Practice problems for final exam

MATH113 - Carlos Chiquete - 5/4/09

Note: the following bunch of problems are **not exhaustive** but they are **essential**.

1 Limits

Obtain the limits and it they do not exist write DNE and state the reason:

(a) $\lim_{x\to -5} \dfrac{x^2-25}{x+5}$

(b) $\lim_{x\to -2} \dfrac{x+2}{x^2-x-6}$

(c) $\lim_{x\to 3} \dfrac{x+3}{x^2-x-6}$

(d) $\lim_{x\to \infty} \dfrac{x^2-25}{x+5}$

(e) $\lim_{x\to \infty} \dfrac{x+2}{2x+5}$

(f) $\lim_{x\to -\infty} \dfrac{x}{x^2+4}$

2 Limit definition of derivatives

Obtain the derivatives of the function through the limit definition

(a) $f(x) = x^2 - 2$

(b) $g(x) = 4x + 2$

(c) $h(x) = \dfrac{4}{x}$

(d) $j(x) = 3x^2 - 3x + 2$

3 Differentiation

3.1 Basic rules

Find the derivatives using the basic rules. (Hint: **simplify as much as possible before differentiating**)

(a) $f(x) = x^2 + 3x + 1$

(b) $g(x) = \dfrac{x^5 + x^2 + 1}{x}$

(c) $h(x) = \sqrt{x} - 2x + \pi$

(d) $a(x) = \dfrac{2}{\sqrt[3]{x}} + \ln(x)$

(e) $b(x) = e^x + 2x + \dfrac{12}{x^2}$

3.2 Product and Quotient Rule

Always simplify before differentiating! In the following, find the derivatives

(a) $f(x) = x^2e^x + e^x$, Get tangent line at $(1, 2e^1)$

(b) $g(x) = (x^3 + 2)\ln(x)$

(c) $h(x) = \dfrac{3x+2}{x^2+6}$

(d) $b(x) = (x^2 + x)(3x + 4)$

(e) $c(x) = \dfrac{e^x}{e^x+1}$, Additionally get tangent line at $(0, 1/2)$

(f) $d(x) = \ln(x)e^x + \ln(e^x)$

3.3 Chain Rule

Find the derivatives, but remember **to simplify the log expressions as much as possible**!

(a) $f(x) = (x^5 + 2x + 2)^4$

(b) $g(x) = (2x + 2)^{-2}$

(c) $h(x) = \ln\left(\dfrac{1}{(x^2+2x)^2}\right)$

(d) $a(x) = e^{2x+2} + e^{-x^2}$

(e) $b(x) = 3\ln((x + 2)(x + 3))$

3.4 Implicit differentiation

Obtain dy/dx or y' for the following equations

(a) $y + x^2 = 7x$, and get tangent line at $(1, 6)$.

(b) $x^2y + 6x^5 = 90$

(c) $y^2 + x^2 = 2x + 2$

(d) $x^2 + y^2 = 8$, Get equation of tangent line at $(2, 2)$.

(e) $3x^3y + 23x + 3y = 2$

3.5 Applications of derivatives

3.5.1 Analyzing functions

Know how to **critical points, relative extrema, open intervals where function is increasing/decreasing and concave up/down, and points of inflection** these for the following functions:

(a) $f(x) = x^3 - 6x^2 - 5x + 10$

(b) $g(x) = \dfrac{1}{x^2+1}$

(c) $h(x) = x^4 - 2x^2 + 10$

(d) $p(x) = e^{-x^2}$

and draw a graph for each with all minima/maxima and inflection points labeled.

3.5.2 Optimization

(a) Sec. 3.4: # 16-22 and # 33-39.

3.6 Related rates

Obtain dy/dt as a function of x, y and dx/dt

(a) $y = 4\pi x^3/3$

(b) $y = 4\pi x^2$

(c) $y = x^2 + x$

(d) $y^2 = x^3 + 2$

Additionally,

(a) For (a), get dx/dt when $x = 3$, and $dx/dt = 2$.

(b) For (c) in the previous section, find dy/dt when $x = 2$ and $dx/dt = 5$.

(c) For (d) in hte previous section, find dy/dt when $y = 3$, $x = 1$ and $dx/dt = 10$.

4 Integration

4.1 Basic rules

Simplify then take anti-derivative:

(a) $\int (x^2 + x^{-2})dx$

(b) $\int \frac{x^2 + 2x + 4}{x^4} dx$

(c) $\int (x^3 + x^2 + x - 1)dx$

(d) $\int \left(e^x + 3x + \frac{1}{x}\right) dx$

(e) $\int \left(\frac{1}{\sqrt{x}} + x\right) dx$

(f) $\int \frac{x^3 + 2x + 6}{2x} dx$

4.2 Integration by substitution

Get anti-derivatives

(a) $\int \frac{x}{(x^2+1)^2} dx$

(b) $\int \frac{x^2}{2x^3 + 1} dx$

(c) $\int \frac{2x^3 + 1}{x^2} dx$ (Hint: not really a substitution problem but notice the similarity to (b)).

(d) $\int 4x^2 e^{x^3} dx$

(e) $\int e^{-7x} dx$

(f) $\int \frac{1}{2x + 1} dx$

(g) $\int (x^7 + 1)(x^8 + 8x)dx$

(h) $\int \frac{x}{x^2 + 1} dx$

4.3 Definite integrals

Remember to get anti-derivative and then use the FTC[1].

(a) $\int_0^1 (7x^6 + 8x^7 + 1)dx$

(b) $\int_{-2}^2 (3x^3 - 2x)dx$

(c) $\int_1^{16} \sqrt[4]{x}\, dx$

(d) $\int_1^{10} \left(\frac{2}{x} + \frac{1}{x^2}\right) dx$

4.4 Integration by parts

Always **simplify** ln expressions...

(a) $\int 3xe^{2x} dx$

(b) $\int x \ln(x^3) dx$

(c) $\int xe^{4x} dx$

(d) $\int 3x^2 e^{-x} dx$

(e) $\int x^5 e^{x^3} dx$ (Hint: do substitution first)

[1] Fundamental Thm. of Calc.: $\int_a^b f(x)dx = F(b) - F(a)$ **if** $F'(x) = f(x)$