

**Working Towards Reform in Mathematics Education:
Parents', Teachers', and Students' Views of "Different"¹**

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This essay is a reflection on several aspects related to my encounters with the concept of reform in mathematics education. I start with an exploration of the question of what is reform, grounded on my work with teachers in a project aimed at promoting reform. I focus on two aspects that seem to be present in most approaches to reform—group work and mathematical discussions—and in particular on implications related to equity and the participation of all students when implementing these practices. This takes me to my research in low-income, Latino communities and my efforts to bridge home/community mathematics and school mathematics. I address the notion of valorization of knowledge and the reactions to “difference,” particularly in the context of immigration. I illustrate aspects of these reactions with data from parents, preservice teachers, and students.

What is Reform?

I always thought reform was this magical place I was headed for and I couldn't wait to get there but now I have conceptualized that I'm not going to a place, I'm involved in an incredible process that will never end. [Journal entry, teacher participant in a project aimed at promoting reform mathematics, year 2]

In this first section I present some of my musings based on my first experiences with “reform.” My first reaction to the symposium theme – mathematics education reform and resistance—was “what? Reform, again? Or shall I say, still?” This reaction lasted a few seconds. As soon as I started reflecting on my experiences with reform over the years and where I see things now, I wondered, “have we made ‘progress’?” And by “progress” I mean in terms of the vision for mathematics education that is outlined by NCTM (1989; 2000).

About twelve years ago I co-directed a teacher enhancement project that had as a goal “reform in mathematics education.” I will refer to this project² as “Reforming Mathematics Education” (RME). During several years I had an opportunity to work with about 100 teachers (grades 3-8). I engaged in conversations about teaching and learning mathematics with them and I also visited the classrooms of several of them. My impressions on what reform seemed to be at the time include: rejection of a textbook (the choices then were mostly the more traditional series); hands on activities; group work; process versus answer. As I wrote in the journal I kept at the time:

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Smorgasbord of activities; doing “pattern blocks”; no apparent road map. Where is the math? Teachers doing activities: is the choice of activity based on the math content or on the attractiveness of the activity?

The participating teachers were at very different places in their thinking on reform. Some had been involved in one of the district mathematics projects and were very knowledgeable of (and big proponents) the materials of a nationally well known mathematics educator. Several of them were in fact facilitators for what I will refer here to as the “Math Project.” To them, the work and materials of this mathematics educator showed them the road to follow towards reform in mathematics education. A feeling of “already being there [that is, in the world of reform]” or “knowing more than the other participants” was often present in their interactions, as the quote from one of these teachers shows, “I am feeling like a ‘reform in math education’ drop in the ocean of educational unawareness.”

Other teachers took the whole reform idea in stride and tried to make sense of it in terms of their own experience and context. For example, Leticia, on the second year of being in our project wrote:

Although there was more hands-on participation, it was important for me that the students not view the activity as a game. It was crucial that students apply the math involved in the activity or better yet, that they see the Math.

I am still convinced that you need to have a balance of skills and concepts. I’m still more comfortable working with algorithms as a learner, but as a teacher I see the importance of conceptualization. I still need to understand Math. [June 1995]

When asked in a journal prompt to specifically reflect on the expression “reform mathematics”, she wrote:

Reform in Mathematics

I’m not sure what that is. I hear it a lot in this RME project, but I’ve yet to see the “reform.” I still hear value judgments on what we should be teaching and how, however the applications are not being implemented. There has never been

an instructor who has taught us how to use the manipulatives. Part of the problem I feel with the manipulatives is our illiteracy in using them. It's just as easy to get all our buckets of manipulatives and put them away in our closets, as it is not to turn on your computer. Manipulatives do scare some of us. Yes, all the activities have involved using the manipulatives for tactile purposes, such as in graphing, but not for much else. ...I know there's Algeblocks, base 10, etc. but how do you use them. **Is reform self-teaching?** (Emphasis added)

In her second year in our project, Karen, a teacher who had participated in professional development experiences as part of the "Math Project" that I referred to earlier, wrote the following in response to the prompt, "What perspectives have changed for me about teaching mathematics this year?"

One thing that has changed about my teaching of math this year is that I'm more selective (or try to be) about the activities I choose. As a matter of fact, I'm less focused on "activities" per se and more focused on the mathematical concepts embedded in the activities. I have become less of a constructivist and I guess more traditional in a sense. For a while, in teaching math I presented activities with very little structure, allowing students to explore concepts and somehow hoping that the students would construct the "rules" or develop the algorithm on their own. I knew how to teach the traditional way to teach a concept, the way I learned, on the board and drill and practice and I knew that that way was very "unpopular" so rather than do something unpopular I just hoped the kids would figure out how to do it. This year I recruited the co-teachers³ who developed lessons, modeled or collaborated with me to help me figure out ways to develop a concept. So, in summary my students this year did more of less. We covered less material better. [June 29, 1995]

To me this quote shows a move away from the smorgasbord approach to reform; let's put out a collection of activities and manipulatives and let's hope the students "get it." This is consistent with Leticia's concern: is reform self-teaching? Instead, Karen

³ As part of the support provided by our project "Reforming Mathematics Education" we had experienced classroom teachers in residence at the University, whose role was to work with the teachers in the project during the year.

seems to be wondering about how to use these activities to develop a concept and to provide a more coherent program of mathematics education to her students. The tension among the participating teachers around the issue of reform versus traditional—and what each of these terms mean—was very noticeable through the institute, as Karen’s journal entry reflects:

When we talk about philosophy, or maybe not even philosophy but a teaching issue or teaching a concept (when, how, etc), we seem to divide into 29, 28 (or however many we are) separate “camps.” We have the hard-core constructivist camp who rolls their eyes at the middle of the road camp who rolls their eyes at the mere thought of teaching “skills” while those who do teach “skills” are trying to look like they don’t. There is tension in the air that is thick enough to cut and I think we’re all feeling somewhat defensive. Something is wrong. (...) I see us battering over whether to teach the algorithm for multiplication of fractions or to teach for meaning. I think it’s a trivial battle and in fact we probably agree more than we differ, but we focus on our differences rather than our solidarity and commitment to try new ways and listen to new ideas. If we are battling among ourselves-us- a group of teachers who applied to be in a program that addresses reform-Who may I ask is the enemy? I think it’s us! Nothing will ever change if each of us thinks we have the answer. [7/6/95]

