The Organization of a Group Mathematics Discussion in a Middle School Classroom

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This study analyzes patterns of interaction among bilingual middle school students while they engaged in mathematical discussions. Using a sociocultural lens on the practice of school mathematics and methods from discourse analysis, this study addresses three questions: 1) What kinds of mathematical discourse practices did the students engage in during peer mathematics discussions? 2) What implicit “rules” appeared to mediate students’ interactions during mathematical discussions? 3) How did intellectual authority function in this small group? We show that, although the students engaged in hybrid Discourse practices during peer mathematics discussions, their discussions largely reflected a “calculational” orientation. We also describe how “intellectual authority” was an important role that mediated the norms of these calculational discussions.

1. Introduction

Recent reforms of mathematics education emphasize that learning to communicate about and to communicate through mathematics are key components of developing mathematical competence (National Council of Teachers of Mathematics, 1989, 2000). Teachers frequently use small group activities to support and develop mathematical communication among students in mathematics classrooms. Researchers have documented the benefits associated with small-group discussions among peers in terms of developing general problem-solving skills (Mercer, 2005), building positive social relations (Cohen, 1994), and also developing discourse practices integral to mathematics (Brenner, 1994, 1998; Yackel, Cobb, & Wood, 1991; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). This study analyzes patterns of interaction among a small group of bilingual middle school students while they engaged in mathematical discussions. Our over-arching purposes are: 1) to investigate the types of mathematical discourse practices
students engage in during small-group discussions in a typical mathematics classroom, 2) to understand the implicit “rules”\(^1\) that shape students’ interactions during peer mathematics discussions in small groups, and 3) to explore the role of “intellectual authority” in a small group discussion.

2. Theoretical Framework

Prior research in mathematics education has called attention to the benefits of mathematical discussions in both whole-class (Goos, 2004; Lampert, 1990) and small-group settings (Yackel, Cobb, & Wood, 1991). For small-group discussions in mathematics classrooms there is extensive research analyzing both the content and form of students’ peer discussions while working in dyads (Forman & Larreamendy-Joerns, 1995; Moschkovich, 1996, 1998; Sfard & Kieran, 2001; Yackel, Cobb, & Wood, 1991). These studies have greatly expanded our understanding of how peer discussions mediate students’ learning of mathematics. However with the exception of a few studies (Jurow, 2005; Pirie, 1991; Webb, Nemer, & Ing, 2006), analyses of students’ mathematics discussions in small groups with more than two participants have not received much attention. Studies such as this one, which employ an ethnomethodological approach and diverse analytic tools, will enhance our understanding of the nature of students’ mathematical discussions. While our overarching theory of mathematics learning is related to situated and sociocultural theories (Moschkovich, 2002a, 2004; Rogoff, 1990), our analysis also draws upon sociolinguistics (with a focus on conversation analysis), discourse analysis, and

\(^1\) We are following Rowland’s (2000) use of “rules” to refer to the regularities of interaction that point to a collection of implicit norms of speech and behavior that members of a common linguistic community use while engaging in interactions. This is not meant to imply these rules are mandatory (in fact they are often made visible when they are broken) or that these rules are to be interpreted as normative judgments of what students ought to do.
analyses of students’ mathematical Discourse practices. Below we highlight major ideas from each of these areas of research.

*Sociocultural Orientation.*

This study of students’ interactions during small group discussions is situated in a sociocultural theoretical framework. Within this tradition, learning is treated as a process of appropriating (Rogoff, 1990) the meanings, goals, and perspectives enacted by members of a community of practice (Moschkovich, 2004). Learning is assumed to be an active process, and the corresponding emphasis in analysis is on activities such as “knowing” and “representing” rather than static entities such as “knowledge” or “representations” (Sfard & McClain, 2002). This orientation highlights that knowing mathematics cannot be separated from practices, which distinguishes this study from previous works on peer discussions in mathematics, which treated knowledge and representations more like static entities that students must acquire or internalize. Furthermore this orientation toward the study of learning within a sociocultural context highlights that the “cultural context” is dynamic and it should be considered as a result of an individuals engaging in cultural practices located within a community. Therefore, attending to context requires attending to layers of community participation (Gutierrez & Rogoff, 2003). Gutiérrez and Rogoff made four specific recommendations for research that adopts this framework: 1) Cultural observations should be made in past tense, to avoid idea that culture is static; 2) Labels should be used as narrative elements, not historical and static; 3) Participants’ and community background should be treated as a constellation of factors, rather than single categories; and 4) Statements about individual observations should be made with great caution, to avoid overgeneralization. In our analysis below we attempt to remain true to these principles of doing research within a sociocultural framework.
Discourse Analysis as Conceptual Framework

Discourse analysis (DA) focuses on the study of social structures through the analysis of communication. DA emphasizes that understanding social structures requires a broad-based study of interaction and people’s uses of language in real settings together with theoretically grounded analytical tools (Gee, 1996; Gee & Green, 1998). Gee’s theory of Discourses and literacy emphasizes that Discourses are more than simply styles of speech—Discourses are inextricably related to people’s identities, socially meaningful roles, and political relations of power (taking a broad definition of political). To make a distinction between discourse-as-speech and the much broader set of relations between speech, other forms of symbolic communication, group membership, political relations, and social/political power, Gee distinguished between “discourse” and “Discourse.” The former refers to a body of speech or text. The latter, “big-D” Discourse, refers to:

a socially accepted association among ways of using language, other symbolic expressions, and “artifacts”, of thinking, of feeling, believing, valuing, and acting that can be used to identify oneself as a member of a socially meaningful group or “social network”, or to signal (that one is playing) a socially meaningful “role”. (Gee, 1996, p. 131)

We show how the students draw upon a variety of Discourse practices while engaging in school mathematics discussions. At times, the students adopted the Discourse practices of a typical school conversation with one student playing the role of teacher and the others acting as students. In still other peer discussions, the students adopted the Discourse practices of professional mathematics (Moschkovich, 2007), challenging the logical foundations of one another’s’ arguments, requiring evidence for claims, and offering counter examples during
debate. In addition to these two academic Discourse practices, the students also drew upon Discourse practices from out-of-school, including references to popular culture and the mixing of Spanish and English.

