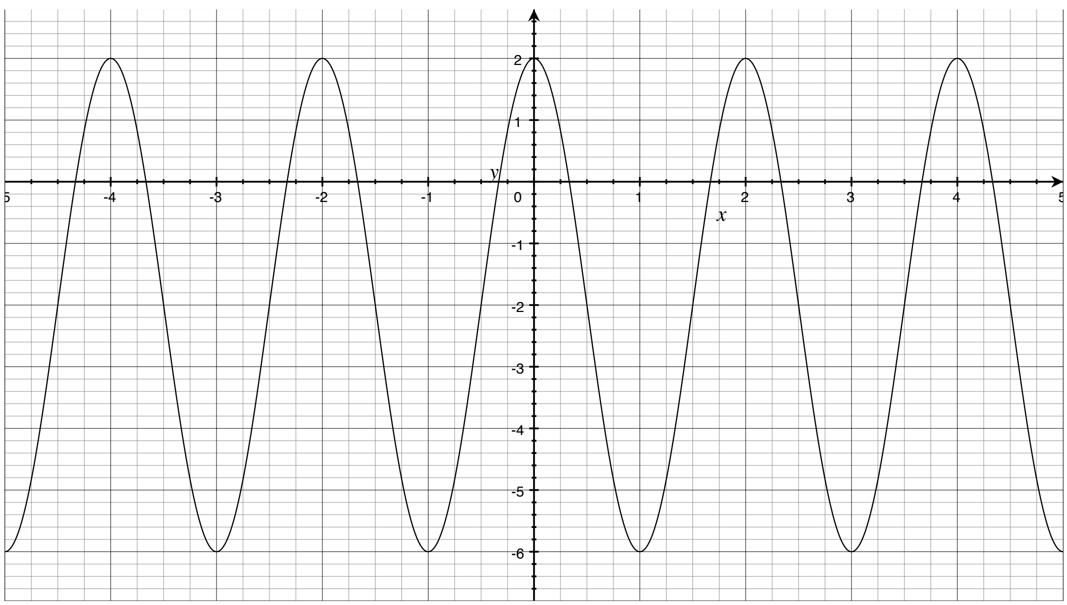
Math 111 - Trigonometry
Practice Exam 2

Warning: This study guide is not all inclusive- there may be material on the test which is covered in the book but not here. This is simply meant to be a supplemental study aid to the homework and class notes

1. Graph two periods of $y = -2 + 4\cos(\pi(x-2))$. Find the amplitude, period, and average value. Label the axes in a way to reflect the important characteristics of the graph (i.e. when the function reaches it maximum, minimum, and average value).



2. Below is a data table for a function involving a trigonometric function.

x	-1	0	1	2	3	4
y	3	7	11	7	3	7

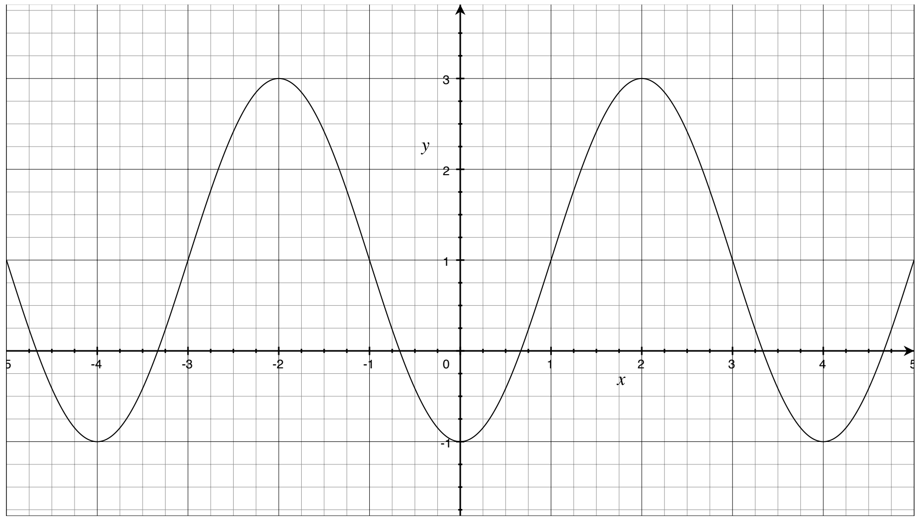
Determine a possible formula for the trigonometric function.

