RTG Paper - The Fluid Drip

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January 13, 2005

1 Introduction

Our research has been concerned with the phenomenon of fluid drip in three liquids - honey, latex, and a cornstarch-water mixture. Our explorations of the phenomenon of fluid drip have taken two directions. On the one hand, much of our work has been devoted to understanding the physical processes at play in fluid drip. How do surface tension, gravity, and viscosity interplay to cause a drop to fall? What determines the shape of a drop and the distance the drop travels before pinch-off? What determines the speed with which the drop falls before pinch-off, and the time at which pinch-off occurs? We have had some modest success in answering these questions.

The second focus of our work has been to discern how the properties of the three fluids we are working with affect the processes of drop formation and snap-off. It is clear from observation that the fluid drip phenomenon is very different in latex, honey, and corn-starch. Some of this difference is explained by the distinction between Newtonian and non-Newtonian fluids. Honey is a Newtonian fluid, while latex and the cornstarch-water solution are not. Accordingly, honey should have certain properties the other two fluids lack. Yet this is not the whole picture. The cornstarch-water mixture clearly behaves quite differently from the latex mixture; one is shear-thickening (the cornstarch mixture) and the other is shear-thinning (latex). Our investigations in this direction have been guided by the discovery of a universal curve describing the radial drop profile of a Newtonian fluid acting in the Stokes flow regime. This universal curve results from a similarity solution for the equations of fluid drip, and depends only on the ratio of the viscosity of the drop to the viscosity of the surrounding medium. It also proscribes power laws for various parameters of the radial drop profile. To quantify the way that fluid drip changes in shear-thinning and shear-thickening fluids, we attempted to fit all three liquids to the similarity solution. Our hope was that the way the non-Newtonian fluids diverge from the model would give us insight into the factors controlling non-Newtonian drip. As explained below, these hopes were not fully realized.

1.1 Honey

Honey is a Newtonian fluid with a low surface tension. According to www.fao.org, the viscosity of honey ranges from 2.6-600 poise. This is dependent on the type and temperature of the honey. It is also mentioned on this website that honey has a low surface tension.

1.2 Cornstarch

From the video provided, it was unclear what proportions of water and cornstarch made up the cornstarch-water mixture used in the experiments we analyzed. Observation of a sample of a similar cornstarch-water solution made it clear that such solutions have very odd behavior.

According to www.sciencebyjones.com, cornstarch and water mixtures are considered Non-Newtonian liquids. Their apparent viscosity depends on whether they are still or agitated. We observed mixtures becoming more viscous when agitated or compressed with a glass rod. Allowing the mixture to dry showed that the cornstarch granules do not dissolve in water; rather, they remain suspended in water.

It is hypothesized that in a cornstarch-water mixture that is left to stand and settle, the granules of starch become surrounded by water. Water's surface tension ensures that the water molecules form a cushioning jacket around the granules. This cushion provides quite a bit of lubrication and allows the granules to move freely. However, if a large shear stress is applied to the mixture, the water is squeezed out from between the granules and the friction between them increases dramatically. Liquids whose apparent viscosity increases under shear stress are called dilatant liquids [SCIENCEBYJONES].

1.3 Latex

We do not have much information on the characteristics of latex. Observations have implied that latex paint is thixotropic; its apparent viscosity decreases with the application of shear stress. This characteristic would imply that the cornstarch-water mixture has an opposite behavior to that of latex. According to www.sciencebyjones.com, most paints are thixotropic.

1.4 Similarity Solutions

A large body of research has been done on the two-fluid drip paradigm, in which a drop of a fluid of viscosity $\lambda\,\eta$ falls in another fluid of viscosity η. λ is thus the ratio of the viscosity of the drop to the viscosity of the surrounding medium.

Since the length and time scales involved in pinch-off are orders of magnitude smaller than other length and time scales affecting the problem, many workers proposed similarity solutions for the dynamics close to pinch-off. In 1998, Lister & Stone proposed that, even when λ becomes very large, viscous and capillary forces dominate the dynamics, so that inertia is insignificant. The resulting balance between capillary forces, viscous forces, and external viscous drag close to pinch-off results in the scalings

$$z \sim (\frac{\gamma t*}{m^{1/2}\mu}), h \sim (\frac{\gamma t*}{\mu}), \nu \sim (\frac{\gamma t*}{m^1/2\mu})$$

Lister & Stone performed numerical simulations of the evolution a variety of radial drop profiles. They then plotted the rescaled radial and axial lengths

$$H = \frac{h}{h_0}, \zeta = \frac{(z - z_0)}{h_0}$$

against each other; they also plotted axial curvature, axial strain rate, and axial velocity against ζ. All graphs but the last collapsed well onto a universal curve. The axial velocity was seen to increase steadily as h_0 decreased. This increase was effectively modeled as a logarithmic function of the time remaining to pinch-off. Lister & Stone explained this logarithmic correction as an axial translation of the point of pinch-off. They hypothesized that this advection was the outcome of a 'tug-of-war' between the surface tensions created by the far-field conical regions on either side of the point of pinch-off.

