Exponential and Logarithm Problems

WebAssign Problems

- WebAssign #11 - 1
  $(-1, 1/2)$ is on the graph, so plugging that into $f(x) = a^x$ gives $1/2 = a^{-1}$. Solving for a by raising both sides to -1 gives $a = 2$.

- WebAssign #11 - 3
  We're given the equation $v(t) = A(1 - e^{-kt})$, and I'm given $A = 62$, $k = .23$ (your numbers may be different). To find the initial velocity, plug in $t = 0$ to get

  $$v(0) = A(1 - e^0) = A(1 - 1) = 0$$

  To find the velocity after 5 or 10 seconds, we just need to plug in $t = 5, 10$ into $v(t)$ and round to the nearest tenth.
  The end behavior here is important: As t gets very large, $e^{-kt} = \frac{1}{e^{kt}}$ will become 1 over a very large number, which is a very small number. So then $v(t) \approx A(1 - 0)$ as t gets very large, and we see there is a horizontal asymptote at A. This should help you figure out which graph is the correct one, as well as part d).

Exponential Functions Worksheet

- #4

  $$\frac{a^4 b^{-5}(1+y)^5}{b^{-2}a^7(1+y)^3} = \frac{b^2(1+y)^2}{b^5 a^3} = \frac{(1+y)^2}{a^3 b^3}$$

- #5

  $$(-z)(3z^4)^2 = (-z)(9z^8) = -9z^9$$

- #10 First write the expression so that each side has the same base.

  $$\left(\frac{1}{4}\right)^{x-2} = 4^{-(x-2)} = (2^2)^{2-x} = 2^{2(2-x)} = 32 = 2^5$$

  So then the exponents need to be equal (since exponential functions are one-to-one) so $4 - 2x = 5$, and solving for x gives $x = -1/2$.

- #12 For these problems, we need to use two points on the graph to solve for the two constants C, b in $f(x) = Cb^x, b > 0$. Usually the y intercept is a good place to start.
  - a) $(0, 2)$ and $(1, 8)$ are on the graph. Plugging in the first point gives $2 = Cb^0 = C$, and the second point gives $8 = Cb^1 = 2b$, so $b = 4$.
  - b) $(-1, -4)$ and $(0, -12)$ are on the graph. Again, the y-intercept will give us $C = -12$, and plugging in the first point gives $-4 = Cb^{-1} = -12b^{-1}$. Isolate b to get $1/3 = b^{-1}$, so $b = 3$.
  - c) $(-1, 12)$ and $(0, 6)$ are on the graph, so $C = 6$, and then $12 = Cb^{-1} = 6b^{-1}$, so $b^{-1} = 2$, and then $b = 1/2$.

- #13 These are similar to #12, except here were just given two points on the graph. a) and b) give you the y-intercept, which automatically give you the scaling C value, so you just need to plug in the other point to solve for b. c) we did in class, and this one does not give you the y-intercept, so we need to get two equations to solve for the two unknowns C, b
  $(1, 12)$ on the graph gives us $12 = Cb$. $(3, 192)$ on the graph gives us $192 = Cb^3$. So our two equations are
  $$12 = Cb \qquad 192 = Cb^3$$
  Now use the first equation to solve for one variable in terms of the other. We'll solve for C in terms of b, but you could do it vice-versa. So $C = 12/b$, and we now plug this expression for C into the second equation to get
  $$192 = (12/b) * b^3 = 12b^2$$
  Now we can solve for b: Divide both sides by 12 to get $16 = b^2$, and take the square root to get $b = 4$; we don't worry about $b = -4$ since we're given $b > 0$. So now that we have b, we can use $C = 12/b$ to get that $C = 12/4 = 3$.
  Alternate Method Notice if we take the first equation $12 = Cb$ and multiply both sides by b^2, we'll get
  $$12b^2 = Cb * b^2 = Cb^3$$
  Now the second equation tells us $12b^2 = Cb^3 = 192$, so we have $12b^2 = 192$ and we can solve for b that way. Then we just plug our value for b, which is $b = 4$, into either of the above equations to solve for C.