Solution: Here are two possibilities. There are others:

$$y = 7 + 4\sin(\frac{\pi}{2}x)$$

$$y = 7 + 4\cos(\frac{\pi}{2}(x-1))$$

3. Consider a function with the below graph:



(a) Find a formula for this function involving sine.

Solution:

$$y = 1 + 2\sin\left(\frac{\pi}{2}(x - 1)\right)$$

(b) Find a formula for this function involving cosine.

Solution:

$$y = 1 - 2\cos\left(\frac{\pi}{2}x\right)$$

4. Find the following exact values

(a) The period of the function $y = \tan(4(x - 3))$

Solution: $\pi/4$

(b) The horizontal shift of the function $f(x) = 1 - 2\sin(3(x - 4))$

Solution: 4 units to the right

(c) The average value of the function $g(x) = 2 - 2\cos(\pi(x + \pi))$

Solution: 2

(d) $\tan(\cos^{-1}(-\frac{5}{13}))$

Solution: -12/5

(e) $\cos(\arctan(\sqrt{3}) + \sin^{-1}(\frac{1}{3}))$ (Hint: Use the Cosine of a Sum identity)

Solution:

$$\frac{2\sqrt{2}-\sqrt{3}}{6}$$

(f) $\sin(\tan^{-1}(\frac{3}{2}))$

Solution: $3/\sqrt{13}$

5. Mark the following true or false.

(a) $\csc(-x) = -\csc(x)$

Solution: True (since $\sin(-x) = -\sin(x)$

(b) The amplitude of $y = -3\sin(5(x+3))$ is -3.

Solution: False (the amplitude is always positive)

(c) $\arctan(z+3) = \arctan(z) + 3$

Solution: False

(d) $\tan^{-1} x = \arctan x$

Solution: True

6. For each of the below expressions, determine if the statement is *possible* or *impossible*. For those that are possible, determine the exact value. Represent all angles in radians.

(a) $\arcsin \pi$

Solution: Impossible (domain of arcsin is $[-1, 1]$)

(b) $\cos^{-1}(-\frac{\sqrt{3}}{2})$

Solution: Possible, and the value is $(5\pi)/6$.

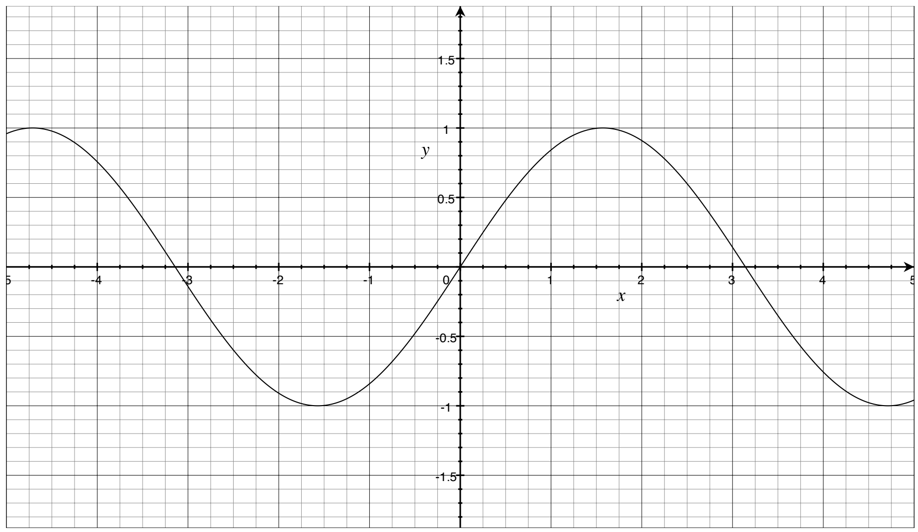
(c) $\sin(-4)$

Solution: Possible, and the value is approximately 0.7568

(d) $\sin^{-1}(30°)$

Solution: Impossible (domain of arcsin is $[-1, 1]$).