In two papers published back to back, Cohen et al. (1999) and Zhang & Lister (1999) further analyzed similarity solutions for the radial profile of the two fluid drop. In the first paper, Cohen et al. presented a similarity solution in the case that $\lambda = 1$. Through dimensional analysis, they concluded that

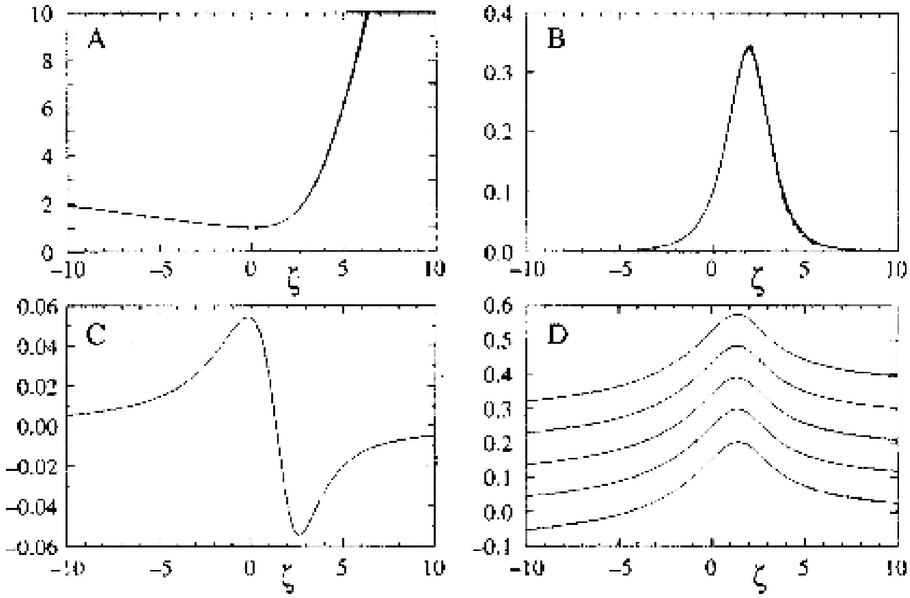


Figure 1: Graphs from Lister & Stone. A) H vs. ζ. B) Curvature vs. ζ. C) Axial strain rate vs. ζ. D) Axial velocity vs. ζ.

the profile of the drop close to pinch-off is a function of λ, scaled by γ (the surface tension of the two-fluid interface), η, and t* (the time until pinch-off). To find a universal drop profile, they substituted the similarity ansatz

$$H(\zeta) = \frac{\eta}{\gamma t^*} h(z,t)$$

$$\zeta = \frac{\eta}{\gamma}(\frac{z^*}{t^*}) + b \ln t^* + \zeta_0$$

where z^* is the distance from the point of minimum width, into the equation of motion for the drop interface. They then solved the resulting differential equation numerically to find a drop profile that matched experimental evidence for several systems with $\lambda = 1$.

They also give experimental and numerical evidence suggesting that the 'steep' slope s_+, the slope on the drop side of the neck, scales like $\lambda^{0.22+-0.07}$, and h_{min} fits the power law $\lambda^{-0.53+-0.05}$.

Zhang & Lister computed similarity profiles for a wider range of λ values. The shallow slope s_-, the steep slope s_+, and the curvature at the point of

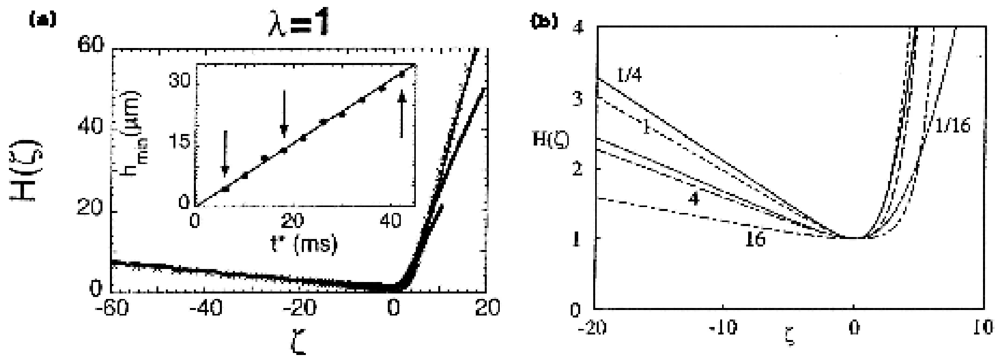


Figure 2: Similarity profiles from (a) Cohen et al. and (b) Lister & Zhang.

minimum radius $\kappa(0)$ were computed for a wide range of λ. It was suggested that, for $\lambda \gg 1$, s_- scales like $\lambda^{-1/2}$, and $\kappa(0)$ scales like λ^{-1}.

Since we are clearly in a regime where $\lambda \gg 1$, we attempted to test whether the similarity profiles of our three systems match the scaling laws given above. For latex and cornstarch, whose viscosity and density are not constant in time, we used the scaled variables H and ζ of Lister & Stone. For honey, whose average viscosity and density are well known, we used the dimensionless variables H and ζ of Cohen et al. For reasons discussed later in this paper, we were unable to recreate similarity profiles like those of Lister & Stone, Cohen et. al, and Zhang & Lister. The divergence of latex and cornstarch from these predictions is not surprising; the divergence of honey is unexpected. We offer some explanations in the Discussion section.

