Name:

Project/Quiz # 11, 11.5 Forced oscillations, due in class August 6, 2008

Consider the non-homogeneous ODE

$$y'' + 0.01y' + 16y = r(x), \tag{1}$$

where the external force is $r(x) = \cos(x)$, for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, extended by periodicity with period $p = \pi$. The goal of the following questions is to lead you to the finding of the steady-state solution of (1). You may also want to read 2.8 to refresh your memory on forced systems.

(a) Find the Fourier series of r

$$\mathcal{F}(r)(x) = a_0 + \sum_{n=1}^{\infty} a_n \cdot \cos(2nx).$$

(b) Find the steady-state solution of the non-homogeneous equation

$$y'' + 0.01y' + 16y = a_n \cos(2nx). \tag{2}$$

Write the solution in the form

$$y_{P,n} = A_n \cos(2nx) + B_n \sin(2nx).$$

(c) The amplitude of the steady-state solution of (2) is $C_n = \sqrt{A_n^2 + B_n^2}$. Compute the first 6 amplitudes, namely C_0, C_1, C_2, C_3, C_4, and C_5.

(d) Graph the steady-state solution and $r(x)$ on the same system of coordinates. Does it make sense? Explain.

(e) Based on your work, would you agree with the statement below? Explain.

> *The steady-state is a superposition of oscillations. If one of the frequencies of these oscillations is close to the practical resonant frequency of the vibrating system then this particular oscillation may be the dominant one.*