

Math 263, Section 5
Third Test Spring 2014
75 minutes; total 125 points.

Name _____

For Questions 1-10, circle *one* answer.¹ (5 points each.)

1. An automaker claims that the gas mileage of their new model car is 35 miles per gallon; a consumer watchdog is skeptical of this claim. What hypotheses should the consumer group use to test the claim?
 - (a) $H_0: \mu < 35$ and $H_a: \mu \geq 35$
 - (b) $H_0: \mu \leq 35$ and $H_a: \mu > 35$
 - (c) $H_0: \mu \neq 35$ and $H_a: \mu = 35$
 - (d) $H_0: \mu \neq 35$ and $H_a: \mu < 35$
 - (e) $H_0: \mu = 35$ and $H_a: \mu > 35$
 - (f) $H_0: \mu = 35$ and $H_a: \mu < 35$

2. A random sample of the costs of forty repair jobs at a muffler repair shop has mean \$127.95 and standard deviation \$24.03. Which of the following is approximately the 90% confidence interval?
 - (a) $\$127.95 \pm \3.80
 - (b) $\$127.95 \pm \4.87
 - (c) $\$127.95 \pm \6.40
 - (d) $\$127.95 \pm \7.68
 - (e) $\$127.95 \pm \24.03

3. Which of the following is a criterion for choosing a t -test rather than a z -test when making an inference about the mean of a population?
 - (a) The mean of the population is unknown
 - (b) The standard deviation of the population is unknown
 - (c) The sample was not a simple random sample
 - (d) The population is not normally distributed
 - (e) The sample size is more than 120
 - (f) There is more than one sample

4. When a virus is placed on a tobacco leaf, small lesions appear on the leaf. To compare the number of lesions produced by two different strains of virus, one strain is applied to each side of a leaf. The side of the leaf is decided randomly. The lesions that appear on each side are counted on eight leaves. To test whether there is a significant difference between the numbers of lesions produced by each strain, what is the number of degrees of freedom of the appropriate t -test?
 - (a) 7
 - (b) 8
 - (c) 11
 - (d) 14
 - (e) 15
 - (f) 16

¹ CB-AP 1997

5. When performing hypothesis tests at significance level α , if the p -value obtained is _____ α , the null hypothesis will *not* be rejected.
- (a) Larger than
 - (b) Smaller than
 - (c) The same as
 - (d) None of the above.
6. A two-sided hypothesis test was done, leading to a p -value of 0.02. What is p -value of the corresponding one-sided test?
- (a) 0.98
 - (b) 0.04
 - (c) 0.03
 - (d) 0.02
 - (e) 0.01
 - (f) 0
7. In a test of statistical hypotheses, what does the p -value tell us?
- (a) The probability that the null hypothesis is true.
 - (b) The probability that the alternative hypothesis is true.
 - (c) Probability of obtaining the data we did if the null hypothesis is true
 - (d) Probability of obtaining the data we did if the alternate hypothesis is true
8. For a chi-square analysis of the data in a two-way table, which of the following statements is (are) true?
- (a) Under the null hypothesis, the expected cell count is

$$\text{Expected cell count} = \frac{(\text{Row total}) \cdot (\text{Column total})}{\text{Overall total}}$$
 - (b) In a table with r rows and c columns, the number of degrees of freedom is

$$df = (r - 1)(c - 1)$$
 - (c) A possible null hypothesis is that there is no association between the row and column variables.
 - (d) All of the above are true.
 - (e) Only (a) and (b) are true.

Use this information for Problems 9 and 10:

Television networks frequently run public opinion polls on issues of concern. During the Iraq War, one network conducted a telephone poll asking a question about the way President Obama was handling the war. At the same time, a second network ran an online poll using a very similar question. The results of the two polls are summarized in the following table:

	Poll		Total
	Telephone	Online	
Approve	339	385	724
Disapprove	780	573	1353
Total	1119	958	2077

We would like to test to see if the two polls are consistent with respect to the proportion who approve of President Obama's handling of the war, that is we want to test $H_0: p_{\text{telephone}} = p_{\text{online}}$