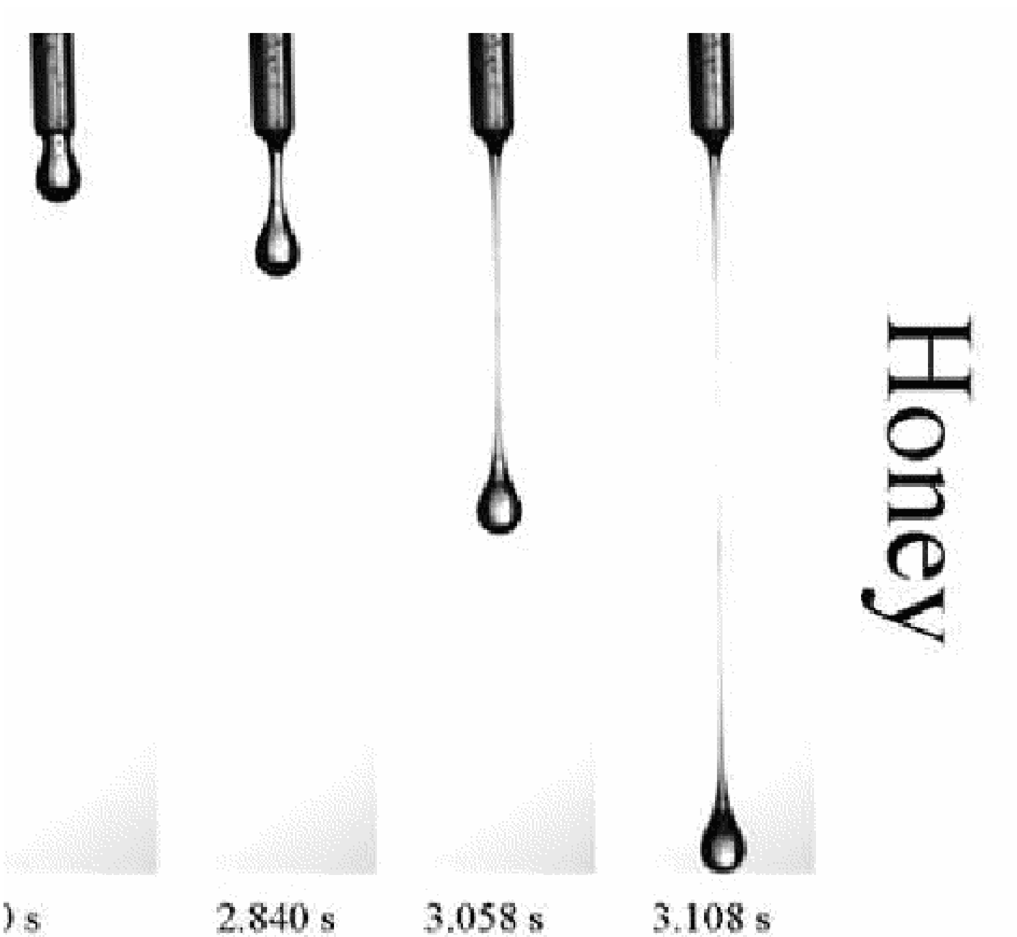
1.5 Iterated Instabilities

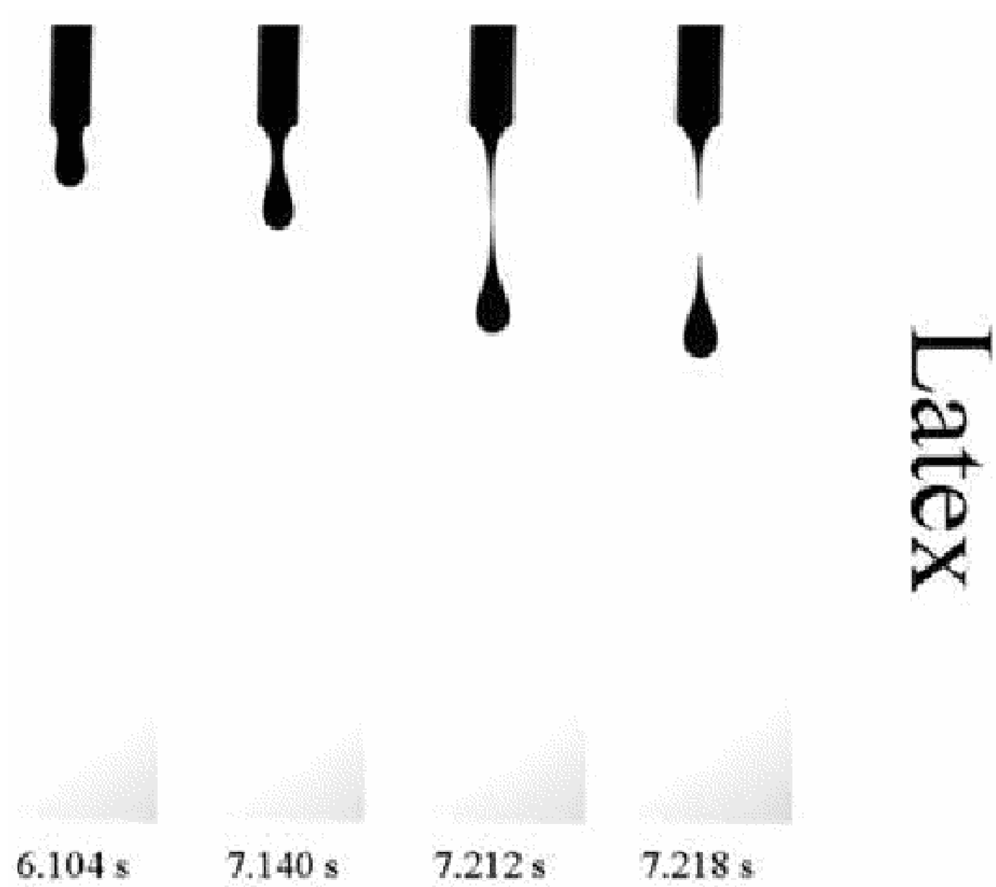
According to X.D Shi, as the value of viscosity η increases, the neck of the drop elongates and forms structures not observable in dripping water. These structures occur as "sub-necks" or irregularities such as bumps in the filament. These structures either do not form in our three test fluids or the resolution of the video is not sharp enough for us to detect them.