Our analysis of the students’ use of Discourse practices follows along three major themes from a cultural-historical activity theory framework (Gutiérrez, Baquedano-López, & Tejeda, 1999): we analyze these students’ mathematical discussions in terms of goals for the activity, rules (made visible in interaction, rather than as formal structures), and the division of labor (or roles).

**Goals.** In the classroom we study here, the students’ peer mathematics discussions have a constantly negotiated order that highlights that these discussions occur at the intersection of both formal and informal Discourse practices. Eggins and Slade (1997) distinguish between casual and pragmatic conversations by noting that in the latter, the goal-directed nature of the conversations is relatively transparent. For example, a conversation between a clerk and a customer at the post office about buying stamps is pragmatic since it is structured around the goal of the conversation—completing a commercial transaction. Characterizing peer discussions in a mathematics classroom as either casual or pragmatic is more challenging since the conversations do have well-defined goals (i.e. find the answers to given problems), but the *socially meaningful roles* and interactional means to accomplish those goals are not as well-defined. In the postal example, the assignment of socially meaningful roles during the interaction is signaled by the participants’ physical location (on one side or the other of the service counter), dress (in a uniform for the postal employee), and actions. The classroom is a formal school setting with institutional goals, explicit rules, and roles for people in the setting (i.e. student, teacher). However, peer discussions in the classroom are not directly regulated by an
institutionally defined authority as teacher-led whole class discussions are (Mehan, 1979). For
this reason we argue here that the students’ peer discussions occupy a “Third Space” between the
official Discourse practices of school and mathematics, and the students’ “informal” Discourse
practices in out-of-school settings (Gutiérrez, Baquedano-López, & Tejeda, 1999).

The (implicit) goals of discussions in school mathematics are related to the debate in
mathematics education centered around the distinction between teaching for procedural
competence or conceptual understanding (Hiebert, 1990). These two instructional goals result in
different orientations in students’ and teachers’ mathematical Discourse practices. Thompson, et
al. distinguished between two general discursive orientations they named “calculational” and
“conceptual” and they analyzed how these orientations are manifest in two teachers’ whole-class
discussions:

   Conceptually-oriented teachers often express the images described above in ways
that focus students’ attention away from thoughtless application of procedures and
toward a rich conception of situations, ideas and relationships among ideas.... The
actions of a teacher with a calculational orientation are driven by a fundamental
image of mathematics as the application of calculations and procedures for
deriving numerical results. This does not mean that such a teacher is focused only
on computational procedures. Rather, his view of mathematics is a more
inclusive, but still one focused on procedures—computational or otherwise—for
“getting answers” (Thompson, Philipp, Thompson, & Boyd, 1994, p. 86).

Thompson et al. also identified a third orientation called a “computational perspective,” which
focuses almost exclusively on the mechanical computations necessary to arrive at an answer with
little or no attention paid to the meaning of numbers derived, or the reasons for doing a particular
calculation. While Thompson et al.’s interest was to show how these orientations are related to the effort to reform the teaching and learning of school mathematics within a teacher-led whole-class discussion, in the context of this study, the distinctions between a calculational and conceptual orientation will provide a useful tool for identifying the goals in students’ mathematical Discourse practices during small group discussions.

The Division of Labor: Rules and Roles. It is a common practice for teachers to formally assign roles to students while working in small groups, and to establish “ground rules” for the group work time (Cohen, 1994). In addition to formally assigned roles and explicit rules for interaction, students also constantly negotiate roles and the rules for interaction within the small group in their moment-by-moment Discourse practices. Our analysis in this paper focuses on how students use Discourse practices to (co-)construct roles and rules for interaction during group discussions. While rules and roles occupy different vertices of the “activity triangle” (Gutiérrez, Baquedano-López, & Tejeda, 1999), we consider them together in this analysis, and we show how these two elements of the activity system are mutually constitutive. For this reason we consider them together here.

Linde’s (1988) study of cockpit discussions in police helicopters illustrates how analysis at the level of discursive moves can be used to reveal implicit social structures and hierarchies that are created by conversational participants in settings that blend elements of formal and informal conversational norms. Her study documented how a pilot and police officer created and reified a social hierarchy within a two-person police helicopter, even though the two officers officially held the same rank within the police department. While each officer showed deference for each other’s expertise during official conversations related to the purpose of a mission, during “down time” Linde showed how the police officer frequently gave (conversational)
authority to the pilot. Specifically, she analyzed the frequency and direction of teasing, the right
to begin and end conversations as well as means used to do so, and the directness of commands.
By analyzing these moves, Linde revealed the co-created social hierarchy of the police
helicopter. While Linde’s study is not specifically related to education, her analysis points to the
variety of methods that can be used to analyze conversations taking place in settings that sit at
the nexus of the formal and informal.

Several mathematics education researchers have documented how whole-class
mathematical Discourse practices can be transformed in classrooms where a reform-oriented
perspective on mathematics guides instruction (Forman, 1996; Lampert, 1990; O'Connor, 2001).
One theme that runs throughout the literature is that part of changing students’ and teachers’
mathematical Discourse practices involves addressing the role of intellectual authority in the
classroom. According to Lampert:

The issue of intellectual authority is central to this comparison between how
mathematics is known in school and how it is known in the discipline. In the
classroom, the teacher and the textbook are the authorities and mathematics is not
a subject to be created or explored (Lampert, 1990, p. 32, emphasis added).

While Lampert (as well as other researchers who use related ideas) does not explicitly define
“intellectual authority,” this description seems to imply that some person or artifact has the right
to decide what kinds of statements are “right,” “sufficient,” or “well formed.” Yackel and Cobb
invoked a similar notion that they called sociomathematical norms, “normative aspects of
mathematics discussions specific to students’ mathematical activity” (Yackel & Cobb, 1996, p.
461). Outside of mathematics education, sociolinguists have used the term “epistemic authority”
to signal a similar role: the conversational participant with “epistemic authority” has unique
rights and privileges, such as the right to initiate repair, in the conversation (Heritage & Raymond, 2005; Razfar, 2005). In our analysis of students’ mathematical discussions, we treat intellectual authority as one of the “socially meaningful roles” (Gee, 1996) that students can invoke through engaging in particular Discourse practices during peer discussions. While this role is not as well defined as other discursively created roles (such as teacher or preacher), we will show how this role of intellectual authority was constructed through participation in multiple Discourse practices, and simultaneously, how this role mediated the discursive rules of students’ peer discussions.