9. A statistic (calculated from the data assuming the null hypothesis) has the value 22.28. If the respondents in both polls can be considered to come from random samples, what is this statistic?
- (a) A t statistic with 1 degree of freedom.
 - (b) A t statistic with 957 degrees of freedom.
 - (c) A chi-square statistic with 3 degrees of freedom.
 - (d) A chi-square statistic with 1 degree of freedom.
 - (e) A z statistic.
10. If the chi-square test is used to test the null hypothesis, the expected cell count in the online poll for those who approve and that cell's contribution to the value of the test statistic are, respectively, about:
- (a) 390.1 and 7.70
 - (b) 333.9 and 6.78
 - (c) 390.1 and 6.69
 - (d) 333.9 and 7.81
 - (e) 362.0 and 1.46

11. (15 points) Wayne Gretzky (“The Great One”) is a famous Canadian hockey player who is the lead scorer in National Hockey League history. He played for the Edmonton Oilers and retired in 1999. In his last season, Gretzky played 41 games and missed 17 due to injury. The statistics for these games are in the table.²

	Number	Mean number of goals per game	Standard deviation
Games with Gretzky	41	4.73	1.29
Games without Gretzky	17	3.88	1.18

Consider the 41 games a random sample of the games the Oilers played with Gretzky and the 17 games a random sample of games they played without him. Is there evidence that the Oilers scored higher when Gretzky was playing? Use a 1% significance level and the following steps to decide:

The variable is quantitative—the number of goals per game. We do a two-sample hypothesis test. Let Population 1 be the games Gretzky played and Population 2 be the games in which he did not play.

- (a) Null hypothesis is that the two populations of games have the same mean number of goals:

$$H_0: \mu_1 = \mu_2.$$

- (b) Alternate hypothesis is that the games in which Gretzky played have a higher mean number of goals: $H_a: \mu_1 > \mu_2$.

- (c) The standard error of the difference in means is

$$SE = \sqrt{\frac{1.29^2}{41} + \frac{1.18^2}{17}} = 0.35$$

Thus the t -value is

$$t = \frac{4.73 - 3.88}{0.35} = 2.43.$$

The t -distribution has 16 degrees of freedom.

- (d) The p -value given by the t -table is between 0.01 and 0.02. With a calculator or computer $p = 0.0136 = 1.36\%$.

- (e) We do not reject the null hypothesis at the 1% level and conclude that we do not have evidence that the Oilers scored a greater number of goals when Gretzky was playing.

² “The Great Gretzky” in *Chance*, Reported in *Statistics: Learning from Data*, R. Peck, Brooks-Cole 2014.

12. (22 points) A classic randomized trial tested the efficacy of the drug sulphinpyrazone in reducing the risk of death in heart attack patients. The treatment group was given sulphinpyrazone and the control group was given a placebo; none of the patients were told which group they were in. The results were:

Group	Outcome		
	Lived	Died	
Treatment	692	41	733
Control	682	60	742
	1374	101	1475

We will look at the proportion that died. However, it is also fine to look at the proportion that lived. Doing this gives different \hat{p}_1 , \hat{p}_2 , \hat{p} values but the same z and p -values.

Let population 1 be the population of heart attack patients who had sulphinpyrazone; let population 2 be the population of heart attack patients who had the placebo.

(a) We consider the proportion of each group who died.

(i) Null hypothesis: Proportion died equal in two groups

$$H_0: p_1 = p_2$$

(ii) Alternative hypothesis: Proportion died is lower in treatment group

$$H_a: p_1 < p_2$$

(iii) We have

$$\hat{p}_1 = \frac{41}{(692 + 41)} = \frac{41}{733} = 0.056,$$

$$\hat{p}_2 = \frac{60}{(682 + 60)} = \frac{60}{742} = 0.081.$$

$$\hat{p} = \frac{(41 + 60)}{(733 + 742)} = \frac{101}{1475} = 0.068.$$

(iv) Using the pooled proportion to find the standard error,

$$z = \frac{0.056 - 0.081}{\sqrt{0.068(1 - 0.068) \left(\frac{1}{733} + \frac{1}{742} \right)}} = -1.91.$$

(v) From the table we find p -value is $0.0281 = 2.81\%$

(vi) Since the p -value is small, we reject the null hypothesis and conclude that the treatment group had a significantly lower death rate.