- #14 Here we're given two tables, one function is linear and the other is exponential. How do we figure out which is which? Well remember for linear functions, when you increase x by the same amount (here by 1), then you increase the y values by some constant amount, since the graph is just a line. This is a long way of saying the *first differences* are constant. So notice in the first table, the x values are all increasing by 1, and the $f(x)$ values are all increasing by 4. That tells you the slope $m = 4$, and now use an equation for a line (point-slope or slope-intercept form) to find the y-intercept. You should get $f(x) = 4x + 4$.
  Exponential functions grow or decay much faster than linear functions. Since we've already determined the first table is linear, we know the second is exponential, so we can go about finding the formula. Use the same technique as in #13 c above, and you should get
  $$g(x) = \frac{16}{3} \cdot \left(\frac{3}{2}\right)^x$$
  Notice again you don't have the y intercept, so you will need to solve two equations for two unknowns, as above.
  So if we didn't know for sure this table was an exponential function, how could we tell? When we increase x by 1, the increase in the $g(x)$ values is not constant (as in linear), but rather increase by a *percent* of the $g(x)$ values. Notice 12 is 8 times 1.5, 18 is 12 times 1.5, 27 is 18 times 1.5. It's no accident that the amount you multiply by is the

same number $b = 3/2$. So instead of looking at the first difference being constant, you look at the *ratio* of function values to be constant.
So to summarize, when you're looking at a table where you're increasing the x by a constant increment (like 1), then for linear functions, you *add* a constant amount to the function values (the slope). For exponential functions, you *multiply* by a constant amount to the function values (the base).

- #15 $-9^2 = -81$
- #16 $(-z)^3(3z^2)^4 = -z^3 3^4 z^8 = -81z^{11}$
- #17 $(-27)^{-1/3} = -(3^3)^{-1/3} = -3^{-1} = -\frac{1}{3}$
- #18 $(125^2)^{1/3} = (125^{1/3})^2 = 5^2 = 25$
- #19 $\left[\frac{x^{-3}y^4}{5x^{-2}}\right]^3 = \left[\frac{y^4}{5x}\right]^3 = \frac{y^{12}}{125x^3}$
- #20 $(4x^3)^2 = 16x^6$
- #21 $(2n^3m^{-1})(3nm)^0 = \frac{2n^3}{m}$
- #22 $\frac{1}{a^{-1}+b^{-1}} = \frac{1}{\frac{1}{a}+\frac{1}{b}} = \frac{1}{\frac{a+b}{ab}} = \frac{ab}{a+b}$
- #26 Another useful fact we'll use about linear and exponential functions here is that *linear and exponential functions are either always increasing or always decreasing.*
  - a) $f(x)$ is not always increasing or decreasing, so this table is neither linear or exponential.
  - b) The differences in $g(x)$ look to be increasing by 1, but notice the x values are increasing by powers of 2, so the function cannot be linear. Remember that $y = b^x$ does not have an x-intercept, and this table has an x intercept at $x = 1$, so this table cannot be an exponential function of the form $y = Cb^x$.
  - c) The x values are increasing by 2, so let's check out the $h(x)$ values. They're clearly not increasing by the same amount, so $h(x)$ is not linear. Let's see if they're increasing by the same percent by looking at the ratios of $h(x)$ values:

$$\begin{aligned} 1.9167/1.3310 &= 1.44 \\ 2.76/1.9167 &= 1.44 \\ 3.9744/2.76 &= 1.44 \\ 5.7231/3.9744 &= 1.44 \end{aligned}$$

    So we see they're increasing by a constant percent of 1.44 when you increase x by 2, so this is an exponential function. So let's find b, C. b is going to be related to 1.44, but how (here we're increasing x by 2, not 1, so this ratio is not b)? I'll use

$$(1, 2.76) \Rightarrow 2.76 = Cb \qquad (3, 3.9744) \Rightarrow 3.9744 = Cb^3$$

So we have two equations in two unknowns. Solving for C in terms of b gives us $C = 2.76/b$, and plugging this into the second equation gives us

$$3.9744 = 2.76/b * b^3 = 2.76b^2 \quad \Rightarrow \quad b^2 = \frac{3.9744}{2.76} = 1.44 \quad \Rightarrow \quad b = 1.2$$

Then we have $C = 2.76/b = 2.76/1.2 = 2.3$. Then our function is

$$h(x) = 2.3 \cdot (1.2)^x$$

– d) Here we see the function values are increasing by a constant amount of 28.72 for every increase in 2 on the x values. So the function is linear, and the slope is $28.72/2 = 14.36$. We're given the y intercept is 3.5, so we have

$$k(x) = 14.36x + 3.5$$