2 Methodology

2.1 Experiment Setup

We were provided with high speed films of honey, latex, and a cornstarch-water mixture drippping from a pipette. The pipette was held in a fixed position in front of a high-speed camera. It was filled with liquid, and the tip of the pipette was filmed as a drop of liquid formed and pinched-off. Our participation in this experiment began with the analysis of these high-speed videos.



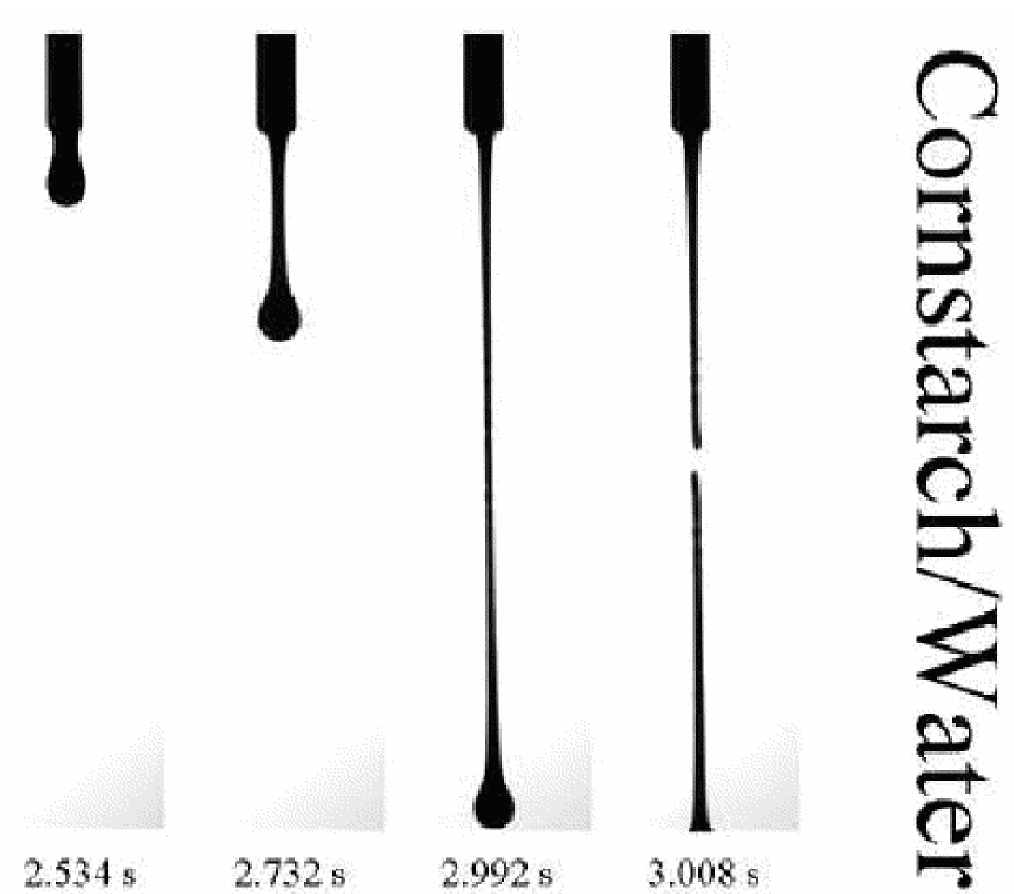


2.2 Hand Measurements

Estimates of the minimum width of the fluid neck (h_{min}) were made by hand, using the program ScionImage. ScionImage allows the user to scroll through a video frame-by-frame, and retrieve the coordinates of specific pixels. These measurements were made over an interval chosen to capture the period of greatest variability in h_{min} before pinch-off. The frequency of the measurements was designed to capture the variability in h_{min} with sufficient resolution while allowing for expedient measurement of the entire measurement interval.

Estimates of the radius of the drop for each liquid were also made by hand, ten measurements were made at increments evenly spaced over the duration of the videos.

Finally, all measurements were converted from having units of pixels and



frames to having the more physically relevant units of millimeters and seconds. The diameter of the pipette was measured and used to scale the data into metric units.