In summary, superficial (but unfortunately, widespread) interpretations of “reform mathematics education” seem to include: use of activities, group work, manipulatives, mathematics talk (I’ll come back to this one), teaching for meaning (what does this mean exactly?), the process is what counts, not so much the answer. Do we think that this was happening 10 years ago, but that it is now over? No, of course not. Here is what Matthew Ondrus (Matt), a CEMELA⁴ post-doctoral Fellow (Ph.D. in mathematics), wrote recently

This is just an informal observation from being in a number of middle school classes. Those of you in the math education business have probably thought about this long before I did, but (given my background) it was somewhat eye-

⁴ CEMELA, Center for the Mathematics Education of Latinos/as, is an NSF-funded Center for Learning and Teaching, under Grant No. ESI- 0424983. The views expressed here are those of the author and do not necessarily reflect the views of NSF.

opening for me to realize this. Teachers of ELLs [English Language Learners] or Latino students hear quite often that they should incorporate manipulatives and group work, contextualize problems, ask students to write, make vocabulary lists, and do various other things. These suggestions may be helpful, but my observations so far suggest that these techniques, by themselves, may amount to little more than another way of doing the same old thing. Rather, teachers must try to deeply understand how students are thinking. Of course, this is much harder than simply handing out manipulatives to students, and it may be even more difficult if the language and background/culture of the students is different from that of the teacher. [April 25, 2006; e-mail communication]

Matt's observation serves as a transition into the issue of reform and equity. I have often heard that using group work and manipulatives are particularly appropriate pedagogical strategies for "minority" students. My argument would be that these seem to be appropriate strategies for ALL students, when successfully implemented. But of course, that is the issue: what do we mean by successful implementation? Do these teachers of Latino students/ ELLs that Matt is referring to have the tools to make it successful? Judging by what Matt observed, this does not seem to be the case. I agree wholeheartedly with Matt on the importance of the need for teachers to "deeply understand how students are thinking." I will get back to the importance to paying attention to students' thinking after the next section on issues related to mathematical discussions and group work. Matt also hypothesizes that doing this "may be even more difficult if the language and background/culture of the students is different from that of the teacher." Why? Should it be or are we using it as an excuse?

Mathematics Talk and Group Work

One important feature of reform is the concept of discourse, "math talk," encouraging students to talk mathematics. What are the mathematical demands of doing this? As a third grade teacher shared with me, "I know how to let students play with language but I don't know how to let them play with mathematics." And what does this look like in classrooms where the children come from different patterns of discourse

(and/or different languages)? Lampert, Rittenhouse, and Crumbaugh (1996) seem to dismiss the potential influence of differences in arguing styles among different cultural groups, when they write, “although we come from different cultural backgrounds, we learn how to respond to such disagreements in ways that accomplish multiple goals, including preserving relationships with people who make assertions that we believe to be unreasonable” (p. 740). Lubienski (2002) addresses the issue of different dispositions towards discussions in mathematics among high and low SES students. She writes, “whereas the higher SES students seemed to approach the problems and discussions with an eye towards the overarching, mathematical ideas I intended to teach, the lower SES students more often became deeply engaged in the context of the problem at hand and missed the intended mathematical point” (p. 116). Cooper and Dunne (2000) illustrate some of the problems that occur when students (particularly working class students) “import their everyday knowledge when it is ‘inappropriate’ to do so” (p. 43). In my own work with preservice teachers I did notice that often students who had been less successful (by traditional standards) in mathematics, tended to be the ones who paid attention to the context of the problems and tried to make sense of them from a real world point of view (but I did not look at the data in terms of SES at the time). I come back to this point of making connections to everyday life later in the paper.

What are the implications of the emphasis on discourse for the participation of all students? Closely related to the emphasis on discourse is the emphasis on group work, as a means to encourage mathematical discussions. And closely related to these issues is the concept of status: how students perceive others and themselves in the classroom plays a role in these reform approaches that try to open up the discourse patterns. As Lampert, Rittenhouse, and Crumbaugh (1996) write, “children do not readily separate the quality of ideas from the person expressing those ideas in judging the veracity of assertions” (p. 740). In Civil, 2002b and Civil & Planas, 2004, we provide several examples of the constraints to the participation of some students. These students were the less popular (judged by their accomplishments at sports) and the non-GATE (Gifted and Talented Education) students in the Tucson case, and the immigrant students and local students of Gypsy origin in the Barcelona case. I should point out that in the Tucson case, there were

29 children, 19 of whom were Latino/Hispanic, 5 Anglo, 4 African American, and 1 Native American; there were 7 students in GATE, 4 of whom were Anglo.

Students' perceptions of where they stand in the class are likely to play a role in their willingness to engage in mathematical discussions. Lampert, Rittenhouse, and Crumbaugh (1996) provide an insightful discussion of possible interpretations for why the fifth graders in their study seem reluctant to engage in mathematical discussions. They wonder whether this could be related in part to their age. The students in my own work (Civil, 2002b) were also fifth graders; the students in Planas' work (Civil & Planas, 2004) were in high school. And, I will add, that I have had similar resistance to speaking mathematically in public among some of the preservice elementary teachers I have worked with. The most memorable case was that of Carol (see Civil, 1998, for more on small group discussions), who wrote in her journal:

Again, I am concerned about this nontraditional method of teaching. It seems to be a good thing for group cooperation and for higher ability math solvers, but not for people like myself. (...) What happens in the group dynamics is that those who understand, have background knowledge, etc. get better and people like myself get worse. I think it's a lot to ask of myself (at 24) and kids, very young ones, socially to appear as a constant failure to his/herself, peers, the teacher.