7. Use the below graph of $y = \sin x$ to approximately find $\arcsin 1$, and mark this point on the appropriate axis.



8. Find the domain and range of the following functions.

   (a) $y = \sin^{-1}(z + 1)$

   Solution: Domain $= [-2, 0]$, Range $= [-\pi/2, \pi/2]$

   (b) $y = 3\arccos(\frac{w}{2})$

   Solution: Domain $= [-2, 2]$, Range $= [0, 3\pi]$

   (c) $y = \tan^{-1}(z) + 1$

   Solution: Domain $= (-\infty, \infty)$, Range $= (-\pi/2 + 1, \pi/2 + 1)$

9. Find an expression for $\sin(\tan^{-1}(\frac{w}{z}))$ that does not involve trigonometric functions.

   Solution:

$$\frac{w}{\sqrt{w^2 + z^2}}$$

10. Find all solutions to $\cos x = \frac{1}{2}$ (*i.e. find all x values that make this statement true*). Which one is $\cos^{-1} \frac{1}{2}$?

Solution:

$$x = \begin{cases} \frac{\pi}{3} + 2\pi n \\ \frac{5\pi}{3} + 2\pi n \end{cases}$$

The only solution that is in the interval $[0, \pi]$ (the range of $\arccos x$) is $x = \pi/3$, so $\cos^{-1}(1/2) = \pi/3$.

11. Consider the following two statements:

    (a) $x = \sin^{-1} y$ always means $y = \sin x$

    (b) $y = \sin x$ always means $x = \sin^{-1} y$

    Is statement (a) true or false? Is statement (b) true or false? Explain.

    Solution: Recall the definition of inverse sine is that $x = \sin^{-1} y$ means $y = \sin x$ AND $x \in [-\pi/2, \pi/2]$. So statement a) is true; the definition tells us that if $x = \sin^{-1} y$, then it must be true that $y = \sin x$.

    However statement b) is false. As an example to see when this is not true, consider $x = \pi$. Then $y = \sin \pi = 0$, but $\sin^{-1} y = \sin^{-1} 0 = 0$, and $0 \neq \pi = x$. In fact any number x that is outside the restricted domain $[-\pi/2, \pi/2]$ of $y = \sin x$ that we used to define inverse sine will show that statement b) is false, exactly because these x cannot satisfy the second part of the definition of inverse sine.

12. Is it true that if $x = \tan 5$, then $5 = \arctan x$?

    Solution: The range of $\arctan x$ is $(-\pi/2, \pi/2)$, and 5 is not in this interval, so the statement cannot possible be true. In general, $\arctan(\tan x) = x$ only if $x \in (-\pi/2, \pi/2)$.