2.3 Extracting Position Data from the Videos

In order to analyze these videos quantitatively, it was necessary to extract certain data from them. Graphs of the axial position of the tip of drop from the beginning of the video until just before pinch-off were obtained using the program Photron. Photron allows certain features of moving videos to be tracked and their position (in pixels) to be recorded. This is accomplished by matching a subset of the first image, incorporating the desired feature, to each subsequent image in the video, and recording the position of the matching subset. Photron then tabulates the extracted data in an excel spreadsheet. The interval of time is discretized according to the speed at which the video is taken. Our videos were all shot at 500 frames per second and accordingly the frames are .002 seconds apart. Our feature selection included a large enough part of the drop's tip so that we could be sure of accurate matching throughout the video, but not so large a part as to make the position estimates at each frame inaccurate.

2.4 Drip Velocities

To obtain graphs of the velocity of each drop, we computed the average velocity between subsequent frames using the formula

$$v(n) = \frac{p(n) - p(n-1)}{t(n) - t(n-1)}$$

where $p(n)$ is the position of the drop tip in frame n, and $t(n)$ is the time of the nth frame. During much of the video, the motion of the drip was so slow that time resolution of the camera could not capture it. As a result, discrete jumps in position were observed where slow, continuous movement was occurring. When the velocity was calculated using the formula above, discrete positive values were also observed. To remedy this situation, a rolling average over 10 frames of $v(n)$ as calculated above was our final measure of velocity.

2.5 Radial Profiles

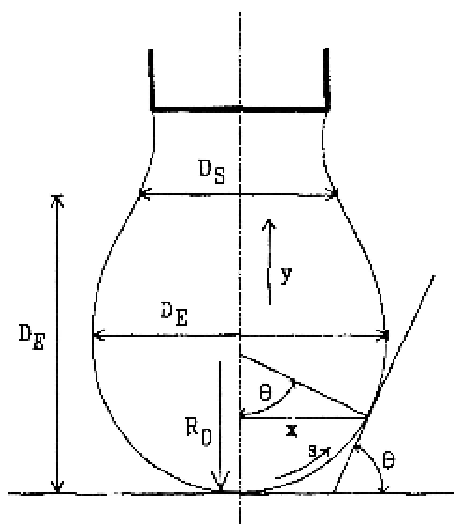
We used Adobe Photoshop to take measurements of the drip profiles close to pinch-off. Photoshop enabled us to perform fine rotations of the frames we chose to measure, allowing us to measure radial height along a direction perpendicular to the axis of the drop. We were able to magnify the frames in question, and snap them to grids of arbitrary size, thereby considerably increasing the accuracy of our measurements. In order to avoid assigning an axis to the drop, drop diameter rather than radial height was measured at evenly-spaced intervals of axial length. The measurements were spaced so as to allow us to make fifty measurements along the length of the drop, from the tip of the pipette to the point of maximum drop diameter. In addition, we used Photoshop to record data used to calculate the surface tension of the drop.

2.6 Surface Tension

To compute the surface tension of the honey-air interface, we followed the 'pendant-drop' method used by Cohen et al., and elaborated by Hansen and Rodsrud (1990). In this method, surface tension is calculated from the equation

$$\gamma = \frac{\Delta\rho g R_0^2}{\beta}$$

where $\Delta\rho$ is the mass density difference between the drop and the surrounding medium, g is the gravitational constant, R_0 is the radius of curvature at the drop apex, and β is the shape factor, defined by this equation. Beta is calculated from the ratio $\sigma = D_s/D_e$, where D_e is the maximum diameter of the drop, and D_s is the diameter of the drop at a distance D_e from its tip. β's dependence on σ is modeled by a cubic equation. All these parameters were measurable to a good degree of accuracy notwithstanding noise.



3 Results

3.1 Dynamics

Graphs of drip diameter versus time show that for honey, the size of the drop remains constant after drip formation. For latex, the size of the drop remains constant until pinch-off; then, it grows slightly. One possibility is that the tail of the drop is more rapidly drawn into the body of the drop of latex, which appears to have a higher surface tension than honey. The size of the drop of cornstarch, in contrast, decreases at a slow rate during the course of the dripping process.