(b) The experiment was designed this way to enable the experimenters to conclude the observed drop in heart attacks was caused by sulphinpyrazone.

(i) The patients were assigned randomly to ensure that the treatment and control groups were as similar as possible; this allows us to conclude that sulphinpyrazone *caused* the significantly lower death rate.

(ii) Patients were not told which group they were in to prevent the Hawthorne effect where knowing they were being treated changes the way patients take care of themselves.

13. (16 points) An AP-GfK Poll conducted Oct. 3-7, 2013, about the Health Exchange roll out³ said 40% of the 1227 adults surveyed said the Health roll out had not gone well.

(a) The margin of error can be found using $p = 0.5$ or $p = 0.4$:

$$ME = 1.96 \sqrt{\frac{0.5(1 - 0.5)}{1227}} = 0.0278 = 2.78\%.$$

$$ME = 1.96 \sqrt{\frac{0.4(1 - 0.6)}{1227}} = 0.0274 = 2.74\%.$$

(b) False, The statement should be “There is a 95% chance that the interval generated by the poll contains the population proportion.” Or “95% of the intervals generated would be expected to contain the population proportion.”

(c) Using the interval (0.068, 0.292):

(i) The point estimate is $\hat{p} = (0.068 + 0.292)/2 = 0.18 = 18\%$.

(ii) Margin of error is $ME = 0.18 - 0.068 = 0.112 = 11.2\%$

(iii) Solve for n :

$$0.112 = 1.96 \sqrt{\frac{0.5(1 - 0.5)}{n}}$$

$$n = \left(\frac{1.96 \cdot 0.5}{0.112}\right)^2 = 76.6 \approx 77.$$

Using $p = 0.18$ in the standard error gives

$$0.112 = 1.96 \sqrt{\frac{0.18(1 - 0.18)}{n}}$$

$$n = \left(\frac{1.96}{0.112}\right)^2 0.18(1 - 0.18) = 45.2 \approx 46.$$

In this case, the two answers are not close because 0.18 is not close to 0.5.

³ <http://news.yahoo.com/poll-health-exchange-rollout-gets-poor-reviews-072742111--finance.html>, Oct 10, 2013.

14. (22 points) In 1998, the Nabisco Company, makers of “Chips Ahoy!” cookies, announced their “1000 Chips Challenge” in which they asked the public to confirm that there were at least 1000 chocolate chips in every 18-ounce bag of cookies. The cadets at the US Air Force Academy accepted the challenge and obtained bags of cookies from around the country.⁴ By first dissolving the cookies in water, they counted the chocolate chips in 42 bags. The mean was 1261.571 chips per bag with standard deviation 117.579.

(a) The variable is quantitative, the number of chocolate chips in an 18-ounce bag of cookies. Let μ be the mean number of cookies in the population of all bags.

(i) The null hypothesis is that there are 1000 chips in a bag. $H_0: \mu = 1000$.

(ii) The alternate hypothesis is that there are more than 1000 chips in a bag. $H_a: \mu > 1000$

(iii) The mean is $\bar{x} = 1261.571$ and the standard deviation is $s = 117.579$. The t -value is

$$t = \frac{1261.571 - 1000}{117.571/\sqrt{42}} = 14.42.$$

(iv) It is a t -value with 41 degrees of freedom.

(v) The p -value is approximately zero.

From a calculator $p = \text{tcdf}(1261.571, 10000, 41) = 5 \cdot 10^{-96}$.

(vi) Since the p -value is tiny, well below a 5% or 1% cut off, we reject the null hypothesis and conclude that there are more than 1000 chocolate chips in each 18-ounce bag.

(b) The cadets asked for bags from around the country as part of getting a good random sample. If there had been some batches of cookies that were very high or very low in chips, they might easily have been delivered to one area. Thus if the entire sample is drawn from one area of the country it may not be representative.

(c) No, this does not confirm that every bag contains 1000 cookies or more. The hypothesis test provides strong evidence that the mean of all bags is over 1000, but with a large standard deviation, some bags might have fewer than 1000.

⁴ From "Checking the Chips Ahoy! Guarantee" by Brad Warner and Jim Rutledge, *Chance*, 1999.