Carol was a student who struggled in this class. She tried to make sense out of the discussions but most of the time preferred to work alone. As she wrote,

I try to understand and in class, I listen and ask questions but most of the time I have absolutely no idea what is going on. And what my peers say to me sounds like a dialect of the Alaskan Eskimo.

Recently, Núria Planas shared with me the following comments about group work, from high school students (ages 14 to 16) in Barcelona. These comments show students' resistance to the idea of working in groups with a particular focus on "local" students (that is, students born and raised in Barcelona and not children of immigrant) and immigrant students (includes children born in Barcelona to immigrant parents). (All the comments below have been translated from Spanish or Catalan into English.)

Local student (Catalan girl – high achiever):

They put us in small groups and they say that this way we will learn more mathematics, but the real reason is that they do it so that those from outside get a chance to practice our language. I don't think this is right because I think that these decisions should be based on the mathematics.

Immigrant student (Moroccan girl- arrived the year before):

I do not like to work in the math groups because I cannot concentrate; everybody talks and I cannot think. Here they do it this way, but it can be done different ways, with more silence.

“Immigrant” student (Moroccan boy born in Barcelona):

In the afternoons (at the mosque) we listen to the math explanations, and in the mornings (at school) we listen to other students. The teacher is there but he is not there because we cannot ask him. It's rather funny.

Local student (Catalan boy):

We spend the day discussing the problems, writing outlines on the board and going back to discuss and talk. I think that we write very little because they [the immigrant students] prefer talking to writing. I would like to have my own set of written notes.

Immigrant student (Chinese girl- Arrived six years before):

I try to have my notes for mathematics like I do for other subjects. But if I am writing, I cannot talk at the same time. We write very little, and I don't know why that is.

As Planas notes, it seems like the local students think that these pedagogical decisions are implemented to help the immigrant student. Yet, the immigrant students do not find these decisions beneficial for themselves.

Building on Students' Thinking: My Transition to Funds of Knowledge

I would like to get back to a focus on understanding students' thinking, or as I like to think about it when I work with preservice and practicing teachers, children, and parents, to the idea of listening to what they have to say. To me, this is key to teaching, listening to students' ideas about mathematics and knowing what to do with that

listening, beyond the “thank you for sharing,” that I have sometimes witnessed in some “reform” classrooms. I am fascinated by trying to understand how students think about mathematics. It is probably this curiosity that guides my approach to teaching. I encourage students to use their own methods to solve problems, to make sense of the problems in their own terms. Often, when I do this, students who did not feel successful with the more standard approaches, offer very elegant solutions (or at least, I thought they were elegant!). As Vicky, a preservice elementary teacher, wrote in her journal, “there is hope yet when I can legally use my methods to solve a problem.” What I noticed was that the students who had been unsuccessful (by traditional school standards) often tried to make sense of the problems by making connections to their everyday experiences (see Schoenfeld, 1991, for a thought-provoking essay on the suspension of sense-making when students enter a mathematics classroom).

This idea of connecting (or not) to everyday life experiences brought me to the literature on situated cognition and street math (Brown, Collins, & Duguid, 1989; Lave, 1988, 1992; Nunes, Schliemann, & Carraher, 1993). In particular the notion that the context in which a task takes place affects performance intrigued me, as I was seeing evidence of this while I was teaching these mathematics content courses for preservice elementary teachers. To me a question became, “how can we build on students’ knowledge and experiences in everyday life, in such a way that these become relevant and useful for the teaching and learning of school mathematics?” My involvement in the Funds of Knowledge for Teaching (FKT) Project allowed me to explore this. (For a comprehensive account of this project, I refer the reader to González, Moll, and Amanti, 2005.)

A primary goal of the FKT project was the development of teaching innovations that build on the background, knowledge, and experiences of students and their families and community. This approach to teaching and learning has many parallels to the idea of place-based pedagogy (Gruenewald, 2003; Long, Bush, & Theobald, n.d.). The emphasis in FKT was on community knowledge. More recently, in our work in CEMELA, we are looking at ways to bring in community knowledge and critical knowledge together in our approaches to teaching mathematics (along with Classical Mathematics). Gruenewald’s

(2003) article on blending “critical pedagogy” and “place-valued education” into “a critical pedagogy of place” is particularly relevant to our current efforts.

The Funds of Knowledge for Teaching was a collaborative research project between university faculty and teachers working in schools where ethnic, racial and language “minority” students were in fact the majority (primarily Latino students (mostly of Mexican origin), but we also worked in schools with Yoeme Indian and African American students). All schools are in working-class / low-income neighborhoods. The basic premise behind these teaching innovations is a rejection of the deficit model for the education of students. Instead, teachers in this project use a participatory approach to instruction in which students and often their family members take an active part in the development of learning modules. A key aspect of the project is hence to learn about these students’ (and their families’) knowledge, experiences, and skills (that is, what we refer as the “funds of knowledge”). The teachers do this through ethnographic visits to the home of some of their students. Questionnaires on the family structure, parental attitudes towards child-rearing, labor history, and household activities, as well as a child’s questionnaire (to learn about his/her interests and participation in activities in the house, community), allowed the teachers to uncover the Funds of Knowledge in the household.

Implications for Mathematics Education

The Funds of Knowledge for Teaching project (and later on, project Bridge⁵, which was specifically focused on mathematics) allowed me to explore issues related to developing teaching innovations that on one hand would build on students’ experiences with mathematics in everyday life, while on the other hand, would allow us to engage students in the kinds of mathematical explorations that I describe as “mathematicians’ mathematics” (see Civil, 2002b, for a description of different forms of mathematics). As a mathematician / mathematics educator, I was attracted to the idea of “mathematics for

⁵ These two projects were funded by The National Center for Research on Cultural Diversity and Second Language Learning and by the Center for Research on Education, Diversity & Excellence (CREDE), through the Office of Educational Research and Improvement (OERI) of the U.S. Department of Education, under Cooperative Agreement No. R117G10022 and PR/Award No. R306A6000. The contents, findings and opinions expressed here are those of the author and do not necessarily represent the positions or policies of OERI, NIEARS, or the USDoe.

the sake of mathematics.” Yet, at the same time, I was aware that in many cases, the schooling experiences for low income, Latino children did not make connections to their everyday experiences and knowledge. Thus from an “ethnomathematics” point of view, and most importantly, from an equity stance, I wanted to develop teaching innovations that would reflect these students’ and their families’ knowledge. This lack of connection to the reality of these students is certainly not unique to my local context. It applies to other low-income communities, to other children of color, and to the rural context, as Long, Bush, and Theobald (n.d.) point out.

