006). Working towards equity in mathematics education: A focus on learners, teachers, and parents. In S. Alatorre, J.L. Cortina, M. Sáiz, & A. Méndez (Eds.), *Proceedings of the Twenty Eighth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, 30-50). Mérida, Mexico: Universidad Pedagógica Nacional.
- Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31*, 175-190.
- Kamii, C., & Kysh, J. (2006). The difficulty of “length x width”: Is a square the unit of measurement? *Journal of Mathematical Behavior, 25*, 105-115.
- Lubienski, S. (2003). Is our teaching measuring up? Race-, SES-, and gender-related gaps in measurement achievement, *Learning and teaching measurement 2003 yearbook* (pp. 282-292). Reston, VA: National Council of Teachers of Mathematics.
- Moschkovich, J. N. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, Special Issue on Diversity, Equity, and Mathematical Learning, 4(2&3)*, 189-212.