This study investigates the following questions: 1) What kinds of mathematical discourse practices (Moschkovich, 2007) did the students engage in during peer mathematics discussions?; 2) What implicit “rules” appeared to mediate students’ interactions during mathematical discussions?; and 3) How did intellectual authority function in this small group?

3. Participants and Setting

To answer these questions, the first author gathered data for this study by observing a group of students in a dual-immersion (Spanish-English) bilingual sixth grade classroom in California for approximately one month. The class used a standard textbook (Maletsky, 2002), and the teacher followed the prescribed curriculum and California state standards for sixth-grade mathematics. The students in this study had two mathematics classes together, one taught in English and one taught in Spanish. The teacher taught this class in English, (though she is fluent in Spanish and frequently used Spanish during individual conversations with students). Our data collection reflected an ethnomethodological approach (Schiffrin, 1994). We sought to capture “everyday” student discourse in a naturalistic setting and we made efforts to not disrupt the flow of activities within the school setting or the teacher’s planned curriculum and activities.
After taping, we created content logs for the four hours of classroom video we had recorded, and then we selected the sections of video in which the students were engaging in sustained mathematical discussions for closer analysis. For the purpose of this study, we adopted Pirie’s definition of mathematics discussion as “purposeful talk on a mathematical subject in which there are genuine pupil contributions and interaction” (Pirie, 1991, p. 143). Pirie’s definition of a mathematical discussion guided us to focus our analysis on a limited subset of the entire data corpus. The first author then transcribed sections of the video in which the students engaged in mathematical discussions (approximately 56 minutes) in detail using conventions from conversation analysis.² Besides the video and transcripts, our other data sources include ethnographic field notes, a semi-structured interview with the teacher, a stimulated response focus-group interview with the students, a short survey on the students’ language preferences and their perceptions of mathematical abilities of the members of their group, and a copy of the students’ written work for the week. This paper reports principally on the analysis of the video data, and the other sources of data were used to support our analysis of the students’ interactions.

4. Findings

We have identified three primary findings from this study. First, although the students’ group discussions involved largely the Discourse practices commonly associated with school mathematics (Moschkovich, 2002b, 2007), the discussions also reflected “hybrid” (Gutiérrez, Baquedano-López, & Tejeda, 1999) Discourse practices: blending formal and informal Discourse practices, school and professional mathematical Discourse practices, and using two languages during mathematical discussions. During the students’ “school mathematics

² Transcripts follow Jefferson’s conventions as detailed in Schiffrin (1994) with minor revisions. The ... symbol denotes one or more turns omitted from this presentation because those turns refer to a side conversation that occurred in parallel to the focal interaction. Also, English translations of utterances in Spanish are give in double parenthesis and quotations (" ").
discussions” in particular, the their orientation reflected the “calculational” and “computational” orientation (Thompson, Philipp, Thompson, & Boyd, 1994) of the exercises they were discussing. Second, the interaction during these calculational discussions were different from interactional patterns documented in everyday conversations (Eggins & Slade, 1997) in several ways: students frequently verbalized their actions (while doing computations), interrupted each other, and used direct contradiction. Finally, “intellectual authority” in the group was a dynamic role that mediated students’ interactions, while simultaneously and dialectically also being created and reified through students’ interactions. Below, we elaborate on the first and third findings, illustrating them with examples. In the interest of space, the second finding is addressed in detail in a separate paper (in preparation).

Mathematical Discourses and Orientations.

Mathematical Discourses: School Mathematics and Professional Mathematics. During these small group discussions, students were engaged in mathematics discussions that largely reflected the discourse practices of school mathematics. However, at times, the students also used some discourse practices of everyday mathematical reasoning as well as elements of professional mathematical discourse practices (Moschkovich, 2007). Moschkovich argues that the discourses practices of school mathematics may align with the discourse practices of professional mathematics (especially in a reform mathematics classroom) but for many students, the discourse practices typical in school mathematics are not the same as the discourse practices of either professional or everyday mathematical settings. Three common discourse practices usually identified with school mathematics are asking known answer questions, following Initiation-Response-Evaluation sequences, and discussing traditional word problems (Moschkovich, 2002b and 2007a).
Students’ use of school mathematical discourse practices during a peer discussion is illustrated below. The discussion took place on the last day of video recording while Amber was completing worksheet exercises asking students to find the prime factorization of numbers. Although Francisco and Amber were supposed to be working as peers, their discussion reflects the organization of a common student-teacher interaction, the three-part IRE pattern (Mehan 1979). Amber acted as the “teacher,” asking known answer questions, while Francisco played the role of her “student.” We have labeled lines that correspond to the IRE sequence using I, R, or E following the speaker’s name and his or her utterance.

**Excerpt 1** [Friday Class, 1:10:00-1:11:52]

Prompt: *Amber and Francisco are working on a review worksheet with exercises that ask for the prime factorization of 63 and 230.*

1. Amber: I Sixty xxx. what makes sixty three? (2) What makes sixty three? (looking at Francisco))

2. Francisco: R xxx not now ((Francisco appears to be addressing Joaquin)). Nine ti:mes seven ((Looking at Amber))

3. ...

4. Amber: (defines) [(and you can make it)

5. Francisco: [well that was easy

6. Amber: E nine times [seven

7. Francisco: [Oh wait no no wait no no

[nine, three times three

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3 Before reading each transcript, the reader may want to do the calculations from the prompt. We believe that familiarity with the calculations will help clarify the meaning of the students’ utterances in the transcripts.
8. Amber: [Oh ha ha ha yeah
9. Francisco: O:::h I told I told you xxx ((dancing in his seat))
10. Amber: zero zero
11. Francisco: ohhh
12. Amber: three to the power of two times seven
13. Francisco: I (. ) I (. ) [I told you ((Continues dancing in seat))
14. Amber: I [What makes two hundred and thirty?
15. Francisco: R twenty three times ten
16. Amber: huh?
17. Francisco: twenty three times ten
18. Amber: (E), I ((20s pause as A writes)). Nothing makes two and nothing makes five.