Of course, these innovations were to take place in the context of school mathematics. That is, in all the cases (at least in the first few years of this work), the students in these innovations had mostly only experienced traditional approaches to the teaching and learning of mathematics. Certainly, the students were not the only ones confronted with change. These innovations also pushed us to rethink what mathematics we should be teaching and what we see as “valid” mathematics. I illustrate some of these issues through three brief examples based on the work with a fifth grade teacher.

Money Module – Take 1

This teacher and I (and one other teacher) had collaborated at a different school around a module on the topic of money (see Civil 1992 for a detailed account of this module). The idea for this module came from a household visit in which the teacher learned about the keen interest that her student had on collecting foreign coins. This event plus the fact that many students at that school had relatives in Mexico (near the border) and were used to travel back and forth between the two countries, hence using two currencies, made her think of developing a learning module around the theme of money. A third grade teacher expressed an interest in jointly developing this module. We planned a series of mathematical activities around the idea of currency, including the creation of a currency for each of the two classrooms; they would make products to sell (“cascarones⁶” in the fifth grade class; paper flowers in the third class) and they would have a commercial exchange between these two classrooms.

⁶ A “cascarón” is a confetti filled eggshell attached to a colorful paper maché cone base.

The two teachers and I held several planning meetings brainstorming the mathematical potential of the module. But what really happened in each of the classrooms was quite different from what I had envisioned or expected. Although some of these ideas were present in the activities, overall the money module focused on children discussing social issues in relation to money (such as welfare, food stamps, buying a car or a house) in the third grade class and on researching topics such as “money, power, and politics” or “foreign currencies”, in the fifth grade class. Yet, in terms of our (or should I say my) mathematical agenda, I did not feel that we succeeded in exploring the potential in this module. I think there were multiple reasons for this “lack of mathematics,” including 1) our views as to what we count as mathematics (I will get back to this point later in the paper), 2) what I have termed elsewhere (Civil, in press) as a need to preserve the purity of the funds of knowledge, and 3) the teachers’ limited experience with innovative approaches to the teaching of mathematics. As the third grade teacher said in an interview reflecting on our attempts to bring in the mathematics in the money module,

I am very aware of my lack of knowledge in math education, period. And I think that’s what inhibited me, not allow me to carry it further, but yet the philosophies are parallel [this is in reference to a prior discussion comparing approaches to the teaching of literacy and the teaching of “reform-based” mathematics], and that’s important to realize that. So, now that I understand that the philosophies are really parallel, that learning occurs when it’s authentic, when it has something to do in the child’s life at that point in time (...) and I think that’s very important in both the literacy and the mathematics, but I had more training on how to do this in literacy and I have not had the training on how to do that in mathematics. (...) So, when you came in, that was the support, the source that I could tap. (...) When Pamela [the fifth grade teacher] and I met informally to discuss it, again most of the discussion was on literacy, we both felt we had more expertise in that area; on the mathematics, it was “well, let me ask Marta, let’s see what Marta does.” And you had to be there for us to even think about these issues.

What I learned from this experience was that teachers needed to have a chance to experience for themselves what “playing with mathematics” may be like. Although “superficial” uses of mathematics may be easily available (counting, measuring, simple arithmetic...), other features of mathematics, such as reasoning, abstracting, generalizing, using the language of mathematics, may be more elusive and hard to make them emerge from the context.

Money Module – Take 2

A year later, with the fifth grade teacher having moved to different school, we decided to replicate the money module, but because we both agreed on focusing more on the mathematics, we introduced the topic with the following problem:

I gave \$1 to the cashier and he gave me back 3 coins change. How much might I have spent?

We chose this problem because we think it opens up the way to some mathematics that we would expect in the context of mathematicians' mathematics--search for patterns, what if...?, what if not...?, generalization. Our teaching innovation is not only about the content of the mathematics being taught but also about the pedagogical approach. Hence, in using problems such as the one given above we wanted to encourage students to work in groups, talk about their findings, look at the different combinations, list them and look for a pattern. Yet, this problem was presented in the context of a traditional approach to the teaching and learning of mathematics that was the norm at that school. Many students were uncomfortable with the proposed problem because it was not clear what was expected from them. It was not a problem that conformed to what many students had grown used to in a school mathematics class. For one thing, there is more than one answer possible; also, one is expected to spend some time on the problem. Yet, the students brought to this task the behavior that characterizes much of the culture of traditional school mathematics (see Davis, 1989). Many of them never came close to any kind of “enjoyment” in looking for patterns and in sharing their findings with their peers. Instead, they wrote down two or three combinations and considered it done. Only one student really engaged into this problem... and who was this student? One of the GATE students. Through this brief example I want to illustrate the kinds of tensions that

occurred as we tried to develop a teaching innovation. The change that we envisioned implied a radical departure from the kind of mathematics learning and teaching that most of these children had experienced up to that point. In my vision of change I wanted to somehow combine what we view as two different kinds of mathematics (mathematicians' mathematics and everyday mathematics) in the arena of yet a third kind, school mathematics.

The games module: "What does this have to do with math?"