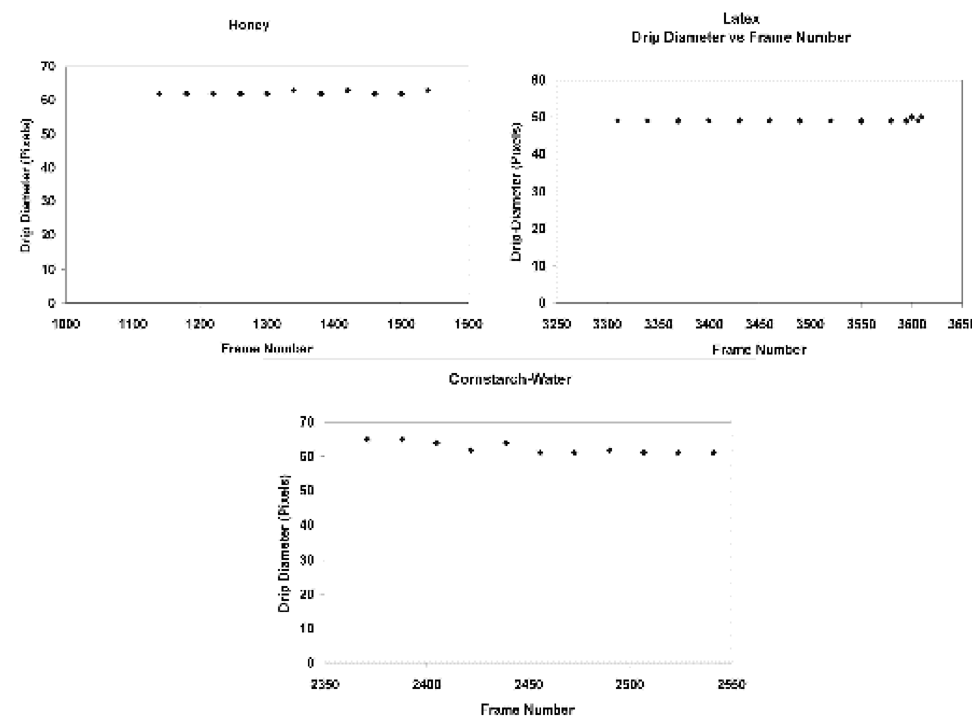


Figure 3: Drop diameter as a function of time

Observation of the films has also shown that the column of honey in the pipette remains stationary once the honey drop has formed. From these facts we infer that the volume of the honey filament remains constant as the drop falls.

Graphing h_0 vs. t^* for honey shows a clear linear relationship between the two quantities. For latex, it seems like h_0 is proportional to $(t^*)^{1/2}$, and h_0 decreases faster than linearly for small t^*; h_0 may also have a linear relationship to t^* at a scale smaller than the resolution of this experiment. For cornstarch, h_0 seems to decrease to a limiting width before pinch-off. When pinch-off occurs, h_0 is still very large — at which point the filament seems to break, rather than pinching in on itself.

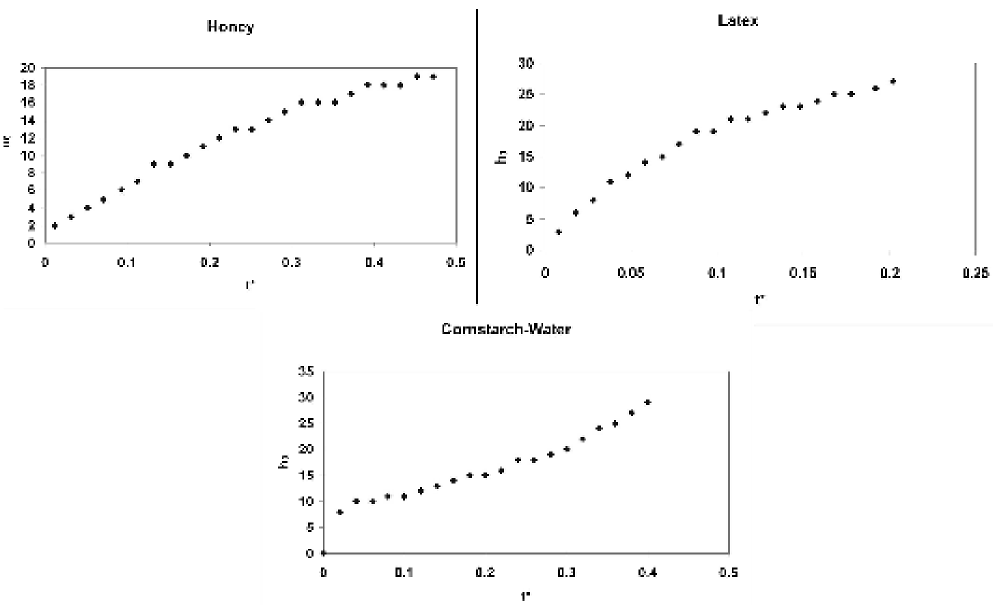


Figure 4: Minimum width versus time to pinch-off

Graphs of h_0 vs. z_0 for both honey and cornstarch show a possibly linear relationship at small t^*. For cornstarch, however, h_0 seems to decay to a constant, positive value.

Initial analysis of our velocity graphs for latex and honey show that drops of these liquids fall in two stages, each of which is characterized by constant acceleration. In the second stage, both drops fall with the same constant acceleration. The cornstarch drop, however, seems to fall with a single constant velocity until pinch-off.

Three similarity profiles were computed for honey; two each were computed for latex and cornstarch. The scales for ζ and $H(\zeta)$ in these graphsmake is clear that near pinch-off, the length scales on which the similarity solution applies vanish relative to the resolution of our videos. It was impossible to compute the parameters s_-, s_+, or $\kappa(0)$ for any of the liquids.