   What makes twenty three?= 
19. Francisco: R =eight times four (1) No eight times three. Eight times three?
20. Amber: E Its (. ) twenty four
21. Francisco: yea
22. Amber: I need twenty three
23. Francisco: Oh
24. ...
25. Amber: Nothing (1) twenty three

To provide some background, based on observations of Amber’s previous work in this classroom, we know that she was able to carry out the computations necessary to find the prime factorization of numbers such as 63 and 230. Also, although immediately before this discussion Francisco had said he was done working for the day, he returned to work and engaged in the
“quiz game” that he and Amber discursively created. During this interaction Amber acted like a
teacher by asking questions to which she already knew the answer, rather than acting like a peer
working together on problems without known answers. Amber also evaluated Francisco’s
responses, instead of leaving evaluation to the teacher. We assume that the questions Amber
asked in lines 1, 14, and 18 were all known-answer questions (or at least within her ability to
answer without help). In contrast with other examples of this group’s peer mathematics
discussions Amber and Francisco’s discussion closely matched the three-part discourse of IRE
sequences associated with traditional, whole-class, teacher-led discussions (Mehan 1979). Amber
initiated questions and evaluated answers, while Francisco responded as a student. Francisco’s
role as “student” in this exchange is highlighted in his responses in lines 9 and 13 when he
danced in his seat and celebrated correcting an error that Amber made (she neglected to factor 9
as $3 \times 3$). This interaction brings to mind the image of a student who celebrates catching his
teacher in a computational error on the blackboard. However, Francisco’s glory was short lived:
in lines 20 and 22 Amber regained control of her “student” by correcting his computational
error!

Although this example shows how this small group engaged in some of the well-
documented discourse practices of school mathematics and whole-class interactional norms, this
was not the case overall. Students in this small group also engaged in dynamic discussions with
multiple participants and disputed or unknown answers, and drew upon a variety of hybrid
Discourse practices (as well as languages and registers) during these discussions. Some of the
Discourse practices used by the students that align with the Discourse practices of professional
mathematics included demanding precision and logical coherence (Forman, 1996), reasoning
with examples, and generalizing (Moschkovich, 2007). At the same time, the students used non-canonical mathematical arguments such as appeals to authority (Harel & Sowder, 1998).

Excerpt 2 below shows an example of a group discussion where multiple participants attempted to devise an algorithm for finding percentages of whole numbers. The initial exercise the students worked on was to find 70% of 90, and Amber’s explanation in lines 8 and 10 was that to find the percentage students needed to multiply the two numbers. Dennis and Francisco appeared to be confused by Amber’s explanation (perhaps because $70 \times 90 = 6300$), and they wanted her to explain what she did to get an answer of 63 (see line 12). At the same time that Dennis and Francisco were looking for an algorithm for computing 70% of 90, Claudia and Amber discussed Claudia’s arithmetic error in the computation of 35% of 45 (Claudia’s initial answer was 14.75). Hence there are two interwoven and overlapping conversations taking place during this excerpt. Since several pairs of adjacent lines in the transcript are part of two distinct conversations, we have used comments after students’ utterances to clarify who the likely addressee is for each utterance. We relied on the students’ gaze and posture, as well as the logical coherence of each utterance in an initiation-response pair in order to ascertain who the intended recipient was for each utterance.

**Excerpt 2: Wednesday session 2 2:55-4:56**

Prompts: This excerpt includes discussions of three exercises from a worksheet: #1 10% of 8 = ____, #4 70% of 90 = ____, and #7 35% of 45 = _____.

1. Francisco:    Can you help me with xxx ((yawning)) Tell me how to do number one
2. Amber:        Wha - we already did number four ((to Francisco))
3. Francisco:    What was number four? ((to Amber))
4. Amber:        Sixty three percent ((to Francisco))
5. Claudia: Twelve thirteen fourteen ((appears to be talking to herself))

6. Amber: It’s: ((possibly to Claudia))

7. Francisco: → Then why is number one eighty percent? ((to Amber))

8. Amber: Cause ten times eight [what is that ((Turns, looking at Francisco))

9. Francisco: → [Oh, so you just multiply it ((To Amber))

10. Amber: Yes. ((to Francisco)) (1.5) It’s [.] ((to Claudia))

11. Claudia: [It’s fourteen point seventy five ((looks up at Amber))

12. Dennis: → How come this sixty three percent then [if you multiply this ((Appears to address Amber based on line 16))

13. Amber: [No (divide) ((looking at Claudia))

14. Claudia: It’s one forty seven [point five ((to Amber))

15. Amber: [No you [yeah ((looking at Claudia, reaches for Claudia’s paper))

16. Dennis: → [This is wrong Hey [Amber

17. Amber: [No it’s [thirty five times forty five ((to Claudia))

18. Francisco: → [Oh, este porque ((“this why”)) you multiply (em) ((Looks across Amber to Dennis))

19. Francisco: [[You add one zero

20. Amber: [[Add up twenty five= ((to Claudia))

21. Dennis: → [[Seventy times nine ((responding to Francisco))

22. Amber: =and then this [is fifteen, seventeen ((to Claudia or reciting algorithm aloud to self))
23. Francisco: → [No, mira porque como ("look because like")] imagine that but quita(le) [los ceros ("take away the zeros")] ((to Dennis))

24. Amber: [The zeros, that’s twenty ((still reciting algorithm))]

25. Joaquin: xxx

26. Amber: and then and then four times three is twelve thirteen fourteen, and then you add it then its five, seven five one ((continues voicing algorithm))

27. Dennis: → Oh, you (gotta) take away the zeros? ((to Francisco))

28. Claudia: Y entonces ("and then") xxx and then [you put xxx ((“then fifteen (for all these)”))]

29. Amber: [Pues quince (por todo estos)]

30. Dennis: → Hey Amber, you want to take away the zeros/ to multiply this=

31. Claudia: No, [you’re doing it wrong that’s why ((to Dennis))]

32. Dennis: [=nine times seven]

33. Amber: umm ¿Qué? ("What?")

34. Dennis: How (do) you multiply? ((to Amber))

35. Amber: Seventy times sixty ((to Dennis, note that the exercise is 70% of 90))

36. Francisco: Seven times [six ((in response to Amber))]

37. Dennis: [Seventy times ninety ((apparently in response to lines 35, 36))]

38. Amber: → like this: seventy times [sixty take that off] ((to Dennis, Francisco))

39. Claudia: [Fifteen sixteen seventeen ((appears to address self))]

40. Francisco: → It’s ninety ((to Amber))
41. Claudia: but I told her its
42. Amber: Well I could do i:t. I did number one ((to Francisco and Dennis))
43. Claudia: [xxx
44. Amber: [xxx
45. Dennis: → So it’s nine times seven= ((to Amber))
46. Claudia: Twelve
47. Dennis: → =is (. ) sixty three ((looking at A)) That’s the answer
49. Francisco: Oh, OK I get it