The year after, we continued pushing for this vision, with yet a different group of fifth graders. Some of this work is described elsewhere (Civil, 2002b). What I want to highlight here is the resistance to change that we encountered among the students and the role of the social context. In an attempt to build on students' interests while at the same time pushing for the mathematics, we implemented a module on the topic of games, where the culminating product would have students working in groups to create a game. As we started the discussion on what makes a game a game and on what they knew about games in general, one of the students said (in a complaining tone), "what does this have to do with math?" This is indeed a very good question! As we addressed his question, the discussion on games led to narrow images of what mathematics is: counting and arithmetic. We introduced several games that are based on probability to develop their understanding of that topic. For example, in one game players put counters on a "number line" numbered 2 through 12. They roll two dice and add the two numbers. If they have a counter on that number, they remove it. The first one to remove all of his or her counters wins. An understanding of the probabilities of the different sums when rolling two dice would inform a strategy on how to place the counters. Some students did realize this; others either did not or seemed to have forgotten it by the time they developed their games, as was the case of "slam dunk" a game developed by two boys.

"Slam Dunk" is a board game with a basketball theme. It is a game for two players. Each player rolls two dice and moves his/her basketball player along a numbered strip. The strip is numbered 2 through 12, and the player can only move in that order, that is: first he/she needs to get a sum of two on the dice, then a sum of three, then a sum of four, and so on. Once one of the players reaches 12 ("if that ever happens" as one of the

developers of this game said in the presentation to the class), they then have to roll the dice and try to get the total indicated by a card in their basket, in order to win the game. This game immediately caught my attention because it built on the counter game I just described and on similar dice games that we had investigated as part of our exploration of probability. During their presentation to the class, they chose two of their classmates to play a few rounds of the game to show how it worked. They called on two boys who rolled the dice a few times without ever getting a two. Brandon, one of the creators of the game, said, “low numbers are hard to get, it gets easier for the higher numbers.” The game went on and on and finally, when one the players had reached just 4, we stopped it because students were getting restless (and we were too!). What caught my attention in this game was its mathematical basis. It is not clear to me what use they made of the fact that they knew that “lower numbers are hard to get.” When I probed Brandon on this, he seemed to know why lower numbers were harder to get (and also numbers at the high end): he was aware of the different combinations, and how for numbers such as 6, 7, 8 there were more combinations than for numbers 2, 3, or 11, 12. He said that this was the reason why they made the game the way they did it. I wonder whether they shared with me the impression that this game could become quite tedious; or maybe it is the case that we just had different concepts and levels of tolerance for “tediousness”; or maybe, they had not quite made the connection between how the sums of the outcomes in the two dice work and their effect on this game. Interestingly, when students (anonymously) commented on the different games, only one student was critical of the game (on the basis that it could get boring because of how long it may take to get a certain result). This was in fact one of the most favorably evaluated games. I believe that this was in large part due to who Brandon was in the class. Brandon was one of the popular students: he was a basketball player; just the fact that the game had to do with basketball was enough for many of the students to evaluate it very positively, as basketball was very important at that school. Furthermore, Brandon was in GATE. I have addressed earlier how a student’s status in the class may give or deny his/her voice (see Civil, 2002b; Civil & Planas, 2004).

Issues to Ponder

The three examples highlight different (although related) issues as we tried to implement mathematics teaching innovations. The first example (Money Module – Take 1) touches on several issues that I have written about elsewhere, in particular the difficulties of developing mathematical learning experiences that while being true to the context (in this case, explorations around money as guided by the children’s interests), are also true to our mathematical agenda (see Civil, 2002a; in press). These difficulties, I argue, have to do largely with our values as to what we count as mathematics, as well as our own academic training that may make it harder to uncover the mathematics in everyday contexts (González, Andrade, Civil, & Moll, 2001). As a teacher in one of our study groups once asked, “if you have too much school mathematics, does it erase our practical mathematics?” In our work we have found the pedagogical transformation of community knowledge into school mathematics learning opportunities to be a non trivial endeavor. Certainly, having a teacher with a deep understanding of mathematics, as well as experience with and knowledge of the practices in the community is an asset, as was the case of the middle school teacher who developed the “Build your dream house” module (Ayers, Fonseca, Andrade, & Civil, 2001). These findings concur with Long, Bush, and Theobald (n.d.) when they write, “Using the local place, however, requires sophisticated pedagogical work on the part of the mathematics teacher” (p. 7), and later on “placing mathematics in authentic local contexts is not a simple task” (p. 9). They end with a call for the need for mathematicians, mathematics educators and rural sociologists to work together. In our work, the need is similar: mathematicians, mathematics educators and language / culture experts in the Latino context need to work together.

The second example (Money Module – Take 2) addresses the notion of classroom norms—social and sociomathematical (Yackel & Cobb, 1996). The particular innovation we were trying to implement at that point was one more along the lines of “standard” reform mathematics, in terms of using open ended problems that would promote mathematical discussions. We viewed that as a first step towards bringing in modules that would be more community based. The main obstacles we encountered were with the students’ resistance to engaging in discussions about mathematical problems. As the teacher explained, they were used to worksheets in mathematics and a discourse around

right and wrong answers (see Civil, 2002b, for more on the issue of changing classroom norms). Also, for a thought provoking article and an international perspective on this issue of norms and valorization, see the study by Planas and Gorgorió (2004) in a high school mathematics classroom with a large number of immigrant students.

The third example (the games module) highlights the issue of what students are willing to count as valid mathematics: Do students view the mathematics in these teaching innovations as “real mathematics”? Students may have indeed been involved in rich mathematical opportunities but if they do not see what they did as the mathematics they should be learning in school, or if the connections to what they may expect to see in the next grade are not made, are we helping these students? As I reflect on our efforts to try to bring change to the teaching of mathematics in these classrooms, I find Spradbery's (1976) work particularly relevant (even though it is 30 years old!). Spradbery refers to a group of sixteen year old students unsuccessful at school mathematics. Outside school, some of these students kept and raced pigeons. The author goes on to describe some of the mathematics embedded in this practice, and then writes:

Although the mathematician may regard certain aspects of pigeon-keeping (along with many of the other daily activities of children) as being ‘mathematical’, such knowledge appears to have little value or status in the classroom. For ‘Maths’ to be ‘Maths’ (or ‘proper Maths’, as a number of children described it) it has to be separated from other everyday knowledge. (p. 237)

Spradbery (1976) describes the opposition among students—who had so far failed at school mathematics—towards an innovative curriculum that was intended to be more liberating by presenting situations for which the students were encouraged to use their own intuitions and knowledge. In our work, we also had students questioning whether what they were doing was mathematics. The games module is one such example. When we started a topic on geometry (after the games module), students were asked to brainstorm in their groups what the word “geometry” meant to them. One of entries read “listening to teachers tell you that geometry is everywhere.” Reading this comment, I wonder what students really think about the claim we often make that mathematics is everywhere. Students (particularly at 10 years of age and older) can be very skeptical.

