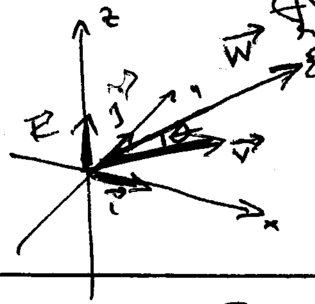
A half-way journey to the notion of Hilbert Space



finite dimensional world	terminology	infinite dimensional world
$V = \mathbb{R}^3$	vector space V	V = a space of functions, for example $C([a,b])$ = all the continuous functions $f : [a,b] \to \mathbb{R}$
$\mathcal{B} = \{\vec{i}, \vec{j}, \vec{k}\}$ $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$	basis $\mathcal{B}$	$\mathcal{B} = \{\xi_1, \xi_2, \dots, \xi_m, \dots\}$ $f = a_1\xi_1 + a_2\xi_2 + \dots + a_m\xi_m + \dots$

Q: What are the interesting features of $\mathbb{R}^3$?

$(\vec{v},\vec{w}) = \vec{v}\cdot\vec{w}$ $= v_1w_1 + v_2w_2 + v_3w_3$ $= \|\vec{v}\|\cdot\|\vec{w}\|\cdot\cos\theta$	inner product $(\,,\,)$	$(f,g) = ???$ This is the key issue? How to define an inner product?
$\|\vec{v}\|^2 = v_1^2 + v_2^2 + v_3^2 = (\vec{v},\vec{v})$	length/magnitude/norm of a vector $\|\ \|$	$\|f\|^2 = (f,f)$ once the inner product has been defined
$\mathcal{B}^\perp = \{\vec{i}, \vec{j}, \vec{k}\}$ $v_1 = (\vec{v},\vec{i})$ $v_2 = (\vec{v},\vec{j})$ $v_3 = (\vec{v},\vec{k})$	orthonormal basis $\mathcal{B}^\perp$	Every vector space with $(\,,\,)$ has a basis $\mathcal{B}^\perp$ consisting of orthonormal vectors. Then the expansion $$f = \sum_{m=1}^{\infty} a_m \cdot \xi_m$$ is called the formal Fourier series of f and its Fourier coefficients a_m's are given by $\boxed{a_m = (f,\xi_m)}$, for $m = 1,2,3,\dots$