Below is the algorithm Claudia used to compute 35% of 45. She apparently made a computational error before she announced her answer line 11.

\[
\begin{align*}
35 & \times 45 \\
175 & \\
140 & \\
1575 &
\end{align*}
\]

The interaction in Excerpt 2 appears to have occurred in three parts. In lines 1-10, Francisco asked Amber for help with exercises one and four. During lines 11-32, there are two simultaneous conversations, one between Amber and Claudia, and a separate conversation between Dennis, and Francisco (and they try to engage with Amber). Finally, in lines 33-49, Amber responds to Dennis and Francisco’s questions about her method for finding percentages.

In the lines marked with an arrow (→) in Excerpt 2 we see examples where Dennis and Francisco used Discourse practices associated with professional mathematics while questioning Amber’s method. In line 7 Francisco attempted to reconcile Amber’s answer for question number 4 with the answer she gave to question number 1. His question in line 7 addressed an
apparent inconsistency in her answers and procedures, and reflected a practice of professional mathematical—requiring consistency between answers and algorithms. His question implicitly raised the issue that her explanation was not sufficient. In line 12, Dennis more directly identified the logical inconsistency in Amber’s reasoning from line 8 (“Cause eight times ten what is that” which might be glossed as “multiply 8 times 10 to get the answer”). He pointed out that $70 \times 90$ is not 63, and in line 16 he used more direct language to address the inconsistency of Amber’s reasoning. These interactions show how students used two Discourse practices of professional mathematics, requiring logical coherence for solutions and demanding precision for calculations. In their discussion of problem #7, Amber regulated Claudia’s actions in much the same way that Dennis and Francisco attempted to get Amber to respond to their suggestions—by pointing out her errors in applying the standard algorithm.

In the latter portion of the excerpt (lines 33-49), the group discussion focused on Amber’s computation to find 70 percent of 90. Presumably Amber misspoke when she said “seventy times sixty” in line 35 (based on the algorithm she used in lines 8 and 10), and this led to an attempted “repair” by Dennis in line 37. Meanwhile Francisco apparently applied his algorithm (drop the zeros and multiply) to the numbers suggested by Amber in line 35. Finally, in line 42 Amber appealed to her own (and implicitly the teacher’s) authority to aver the correctness of her solution since she answered number one correctly. (Off camera, Amber checked her answer to question #1 with the teacher, another appeal to authority). Amber’s appeal to authority is a reasoning practice not commonly associated with professional mathematics (though perhaps common in school mathematics [Harel & Sowder, 1998]). By the end of this excerpt, Francisco apparently left the discussion convinced that the algorithm for computing percentages was to take the numbers given in the question, drop any zeros and then multiply them. A minute after
the end of this excerpt Francisco checked his answer for exercise #2 (25% of 60 = ____ ) by suggesting “it’s 25 times 6.” (Interestingly, his flawed algorithm might result in a correct answer in this case: 25 × 6 = 150. Then, drop the zeros: 150 = 15. Unfortunately, his method is not generalizable).

*Calculational Orientation.* This small group spent the majority of their time discussing the calculations necessary to reach an answer rather than addressing more conceptual aspects of their activity, such as the meaning of the calculations they were doing, explanations for why the calculations were correct, or examination of errors. Further, the types of questions students answered appeared to shape the type of mathematical discourse practices in which they engaged. A majority of the prompts for the students’ group work were “exercises” (Schoenfeld, 1992), and the students used a subset of school mathematics discourse practices (Moschkovich, 2007) reflecting a calculational orientation (Thompson, Philipp, Thompson, & Boyd, 1994) while they engaged in their discussions. The excerpt below illustrates the calculational orientation of their discussions:

**Excerpt 3** [Monday, 45:05–45:31]

*Prompt:* “If six pairs of socks cost $4.50, how much will 9 pairs cost?” The text then listed four answer choices: “F. $6.75, G. $7.25, H. $9.00, J. Not Here.” (Maletsky, 2002 p. 393 Exercise #4)

1. Lorenzo A: ((Amber walks around the table and stands to the right of Lorenzo A. who is writing on a shared paper)) How would I do?

... 

4. Amber: Ah a um let me see

5. Lorenzo A: Number four

6. Amber: Number four (for the) (1) then you put (2) nine up xxx equals nine (3.5)=
8. Amber: =Now put a:: number-a letter (1) alright, and then, let's se[e:: six

9. Francisco: [Oh my God, you guys have the problem up there ((points to blackboard))

10. Lorenzo A: Six divided by (four-fifty)?

In this excerpt, Amber walked Lorenzo through step-by-step instructions on how to set-up the problem using equivalent ratios, and her instructions mirrored the steps for setting up the proportion

\[
\frac{9}{x} = \frac{6}{4.50}.
\]

In line 8 Francisco interrupted Amber’s step-by-step instructions to tell Lorenzo and Amber that the solution to the problem was displayed on a poster on the wall. (Francisco’s interruption and the subsequent re-orientation of the students’ action after the end of this excerpt illustrates that the students in this group perceived the goal of this activity to be “finding the answer.”) Prior to Francisco’ interruption, however, Amber responded to Lorenzo’s original question “How would I do [the problem]” (line 1) with detailed step-by-step instructions for setting up a proportion, relying on physical descriptions for placing the elements on the page, rather than using a more conceptually-based explanation using the meanings of the numbers in the exercise. As Amber gave directions to Lorenzo she did not elaborate a rationale for each step, and she demonstrated the mechanical steps necessary to complete the exercise.