In summary, what mathematics should students be doing? In the reform discourse there is reference to the use of “authentic tasks,” what, however, is meant by this? Who determines what is authentic and for whom? Should students engage in mathematically rich investigations and try to recreate in the classroom some kind of a mathematicians’ culture? Or should students be engaged in thematic approaches to learning, where mathematics is embedded in the context of a project? If so, who determines what kinds of projects to use? Should mathematics in school try to build on or even mirror outside-school mathematics in an effort to make mathematics learning relevant to the students’ everyday experiences? And who determines what is relevant? Should mathematics be studied in the context of issues that are socially and politically relevant to the community where the students live?

I have been pondering about these questions for many years... and I know others have. As I continue reflecting on my work and that of others concerned with similar issues (i.e., the inequities in the mathematics education experiences, particularly in some communities), the whole issue of valorization of knowledge that I have referred to throughout this paper, is what strikes me the most. Abreu (1995, 2002) has written extensively on the concept of valorization of mathematical practices and on the potential for conflict situations as children (in particular, language / ethnic / racial “minority” and low-income children) experience two kinds of mathematics—home mathematics and school mathematics. Abreu’s work has been very influential in my own thinking, not only as I look back at the comments students (preservice elementary teachers in this case) made as they talked about their way to do mathematics versus the “proper” way (Civil, 1990), but much more recently, in my work with parents, and in particular with immigrant parents. Underlying much of this work is the notion of “difference.” I turn to this in the next (and last) section of this paper.

Handling Different Approaches to Doing Mathematics

Hearing parents talk about how different the teaching and learning of mathematics is now as compared to when they went to school is nothing new. (Well, we can wonder in some cases to which extent they are that different, but that is not so much my point here). Basically, generational differences in pedagogical approaches and in content are probably

to be expected. I will argue, however, that when these differences are viewed in the context of valorization of knowledge, in particular in communities that experience oppression and inequities of one form or another (as is the case of the Latino, working class communities where my work is located, or the case of immigrant communities in Barcelona, where Planas' work is located), they take on a different light. These differences cannot (should not) be pushed aside as something that just happens. Perhaps the fact that I grew up in a different country to the US, with a different educational system (and with me, specifically, attending yet a different school system within Barcelona), and that I experienced changes at other levels too (e.g., major political change in the mid 70s, which among other things brought back the Catalan language as the official language of instruction in Barcelona (and in the rest of Catalonia)) have made me more sensitive to the notion of change and difference, and with the valorization of knowledge that is associated with "difference." I would like to conclude this paper with a few glimpses at what parents (particularly immigrant parents), preservice teachers, and students say about the different approaches to doing mathematics and give some final remarks on reform and equity.

The Voices of Parents

For the last ten years I have been conducting research⁷ around issues of parents' views on the teaching and learning of mathematics (Bratton, Quintos, & Civil, 2004; Civil & Andrade, 2003; Civil, Planas, & Quintos, 2005; Civil, Bratton, & Quintos, 2005; Civil & Quintos, 2006; Civil & Bernier, in press). One of my first interactions with parents around reform in mathematics education occurred at a parents' meeting in which a group of mothers in the high school group expressed their concerns about their children's mathematics education. The district had adopted a reform-based series based on an integrated approach to mathematics. The mothers I heard speaking out in that group made it clear that it was not they did not like the new curriculum. What they were concerned about was that their children were bringing homework that did not look

⁷ Project MAPPS (Math and Parent Partnerships in the Southwest) is funded by the National Science Foundation under grant ESI-99-01275. The views expressed here are those of the author and do not necessarily reflect the views of NSF.

familiar and they were not sure how to go about helping them. A key problem, in our view and in these parents' view, with the implementation of that curriculum was that there were not enough textbooks in the district and thus students were not able to have their own book to bring home. The school district is largely Hispanic (82%) (mostly of Mexican origin) and with 81% children on free or reduced lunch. The district has changed from reform oriented to non reform to now perhaps back to reform in about seven years. Trying to uncover parents' perceptions of "reform" becomes a particularly hard task when we (as mathematics educators) cannot even keep up with what is being implemented, how and when.

There has been, however, a constant theme throughout our years of research into parents' perceptions about the teaching and learning of mathematics. That has do with immigrant parents' views on how their children are being taught here versus the schooling traditions in their country of origin (which in our context it is Mexico in most cases). As everybody else, parents bring their valorizations of knowledge to the discussion. Let me illustrate this point with an interaction between two mothers—both immigrant:

Lucinda: Well, what I say is, for example my daughter tells me "come to learn how they teach here, come see that I am right," when we are upset at each other here around the table, and sometimes she is the one who makes me upset, because I want to explain things to her as I know them, and I tell her, "mija, the way I explain it to you, I know it's much better for you," but she sticks to her [way].

Gabriela: But for one thing, here we are in the U.S. and here is where they are going to grow up, they are going to study here, and I wanted to do the same thing as you, but then I say, but why, if they are teaching him things from here, and he is going to stay here, and so, one wants to teach them more so that they know more, but what they are teaching them is because they are going to stay here, and they are going to follow what they teach them here.