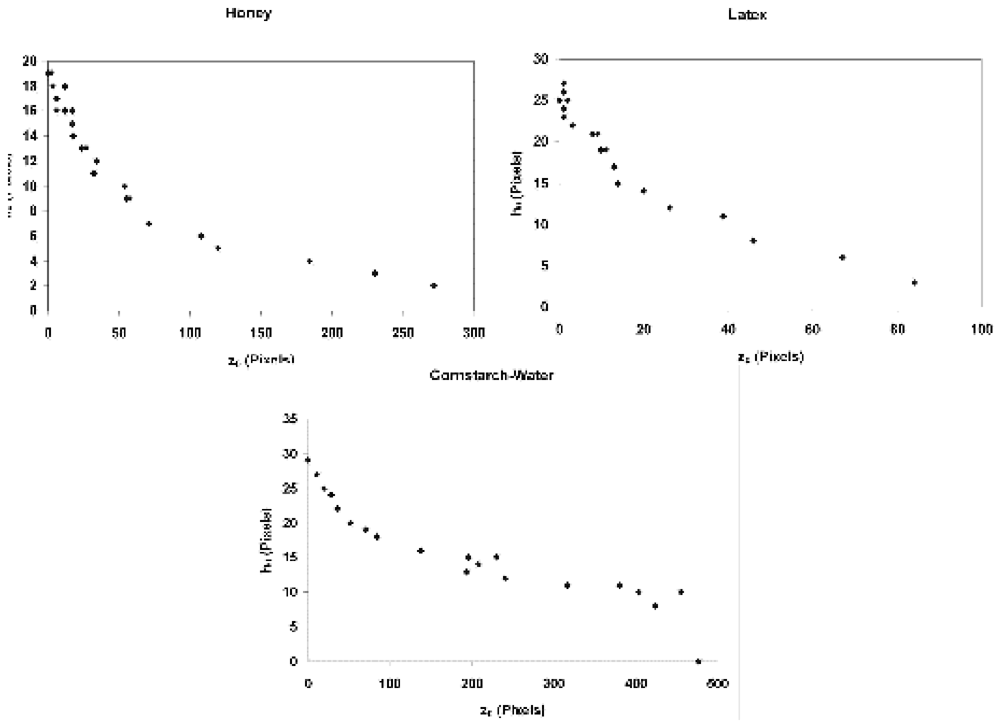


Figure 5: Minimum width versus location of minimum width

However, examination of the similarity profiles for honey does lead to the observation that, at small t^*, several potential pinch-off sites seem to have appeared.

4 Discussion

Initial viewings of the films led us to the observation that the point of pinch-off would be crucial in our analysis. The general progression of all three films is the same: a drop forms at the tip of the pipette, and begins to move quite slowly. After the drop reaches a certain point, its velocity increases and the drop moves quickly toward the pinch-off point. The lengths of the steps in this progression vary from liquid to liquid, as did distance from the pipette to the breaking point and the total time of the process.

Our analysis of the velocity graphs of the latex and honey drops has prompted us to hypothesize that the drops come under the influence of grav-

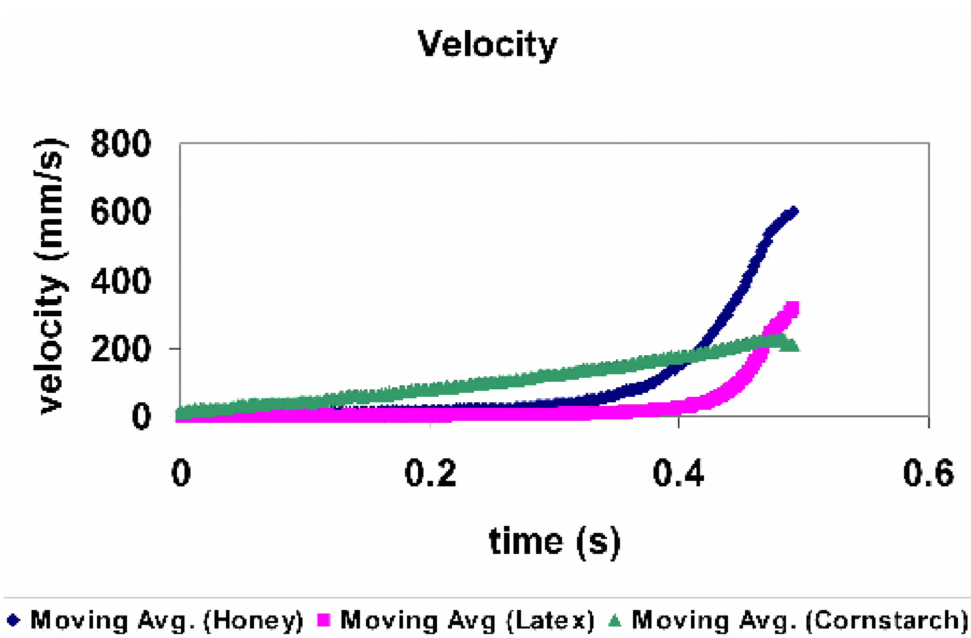


Figure 6: Velocity

ity even before pinch-off occurs. We hypothesize that the first phase of the fall represents the acceleration due to a balance between gravity and another force, possibly surface tension. Thus, surface tension balances gravity, holding the drop close to the aperture, until the drop has fallen sufficiently that surface tension is negligible compared to gravity; then, the drop falls under the influence of gravity alone. For cornstarch, this is clearly not the case. In fact, it appears that the dynamics of the cornstarch drop are dominated by surface tension up until pinch-off.

We can also confirm that, for honey, a Newtonian fluid, the minimum width of the filament decreases linearly with its length and with time. This is in line with scaling arguments proposed by Cohen et al. (1999).

These two facts give us hope that the fall of a fluid drop might be modeled simply as the fall of a mass (the drop) under the influence of an elastic force (surface tension, perhaps modeled as a spring). While this is possibly true for honey, and maybe even latex, it is certainly not valid in the case of cornstarch.