Throughout the data set, students’ mathematical discussions often focused on the computational steps necessary to arrive at an answer. Students generally answered their group-mates’ questions and requests for help with calculational or computational responses (“this is how you…”), rather than conceptual answers (“this is why you…”) or explanations (“this works
because…”). Our observation of this orientation in their language is not meant to impugn the mathematical practices or level of competence of the students in the focus group—these observations are limited to the students’ interactions in this setting, at the time of the recording, in response to the constraints and affordances of the given exercises. As Webb, Nemer, and Ing (2006) argue, in small group discussions, students often use the discourse practices that they have appropriated from their experiences of school mathematics. Furthermore, the exercises that served as the topic for these discussions, standard textbook and worksheet exercises, implicitly set the goal of finding the correct answer by doing the necessary steps and then filling in the blank, or choosing the correct answer from a multiple choice list. The exercises constrained the discussions and did not create affordances for engaging in sustained discussions of the meaning of questions, algorithms, quantities, the reasons for using one procedure or another, or explanations of errors.

**Conversational Rules: The Role of Intellectual Authority within the Group.**

Above we have shown how the students used hybrid Discourse practices during their (calculational) mathematics discussions, drawing on resources from everyday conversation, multiple languages, as well as some of the Discourse practices of professional mathematics. Our search for the interactional “rules” for this group’s mathematical discussions led us to the conclusions that the rules of students’ mathematical discussions differ from the interactional rules of everyday and casual conversations (Eggins & Slade, 1997; Grice, 1975; Sacks, Schegloff, & Jefferson, 1974). For example, the students frequently used direct contradiction, which Eggins and Slade (1997) claim is rarely used in casual conversation. Likewise turn-taking and speaker transitions appeared more contested than the relatively smooth norms for managing
turn-taking explicated by Sacks, Schegloff, and Jefferson (1974). (These findings are explained in more detail elsewhere [in preparation]).

Since these peer mathematical discussions are pragmatic discussions focused (in part) on the goals defined by the exercises that served as the basis of the discussions, the claim that students’ peer mathematics discussions are different from everyday conversations is not necessarily surprising. However a theme that emerged in the course of our analysis is that “intellectual authority” within the group largely mediated interactional norms in the students’ discussions.

Intellectual authority was a “socially meaningful role” (Gee, 1996, p. 131) created and maintained through students’ use of Discourse practices (it may be akin to a form of symbolic capital (Bourdieu, 1991)). This section of the analysis emphasizes that the role of “intellectual authority” is made manifest in Discourse practices through the use of particular conversational moves on the level of the utterance such as asking questions or teasing (Linde, 1988). Simultaneously, assuming (or being constructed as) the intellectual authority in the group allowed for the use of particular interactional and discursive moves (such as interruption, using direct contradiction, passing judgment on others’ contributions). Thus our analysis of intellectual authority parallel’s Linde’s analysis of the negotiation of authority among police helicopter crews (Linde, 1988). Two surprising findings are that IA is not necessarily the same as having the correct answer (see Excerpt 5), and nor does it necessarily align with the students’ perceptions of who is best at mathematics within the group (see Tables 1 and 2).

Students in this small group continually constructed and negotiated who (or what) served as an “intellectual authority” (Lampert, 1990) for the group. This interactional pattern stands in contrast with Lampert’s description of the locus of intellectual authority in traditional
mathematics classrooms where the teacher and the text are usually treated as absolute authorities. In this setting, where the majority of the students’ discussions reflected a calculational orientation, the phrase “intellectual authority” may be grandiose, and perhaps “computational authority” may be a more appropriate term. However, for the sake of this paper, we will use the term “intellectual authority” to maintain congruence with the literature, while aware that this phrase may over-state the nature of students’ construction of authority within the group.

The Construction and Negotiation of IA in Interaction. There were several instances when students overtly negotiated intellectual authority. For example, during the first session on Wednesday, prior to the interaction detailed in Excerpt 4 below, Amber and Claudia had a conflict when they arrived at different answers to the same question. Ultimately Claudia convinced her group-mates of the correctness of her solution (through an appeal to the teacher’s authority), and after that point, the students treated Claudia as an authority for most of that session. In lines 2-6 of Excerpt 4 we see how a dispute over intellectual authority ensued when both Amber and Claudia began to answer Francisco’s question from line 1. Though Amber began answering the question, Claudia interrupted her and talked over Amber’s explanation. By line 7, Amber ceded the “floor” to Claudia, and her back-channeling agreement (“uh-huh”) affirmed both Claudia’s solution as well as Claudia’s right to give a solution. Claudia also appeared to guard her position as an authority in this exchange by ignoring Francisco and Dennis’s interjected answers to the subtraction step 60 – 56 in lines 11 and 12, and she carried out the subtraction algorithm with all of its steps.

**Excerpt 4** [Wednesday session 1, 25:20-25:55]

Prompt: \[
\frac{6}{8} = \underline{\quad} \%
\]

1. Francisco: ah- why six over eight seventy five percent?
2. Claudia: ((bobbing head slowly)) Divide it.

3. Amber: OK watch. Six goes inside divide by eight [xxx=

4. Claudia: [Por eso ‘ira ("that’s why, look")] ((note the interruption of Amber))

5. Amber: =and [that

6. Claudia: [goes in the casita ("little house")), you take out eight you put a
decimal porque no se puede ("because you cannot") [put a decim-you
put a zero= ((Claudia continues talking over Amber))

7. Amber: [Uh-huh ((Amber
cedes the floor))]

8. Claudia: =ocho por siete ("eight times seven") put a seven here it’s fifty-six

9. ...