One could say that this is normal generational discourse—parents trying to show

their children how they were taught because they feel that it was a “better” way. But there is more to this when one looks at it in the context of immigration and trying to navigate two cultures (at least) (and in our case, two languages). We argue, however, that these differences in approach take on a different light when those affected are low-income, immigrant families, whose knowledge has historically not been recognized or valued by institutions such as schools (Abreu, Cline, & Shamsi, 2002). What we witness are situations of potential conflict between children and parents as they try to adjust to the new setting while trying to keep connected to the former. As Suárez-Orozco & Suárez-Orozco (2001) write in their study on immigrant children in the U.S., “(I)mmigrants are by definition in the margins of two cultures. Paradoxically, they can never truly belong either ‘here’ nor ‘there.’” (p. 92). These authors write about the identity issues that immigrant children confront when caught between their parents’ culture and the culture in their new country.

This feeling of “caught between two cultures” impacts parent-child interactions around mathematics. One recurrent topic is that of the division algorithm. All the parents we have talked to who learned how to divide in Mexico comment on their method being more “efficient” and “cleaner.” A basic difference between the way they learned and the “traditional” approach to long division in the U.S. is that in Mexico they do not write down the subtraction, “we do it in our heads”, and they only write down the result (the answer). This is what Marisol and Verónica said about the division algorithms:

Marisol: When I looked at how he [her son] was dividing, he subtracted and subtracted and that he wrote all the equation complete I said, I even said, “this teacher wants to make things complicated. No, son, not that way! This way!” And he learned faster with this [Marisol’s] procedure.

Verónica: I tried to do the same with my child with divisions, that he didn’t write everything, but he says, “no, no, mom, the teacher is going to think that I did it on the computer.” “You don’t need to write the subtraction son,” I say, “you only put what is left.”... “No, no, my teacher is going to think that I did it on the computer, I have to do it like that.” “Ok, you think that..., but I want to teach you how we learned.” And I did teach him, but he still uses his method,

and that way he feels safe that he is doing his homework as they told him to. The same thing with writing above what they borrow and crossing it out, I tell him, “and I remember our homework did not have to have any cross-outs,” whereas his does....

This potential conflict is exacerbated when we add the language factor, as Verónica points out:

It takes a lot of work when it is difficult to translate something for me, to tell me, so he [her son] prefers to go early [to school] or ask someone else and that is something I don't like. He doesn't feel very sure that I understand him because the problem is written in English. I don't know how to read it and he doesn't know how to translate well for me because he speaks Spanish and reads Spanish, but we say different things for the same words and questions. I think he thinks I studied differently.

Verónica was frustrated because she knew the mathematics (she had attended college in México), yet she felt that her son was not valuing her knowledge. More explicit on this was Mónica:

Last night my son said to me that school from Mexico was not valued the same as school here, that is, it doesn't count. What I studied there doesn't count here, and he said, “Mom, do you know this problem?” “No, to tell you the truth, I don't,” I said to him. “See,” he says, “see, that's why I said that they didn't teach you [there] what they teach me here.” He knows that what is taught here is different from what is taught there and so he says, “why would I ask my mom for help if she's not going to know?” So, there is a barrier.

Because of our location near the border, many of these families have frequent contact with relatives across the border in México. Most parents (and in our current study in progress, children too) whom we have interviewed comment that they find the level of mathematics teaching more advanced in México.

Ernesto: I think that the educational level, in the case of my son, the schools are very (basic), the level in México is much higher. I'm saying that because I have nieces and nephews there and here and there, I see that they have learned more things at school. [Referring to a nephew in México] He's in fourth grade and my

son is in fourth grade too. What they're giving my son now, he (the nephew) learned in second grade. So, the educational level is lower and they learn more slowly than they learn in Mexico.

Bertha: No, I'm not happy. I feel that there is repetition of a lot of things; I don't understand why the teaching is so slow, I don't like it, I don't like the system, I don't like it at all. When we go to México ... my nieces and nephews or my husband's nieces and nephews, they are children that are more or less the same age as Jaime [their son] and I see that Jaime is behind. Here they tell me that Jaime (is) really excellent.

I conclude this section with a quote from Lucinda, the mother with whom I opened this discussion. In the earlier quote above, she talks about how she and her daughter (a fifth grader) were having differences of opinion when it came to ways to do mathematics. Lucinda wanted to show her the way she learned in México and her daughter was not interested and wanted to "stick" to the way they do things here. Yet, in the quote below, Lucinda reflects on how when they first arrived, the daughter was the one unhappy with the system in the U.S., and the mother was telling her how she was now going to learn "the way from here" (the U.S.)

Lucinda: When I came from there [México], [my daughter] was in 3rd grade; when we came here, she said that the school looked like play, "why, mijita?", "Because they are making me do $4 + 3$, mom, I don't want to go to this school. It's weird." And I would tell her, "but you are going to learn the way from here," well, at that time, that's what I thought, but then I visited my relatives [in México] [and then she goes on to talk about the difference in levels México vs. U.S.]

These two quotes from Lucinda point to the complexity of the situation, as people try to adapt to and make sense of situations that are different from what they have previously experienced. In our local context of immigration, we wonder about the effect that this adaptation to the new context has on the family dynamics.

The voices of Preservice Teachers

The first part of this paper focused largely on teachers' comments towards reform in mathematics education. A characteristic in many of the current reform programs is that of introducing alternative algorithms. In this section I look at that component, and in particular at comments from preservice teachers in content courses for elementary teachers, as they reacted to algorithms that were different from what they were used to.

To me, a key aspect of embracing reform is the idea that students bring their own ways to do mathematics and that as teachers, we should build on those, share the different ways, discuss them, etc. One area where this is certainly the case, particularly when students come from different countries, is algorithms for the four basic arithmetic operations. This is certainly the case with parents from Mexico and the division algorithm, as I pointed out in the previous section.

After having seen an algorithm for subtraction in which the child goes into negative numbers, instead of the traditional "regrouping," a preservice teacher wrote:

I do believe that you could eventually convince him that learning to carry is easier and leaves less room for error.

We should discuss: why would "learning to carry" be easier?