The similarity profiles obtained for honey make it clear that in this experiment, the scale on which the similarity profile should appear is much

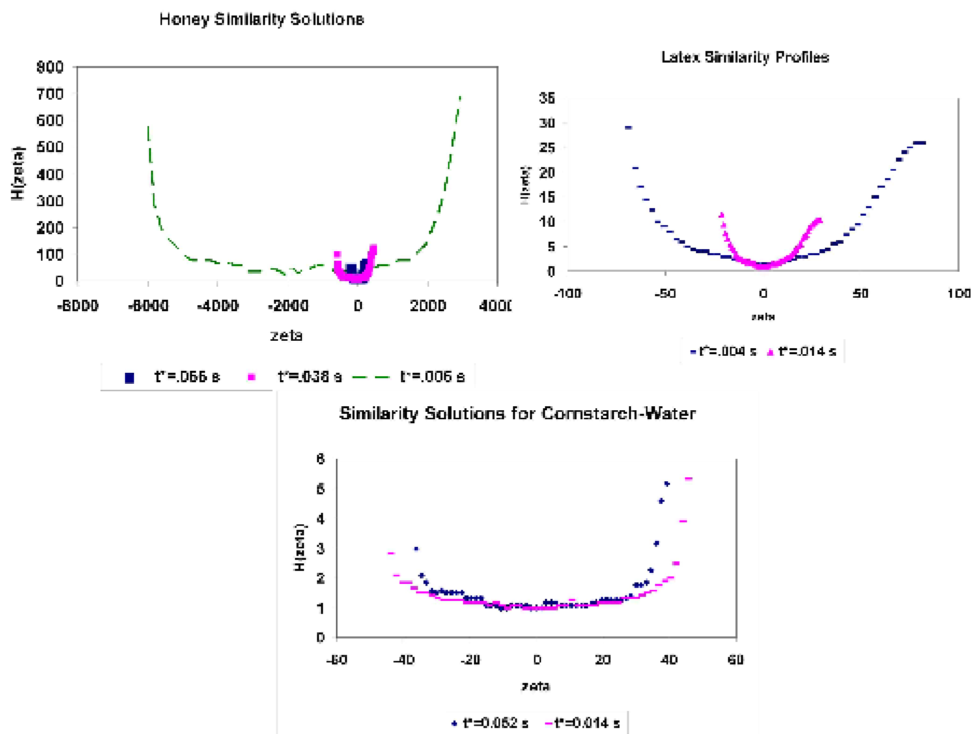


Figure 7: Similarity Profiles

smaller than the resolution of the videos. Particularly for small t^* — the time domain we are interested in — accurately plotting H(ζ) in the interval [-10,10] would require measurements orders of magnitude finer than those we were able to obtain from the films.

However, observation of the similarity profiles for honey does yield some useful qualitative information. From both the videos and the drop profiles, it appears that pinch-off does not occur at one location, close to the drop. Rather, pinch-off seems to occur simultaneously at many locations along the filament connecting the drop to the pipette. A possible explanation is the low surface tension of honey relative to other fluids. Honey's low surface tension may mean that capillary instabilities on the fluid thread will only propagate when the fluid filament is very thin, and thus highly curved and having a high surface tension.

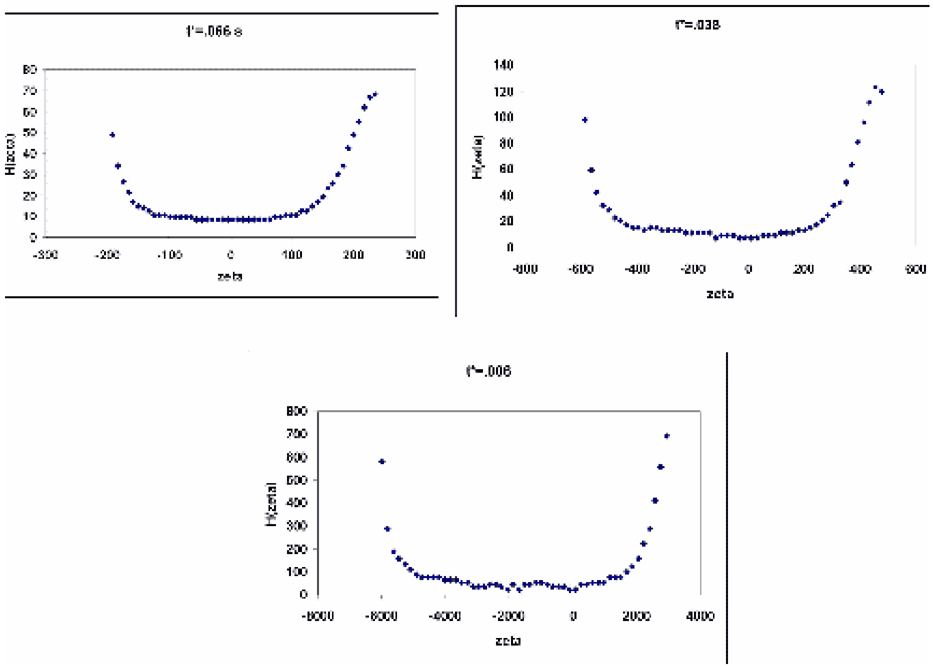


Figure 8: Similarity profiles for honey

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