10. Dennis: four=

11. Francisco: =(and that’s) four

12. Dennis: [[four

13. Claudia: [[five it’s ten is four

14. Francisco: Oh yeah, seventy five percent

15. Claudia: y (luego) ("and then") [xxx zero

16. Dennis: [Zero then five

17. Claudia: y luego es ("and then it is")] seventy five percent
Building on Linde’s (1988) analysis of authority, one way to identify who the participants treat as an intellectual authority is to analyze who is asked questions about mathematics. Table 1 shows that with the exception of the first Wednesday session (which is highlighted above in Excerpt 4), Amber was asked significantly more questions about mathematics than all of the other students combined. The data presented in Table 1 also show the extent to which the students relied on the teacher as an intellectual authority. Though the teacher only checked in with the group occasionally, students asked her many questions—so she is proportionally overrepresented in Table 1. Also, during the second session on Wednesday, Amber frequently asked the teacher questions off-camera, and that is why the number of questions to the teacher appears as zero in the table, even though the students did ask her questions.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Number of questions about mathematics asked to each student during a group discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mon.</td>
</tr>
<tr>
<td>Amber</td>
<td>16</td>
</tr>
<tr>
<td>Claudia</td>
<td>--</td>
</tr>
<tr>
<td>Dennis</td>
<td>3</td>
</tr>
<tr>
<td>Francisco</td>
<td>2</td>
</tr>
<tr>
<td>Joaquin</td>
<td>0</td>
</tr>
<tr>
<td>Lorenzo A.</td>
<td>0</td>
</tr>
<tr>
<td>Lorenzo Y.</td>
<td>0</td>
</tr>
<tr>
<td>Teacher</td>
<td>5</td>
</tr>
<tr>
<td>Totals</td>
<td>26</td>
</tr>
</tbody>
</table>

Note there were two separate math sessions on Wednesday

*a* Cells labeled with -- indicate a student's absence

*b* Lorenzo Y. missed a significant portion of the Monday and Wednesday 1 sessions

*c* The teacher only joined the group on an occasional basis

---

4 For this section we coded all utterances where students asked a question related to mathematics, interpreted broadly. This presumes that the students are not asking each other “known-answer” questions, which generally appeared to be true with the exception of the interaction in Excerpt 1. For example, we coded questions such as “why six over eight seventy five percent?” (Excerpt 4, line 1) as a question about mathematics, but we did not include questions such as “how do I spell your name?” in the count of mathematics questions.
IA Affords the use of Discourse practices. The relationship between intellectual authority and interactional rules for mathematical discussions is reflexive: in addition to being constructed through the students’ Discourse practices, intellectual authority also mediated the sociolinguistic rules of these discussions. For example, during Excerpt 5 below, Amber was positioned in the role of the intellectual authority, and her interaction with Francisco both reflected and reified her role. In her responses to Francisco’s initial question, Amber actively constructed her authority in two ways: by providing Francisco with a very quick procedure to arrive at the answer (line 3) and by passing judgment on the quality of his question through her use of the tag “Du:h.” In both tone (with the elongated syllable) and content, this tag made Francisco’s question seem silly, and reified the “unassailable” status of her answer (which is ironic since her answer did not actually correspond to the mathematically correct solution to Francisco’s question). Francisco challenged Amber’s role as intellectual authority with his response in line 6, but Amber interrupted him (a right of authority). Again, she used an elongated syllable to communicate her opinion of the quality of his contribution. In line 8 Francisco contradicted Amber once again, but she forcefully re-asserted her dominant position in lines 9 and 11. Perhaps the most fascinating part of this exchange is in line 15, where Francis not only ceded authority to Amber, but also told the teacher that he got the answer with Amber’s help, despite the fact that he had (nearly) the correct answer in line 6!

**Excerpt 5** [Monday class 44:27-44:56]

**Prompt:** “At 6:00 AM the temperature was -4°F. By 6:00 PM, the temperature had risen 17°. What was the temperature at 6:00 PM?” (Maltesky, p. 393, Exercise #8).

1. Francisco: But who ge-Do you get number eight at all?
2. ...

3. Amber: Seventeen minus six (.) Du:h

4. Francisco: Oye ("listen" or an interjection), I found it already

5. ...

6. Francisco: Look. Subtract ss seventeen minus negative four [xxx

7. Amber: [Thats that I sai::d

8. Francisco: You said subtract thrirtee:n!

9. Amber: No, I said seventeen!=

10. Teacher: They’re almost do[ne ((addressing class))

11. Amber: [=(minus) negative four

12. Francisco: Now I get [it

13. Lorenzo A.: [Ahhh

14. Amber: I got number eight ((looking up at teacher))

15. Francisco: I got number eight too, but (kinda) she’s helped me

In contrast to Excerpt 5 where Amber acted as authority throughout the interaction, during times when intellectual authority in the group was disputed (i.e. during the first 7 lines of Excerpt 4 above, or in Excerpt 2), the students’ discussion reflected a confrontational pattern marked by frequent interruptions, contradictions, and jockeying for the conversational “floor.”

IA is Distinct from Having the Right Answer or Perceived Mathematical Ability. The construction of intellectual authority did not appear to be directly related to either the students’ stated perceptions of who was best at mathematics, or to who actually had the correct answer to a given question. In a survey after the week of video-recorded classes, we asked the students in
this group to rank their group-mates in terms of their mathematical skills. The number of each
student’s first and second place rankings is recorded in the Table 2:

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amber</td>
<td>3</td>
</tr>
<tr>
<td>Claudia</td>
<td>2</td>
</tr>
<tr>
<td>Dennis</td>
<td>5</td>
</tr>
<tr>
<td>Francisco</td>
<td>1</td>
</tr>
<tr>
<td>Joaquin</td>
<td>1</td>
</tr>
<tr>
<td>Lorenzo A.</td>
<td>0</td>
</tr>
<tr>
<td>Lorenzo Y.</td>
<td>0</td>
</tr>
</tbody>
</table>

*aLorenzo Y was not present the day of the survey, so he did not vote, though
the other students did include him in their ranked lists.

If we assume that students constructed authority based on who they ranked best at
mathematics, then Table 2 suggests that Dennis should have acted as, and been treated as an
authority for this group. However, Dennis did not appear to play a significant role as an
intellectual authority for the group in terms of being asked questions about the mathematical
work of the group (see Table 1) or using direct contradictions (this analysis is detailed
elsewhere). Perhaps most surprising is the pattern that the construction of authority was not
connected to who actually had the correct answer to any of the given problems. For example, in
Excerpt 5 above, Francisco ceded authority to Amber in line 15 by giving Amber credit for
helping him solve the problem, even though he had contributed the (nearly) correct answer
earlier in line 6. Likewise, in separate interaction during the first Wednesday session, Joaquin
offered Francisco a correct algorithm for when to add a decimal in the long division algorithm,
but the group ignored his contribution. This pattern contradicts conclusions from previous
analyses of group work in mathematics classes which describe how individual students with the
highest grades and best performance in mathematics assume the role of leader in a small group
(see review in Cohen, 1994). It is possible that this group is unique in some way, or different from the groups documented in other studies. It is also possible that methodological differences account for this discrepancy.