After having seen a Left to Right algorithm for subtraction, another preservice teacher wrote:

From left to right, wouldn't kids get confused? If I showed them from left to right, I would think that when you got to the real thing, that they would get upset or would get confused.

We should discuss: who decides what the "real thing" is?

Of particular concern to me is how / whether we are preparing teachers to address different approaches, particularly when those different approaches may be coming from low-income, immigrant children. The next section is very telling in terms of how "local" students react to these other ways. But I close this section with a commentary on what preservice elementary teachers wrote in reaction to their reading of "mathematical notations and procedures of recent immigrant students" (Perkins & Flores, 2002). A few of the preservice teachers wrote comments along the lines of the need for immigrant students to learn the way arithmetic is done here. As one of them wrote, "this is nice but

they need to learn to do things the U.S. way.” Is it valorization of knowledge (as in one way being better than the other)? Or is it that they were concerned about their own understanding of these different ways, as one of the preservice teachers hints in the comment, “how can we be expected to know all these different ways?”

The Voices of Students

In earlier sections of this paper, I have illustrated some of the students’ reactions to pedagogical approaches characteristic of reform (e.g., small group discussions), as well as to teaching innovations that tried to involve them in “mathematicians’ mathematics” explorations or in contexts bridging to their everyday experiences. In this section I come back to Planas’ research in Barcelona in high school with large numbers of immigrant students. I shared earlier in the paper the reactions from both non-immigrant (“local”) and immigrant students to the concept of group work. Here my focus is on the local students’ comments on the approaches to mathematics that immigrant students bring to the classroom (Planas, 2005).

In her research study, Planas (2005) interviewed twelve 16 year-old non-immigrant students from the same classroom in a high school in Barcelona that had a high percentage of immigrant students (60% of the students were from Morocco). In that particular classroom fourteen out of twenty-eight students were immigrants, nine of them first generation. There were seven students from Morocco, three from the Dominican Republic, two from Pakistan, and two from Bangladesh. The school, as is the case with schools with high numbers of immigrants in Barcelona, is a very low-income neighborhood. Planas’ research is particularly insightful in that it seeks to understand issues related to immigrant students in the mathematics classroom, not only from the point of view of these students, but also from the point of view of the “local” students (see Planas & Civil, 2002; Planas & Gorgorió, 2004). Planas’ (2005) findings point to a deficit view on the part of the local students towards their immigrant peers. Several of the local students make comments about the extra effort it takes to learn mathematics in classes with immigrant students, in part because the local students have to help them because the immigrant students do not know the official languages (Spanish and Catalan) well, or because their ways of doing mathematics are different. I just want to highlight a

few of the local students' comments on this latter point—their reaction to different ways to do mathematics:

Their [immigrant students] comments help us make sense of the situations before starting solving the problems, but anyway, we cannot always start making sense of it like they do. Our maths are what they are. And theirs... they are fine, but sometimes they just don't fit in.

We are not in the classroom to learn their mathematics but to learn ours. That's what the exams are about. (...) I am not expected to learn Murshed's way of subtracting.

I learn from what others say and do, but you see, in the case of the immigrants, you must be very careful for your own benefit. They learned some mathematics differently and you must know what to learn from them (...). Most of them easily learn our ways.

These students' comments are not very different from those given by the preservice teachers and the parents earlier in this paper. There is this feeling that there are these different forms of mathematics (it is of course true that there are different ways to do mathematics... that is what makes it exciting to me, to have these different ways), yet the feeling one gets is that one of these forms is given higher value than the others, be it school mathematics over everyday mathematics, or "U.S." (or "Spanish" or "Mexican") algorithm over "others'" algorithm. Whose knowledge is being valued, is to me a key question in any discussion of reform mathematics.

Final Remarks

So, are we there yet? I opened this essay with a quote from a teacher who originally thought that reform was "this magical place I was headed for." More than ten years later I feel like we are still on this journey, a journey full of bumps and detours. At the moment, perhaps due to the current emphasis on accountability and high-stakes testing, I seem to notice an approach to instruction that is narrower and more skill-focused than a few years

ago. But in many of the classrooms that I visit, I do see students working in groups and teachers trying to engage in some kind of mathematical discourse.

I still believe that many of the principles behind reform move us towards giving access to mathematics to more students, and particularly to students who historically have had limited access to a larger vision of what mathematics teaching and learning could be like. To me, two related key issues that we need to consider in our attempts to change are the valorization of knowledge and our reactions to difference. Karen, one of the teachers in the Reforming Mathematics Education project discussed earlier in this paper wrote, “we focus on our differences rather than our solidarity and commitment to try new ways and listen to new ideas.” Although Karen was referring to the group of teachers in the project and their different teaching philosophies, this statement can be applied to parents, teachers, and students, as the section on handling different approaches to doing mathematics illustrates. What does this “commitment to try new ways and listen to new ideas” look like when we bring in SES, race, ethnicity, different languages, immigration status, rural / urban settings? As I wrote earlier, I am particularly intrigued by how people (parents, teachers, children, college students) think about mathematics. It is in part this interest in how people think that brought me to the Funds of Knowledge for Teaching project. To me, reform in mathematics is not “only” about content and pedagogical strategies such as group work, hands on activities, discussions. As Spradbery (1976) writes,

There has been little consideration of what is Mathematics or how it reflects and maintains the interest of certain members of a set of competing social groups. Even the progressive ‘discovery’ methods of the primary school... continue to maintain the traditional roles of, and divisions between, teacher and taught. (p. 236)

Thirty years later, this quote is still valid if we change ‘discovery’ to ‘reform.’ Earlier in this paper I raised several questions about what kinds of mathematics should the students be doing. I wonder about the meaning of terms such as “authentic” and “relevant.” One of the issues that I often bring up in our efforts to connect with the mathematics in the community is “what mathematics is that?” I have not yet resolved this issue or the earlier questions. I do think that part of the resolution may be in expanding

our discussions of content and pedagogical strategies to include the community we work in (parents and children, in particular) and build on their forms of knowledge and how that knowledge is “acquired” in these communities (Civil, in press).

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