ERRATA & NOTES
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An Introduction to Structured Population Dynamics

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▶ *Page 3.* A and C are square matrices.

▶ *Page 7.* If $P\frac{x(t)}{p(t)} = 0$ for some t,then $\lambda = 0$ would be an eigenvalue associated with a nonnegative eigenvector $\frac{x(t)}{p(t)}$, in contradiction to Theorem 1. Therefore, $P\frac{x(t)}{p(t)} \neq 0$ for all t.

▶ *Page 8.* In Definition 1, "... nonnegative right eigenvector $u \geq 0$, then ...".

▶ *Page 8.* Delete the sentences:

"By Definition 1, n is given by (see [179])

$$n = \max_{x \in R_+^m/\{0\}} \frac{\left|F\left(I-T\right)^{-1}x\right|}{|x|}.$$

From this formula we can obtain a biological interpretation for n as follows."

▶ *Pages 9-10.* Replace the entire paragraph following (1.12) with the following paragraph :

"The dominant eigenvalue of $F(I-T)^{-1}$ is the dominant eigenvalue of the $j \times j$ irreducible sub-matrix

$$S = \begin{bmatrix} f_1^\tau e_1 & \cdots & f_1^\tau e_j \\ \vdots & \ddots & \vdots \\ f_j^\tau e_1 & \cdots & f_j^\tau e_j \end{bmatrix}.$$

from the upper left hand corner; thus (see [179])

$$n = \max\nolimits_{y \in R_+^j/\{0\}} \min\nolimits_{1 \leq i \leq j} \frac{(Sy)_i}{y_i} = \min\nolimits_{y \in R_+^j/\{0\}} \max\nolimits_{1 \leq i \leq j} \frac{(Sy)_i}{y_i}. \qquad (1.12)$$

For a class distribution of individuals $x \in R_+^m$ the expected distribution of offspring over the course of their life times is $F(I-T)^{-1}x$. If we consider the expected distribution of offspring from a group of newborn members of the population, then x has nonzero entries in only its first j entries, so that

$$x = \begin{bmatrix} y \\ 0 \end{bmatrix}$$

where $y \in R_+^j$ is the distribution of the newborns. Then the distribution of expected offspring from this group of newborns is

$$F(I-T)^{-1}x = \begin{bmatrix} Sy \\ 0 \end{bmatrix}$$

and the total expected i-class offspring is the i^{th} component $(Sy)_i$. Then $\min_{1\leq i\leq j}(Sy)_i/y_i$ is the smallest expected per capita obtained in all the newborn classes. From the first formula in (1.12) we see that n is the maximum of this minimum, where the maximum is taken over all possible newborn class distributions y. This maximum is attained by class distributions proportional to the eigenvector u associated with n as the dominant eigenvalue of $F(I-T)^{-1}$. (A similar interpretation of n is obtained, with maximum and minimum interchanged, from the second formula in (1.12).)"

► *Page 18.* Theorem 1.2.1, part (b): "If $r > 1$, then $x = 0$ is unstable. If, in addition, (1.20) is point dissipative, i.e. (1.23) holds, and if $P(x)x = 0$, $x \geq 0$, implies $x = 0$, then (1.20) is uniformly persistent with respect to the extinction equilibrium $x = 0$."

► *Page 41 (bottom line).* Replace D(C,S) by D(S,C).

► *Page 45.* Correct subscripts as follows:

$$\beta_1 s(0) + \beta_1 (1 - s(0)) \text{ should be } \beta_1 s(0) + \beta_2 (1 - s(0))$$
$$f_1 s(0) + f_1 b (1 - s(0)) \text{ should be } f_1 s(0) + f_2 b (1 - s(0))$$

► *Page 104.* In next to last equation replace $w(t)$ by $p(t)$.

► *Page 105.* Corrected equation (3.8):

$$q(t+1) = rh\left(\frac{v}{|v|}q(t)\right)q(t), q(0) = |x(0)| > 0.$$

► *Page 106 (Theorem 3.1.2).* Replace $|\varphi(t) - v| < \delta$ by $\left|\varphi(t) - \frac{v}{|v|}\right| < \delta$. In first sentence of the last paragraph, $h(t) = 1 + cp$ should be $h(t) = (1 + cp)^{-1}$.