5. Implications for Instruction & Significance

Our study of these students’ peer discussions revealed that the students employed hybrid Discourse practices, including the use of Discourse practices commonly associated with school mathematics, the use of Discourse practices of professional mathematics, the use of two languages, as well as the use of references to popular culture and out-of-school Discourse practices. Some studies of students’ reasoning in mathematics have emphasized that students must learn to reason (and use Discourse practices) like professional mathematicians (Harel & Sowder, 1998). We agree that an instructional objective of using peer discussions is to help students develop the use Discourse practices valued in both school and professional mathematics. At the same time, we view the students’ use of multiple Discourse practices during their discussions as a resource that may useful for the development of mathematical Discourse practices. Gutierrez et al. emphasize that Hybridity and Third Spaces are productive sites of learning (Gutiérrez, Baquedano-López, & Tejeda, 1999), and we concur in this orientation.

Giving explicit attention to how intellectual authority is constructed and negotiated during group discussions in mathematics classrooms may be one way to transform how relations of power are enacted among students, and between students and teachers during mathematical discussions. Yackel and Cobb illustrated how “sociomathematical norms” developed in a lower grade classroom over the course of the year (Yackel & Cobb, 1996). Like the development of sociomathematical norms, if intellectual authority is explicitly discussed, negotiated, and built in
a collaborative classroom environment, then it may be possible to reconfigure peer discussions in mathematics classrooms.

As we noted in the findings about mathematical Discourse practices, the majority of the students’ discussions revealed a calculational orientation in their group discussions of the given exercises. One possible avenue for future exploration might be to study how changing the types of questions that the students are asked to solve, or explicitly altering the goals of the activity and the roles (division of labor) in the small group results in changes to the Discourse practices that students use during discussions. For example, if the students worked on problems or modeling activities that had multiple valid interpretations, or several viable solutions, then the orientation of the students’ discussions may shift away from the goal of finding one right answer (as a multiple choice question explicitly demands) to a goal of providing answers and justifying solutions using agreed upon (and negotiated!) reasoning techniques. Both Lampert and O’Connor have documented how open-ended questions and explicit attention to conversational norms can result in rich mathematical discussions in a whole-class setting (Lampert, 1990; O’Connor, 2001). Future research may consider how their results may be replicated in small-group discussions.

An important consideration that is beyond the scope of this analysis is a systematic treatment of how these students’ broader social and cultural practices (Gutiérrez & Rogoff, 2003) (i.e. in terms of gender, race/ethnicity, home language practices, etc.) impacted the sociolinguistic characteristics of the group’s mathematical discussions. Further research in this area may seek to show the connection between the “constellations” of students’ practices, and their interactions during small group discussions.
As Gee and Green have noted, one fundamental theme in discourse analysis is *reflexivity*: Discourse practices are shaped by the contexts within which they are situated, and at the same time, Discourse practices renew and reify the context (Gee, 1996; Gee & Green, 1998). In this study, we have found a reflexive relationship between students’ small group discussions, the construction of intellectual authority, and the practice of school mathematics. Certain discursive moves appeared to create the position of “intellectual authority” while at the same time, playing the role of intellectual authority for the group made available certain discursive moves. Figure 1 illustrates how certain discursive moves create a position of intellectual authority, and conversely, how the role of intellectual authority makes available certain discursive moves. For instance, the right to judge the validity of another person’s contribution is an affordance of intellectual authority. Giving another student credit for solving a problem reifies her (role of) intellectual authority.

![Diagram](image)

*Figure 1: The reflexive relationship between discourse practices and intellectual authority*

Yet, this reflexive relationship between intellectual authority and Discourse practices only makes sense when it is considered in relation to the broader context within which the conversation is situated: the practice of school mathematics. Below, we attempt to represent this three-part reflexive relationship. Figure 1 shows how intellectual authority is constructed in
micro-interactions between students, but the reflexive relationship illustrated in Figure 1 fails to account for why students’ peer mathematics discussions are different from people’s interactions documented in other settings such as casual conversations, or other institutional interactions. Figure 2 illustrates the three-way reflexive relationship between intellectual authority, school mathematics as a cultural practice (or cultural model), and Discourse practices enacted in students’ peer discussions. In this figure we can see more clearly that the reflexive relationship is not just between Discourses and roles, but between discourses, roles, and the setting within which Discourses are situated (Gee, 1996; Gee & Green, 1998). The practice of school mathematics makes available roles such as “intellectual authority” by emphasizing that every question has one right answer, and the goal of the activity is to find that answer. Likewise, the setting of school mathematics makes available, and it is reified by certain discourse practices (such as asking known-answer questions, IRE sequences, and doing traditional word problems [Moschkovich 2007a]). In figure 2 we have included examples of how the practice of school mathematics creates a position of intellectual authority, and an example of how students’ discourse situates the conversation as part of school mathematics.
6. Summary

This research has illustrated the characteristics of (calculational) school mathematics discussions among a group of bilingual middle school students. We analyzed the students’ mathematical discussions and found that, although students primarily engaged in a subset of school mathematics Discourse practices with a “calculational” or “computational” orientation, these discussions also reflected multiple and hybrid (Gutiérrez, Baquedano-López, & Tejeda, 1999) Discourse practices. This finding is encouraging because it points to how students already use some of the Discourse practices associated with professional mathematics, so this is a resource teachers build upon. Practitioners and researchers need to carefully consider how to arrange small group discussions so that they afford and build on multiple types of mathematical Discourse practices, not only the calculational orientation that is related to the goals of typical of school mathematics exercises but also practices associated with professional mathematical Discourse.

We built upon the construct of “intellectual authority” (Lampert, 1990) and described how IA was continually negotiated and reified through the students’ interaction. We also showed how IA served as a meaningful role for the students in this group, and thus it allowed the person constructed as the intellectual authority to use particular conversational moves such as passing judgment on others’ contributions. Finally we showed that IA was not necessarily aligned with who had the correct answer to a given problem, or to who the students perceived as most competent in mathematics. These findings indicate that explicitly addressing how authority is
constructed in small groups and reified though interactions may help make small group work more aligned with the pedagogical aims of using small group discussions.

***

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