13. For what values of x will $\sin(\arcsin x) = x$?

    (a) all x

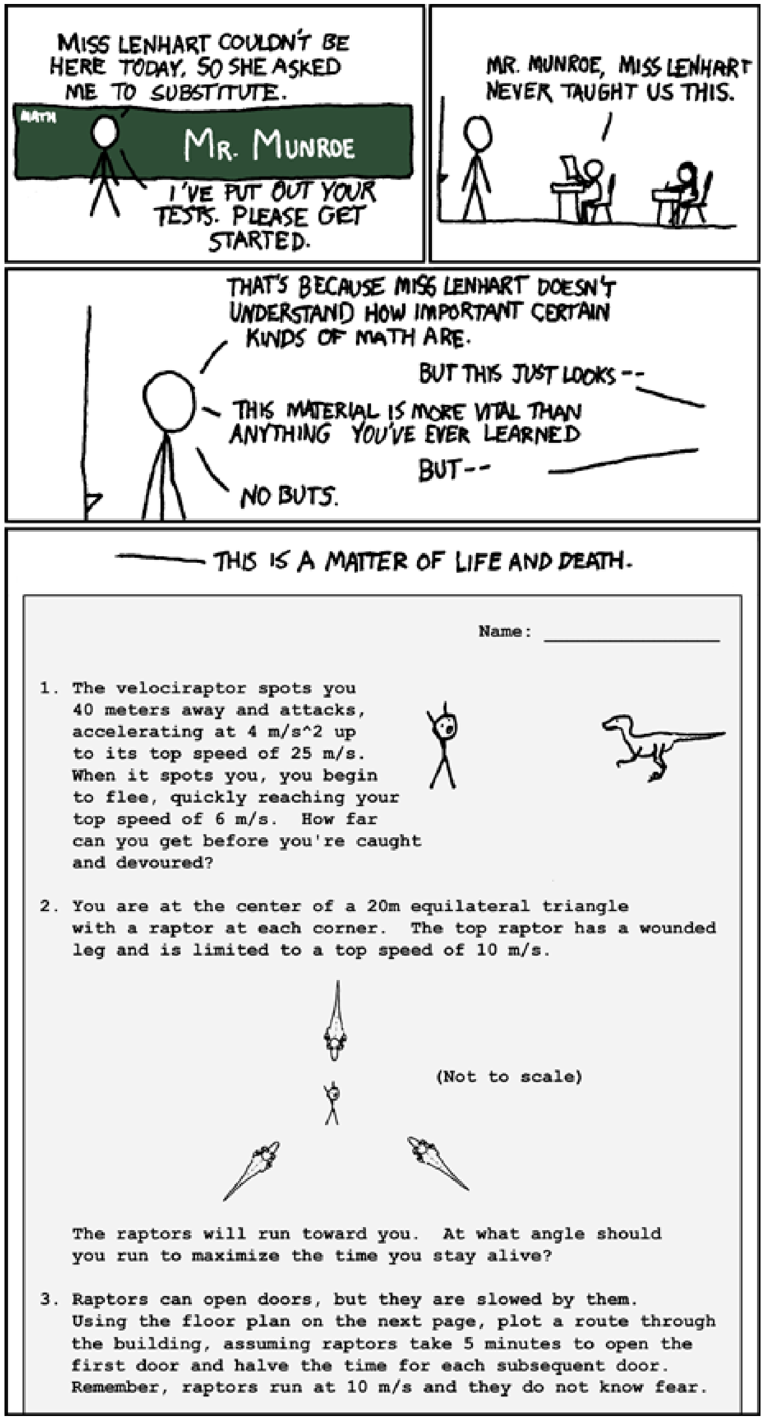
    (b) $[-1, 1]$

    (c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

    (d) $[0, \pi]$

    (e) none of the above

    Solution: This statement is true for all x in the domain of $\arcsin x$, which is $[-1, 1]$.


MISS LENHART COULDN'T BE HERE TODAY, SO SHE ASKED ME TO SUBSTITUTE.
MATH
MR. MUNROE
I'VE PUT OUT YOUR TESTS. PLEASE GET STARTED.
MR. MUNROE, MISS LENHART NEVER TAUGHT US THIS.
THAT'S BECAUSE MISS LENHART DOESN'T UNDERSTAND HOW IMPORTANT CERTAIN KINDS OF MATH ARE.
BUT THIS JUST LOOKS --
THIS MATERIAL IS MORE VITAL THAN ANYTHING YOU'VE EVER LEARNED
BUT--
NO BUTS.
THIS IS A MATTER OF LIFE AND DEATH.
Name: ________________
1. The velociraptor spots you 40 meters away and attacks, accelerating at 4 m/s^2 up to its top speed of 25 m/s. When it spots you, you begin to flee, quickly reaching your top speed of 6 m/s. How far can you get before you're caught and devoured?
2. You are at the center of a 20m equilateral triangle with a raptor at each corner. The top raptor has a wounded leg and is limited to a top speed of 10 m/s.
(Not to scale)
The raptors will run toward you. At what angle should you run to maximize the time you stay alive?
3. Raptors can open doors, but they are slowed by them. Using the floor plan on the next page, plot a route through the building, assuming raptors take 5 minutes to open the first door and halve the time for each subsequent door. Remember, raptors run at 10 m/s and they do not know fear.