► *Page 107.* Replace $p(t)$ by $q(t)$ in the offset equation to obtain

$$q(t+1) = rq(t)\exp(-cq(t)), \quad q(0) = |x(0)| > 0$$

► *Page 108.* In last display, corrected subscripts for t_{ij} are $t_{ij} = \pi_j \tau_{ij} \gamma_j$.

► *Page 109 (second line).* ".... a common resource availability function $u \geq 0$ and the survival ....."

► *Pages 110-112.* θ_{cr} should be θ^{cr}.

► *Page 160 (first two lines).* "In other words, Lemma A.2.2 applies to (A.12) and hence to (A.10)-(A.11) when ....". In Theorems A.2.3 and A.2.4 change (A.10) to (A.12).

► *Page 167 (Lemma C.0.1).* "Suppose $q(t)$, $q(0) \neq 0$, satisfies the scalar ..."

▶ *Corrected page 91:*

An illustration of a Hopf bifurcation to a limit cycle (followed by further bifurcations to chaos) can be seen in FIGS. 2.1 and 2.2 where solutions of the McKendrick model (2.6), with $a_M = \infty$, are plotted for

$$\begin{aligned} \delta &= \text{constant} > 0 \\ \beta &= b u_T(a) \exp\left(-cp(t-a)\right), \quad b > 0, \quad c > 0. \end{aligned}$$

Here $u_T(a)$ is the unit step function at $T > 0$ and hence in this model only individuals of age $a \geq T$ are fertile. The nonlinear density effects in this model occur in an individual's fertility rate which is dependent on the total population size at birth. The model equations

$$\begin{array}{l} \partial_t \rho + \partial_a \rho = -\delta \rho \\ \rho(t,0) = b \int_T^\infty \exp\left(-cp(t-a)\right) \rho(t,a) da \end{array}$$

lead to the system of delay differential equations

$$\begin{array}{l} p'(t) = -\delta p(t) + B(t) \\ B'(t) = -\delta B(t) + be^{-\delta T} B(t-T) \exp\left(-cp(t-T)\right) \end{array} \tag{2.24}$$

for total population size $p(t)$ and the *total birth rate* $B(t) \doteq \rho(t,0)$.

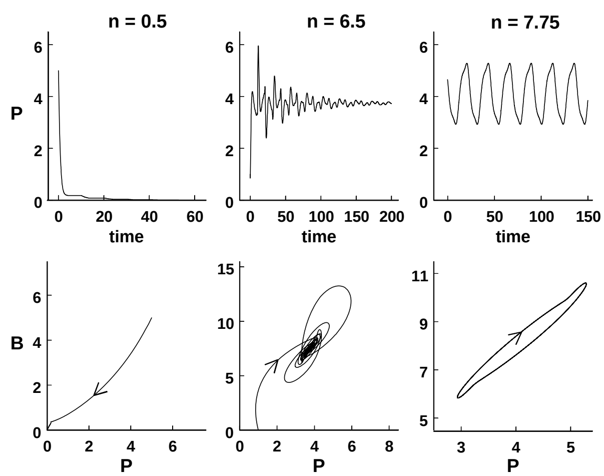

FIG. 2.1. For the delay equations (2.24) the inherent net reproductive number is $n = b\delta^{-1}e^{-\delta T}$. The population goes extinct for $n = 0.5$, equilibrates to a positive equilibrium for $n = 6.5$, and (after a Hopf bifurcation occurs) oscillates periodically for $n = 7.75$. Plots are shown for $\delta = 2$, $c = 0.5$ and $T = 10$.

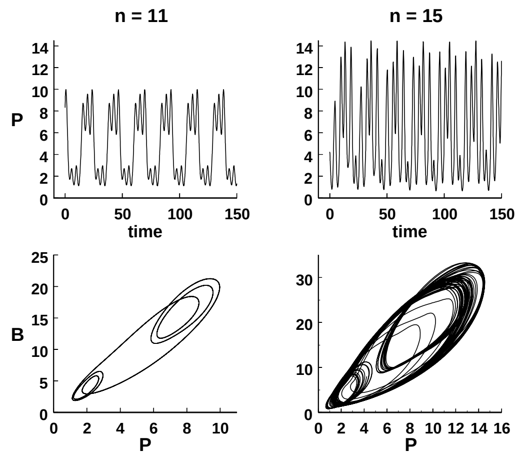

FIG. 2.2. With further increases in n, the periodic solution in Fig. 2.1 becomes more complicated for $n = 11$ and an apparent chaotic oscillation occurs for $n = 15$.