Name: Solutions

Quiz # 12, August 6, 2008

1. ~~(5 pts) Find the inverse Laplace transform of $F(s) = \frac{4}{s^2 - 5s}$.~~

2. (10 pts) Solve the IVP: $y'' + 4y = \cos(3t)$, $y(0) = 1$, $y'(0) = 0$.

Apply Laplace transform to both sides of the ODE:

$$\left(s^2 \cdot \mathcal{L}(y) - s\right) + 4 \cdot \mathcal{L}(y) = \frac{s}{s^2+9}$$

$$\mathcal{L}(y) = \frac{s}{s^2+4} + \frac{s}{(s^2+9)(s^2+4)}$$

Method 2: Applying #14 and #24 from the table, we get:

$$y(t) = \mathcal{L}^{-1}\left(\frac{s}{s^2+4}\right) + \mathcal{L}^{-1}\left(\frac{s}{(s^2+9)(s^2+4)}\right)$$

$$= \cos(2t) + \frac{1}{5}\left(\cos(2t) - \cos(3t)\right)$$

$$= \frac{6}{5} \cdot \cos(2t) - \frac{1}{5}\cos(3t).$$

Method 1:

$$\mathcal{L}(y) = \frac{s^3 + 10s}{(s^2+9)(s^2+4)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+9}$$

Solving for the coefficients, one gets: $A = \frac{6}{5}$, $C = -\frac{1}{5}$, $B = D = 0$.

Consequently $\mathcal{L}(y) = \frac{6}{5} \cdot \frac{s}{s^2+4} - \frac{1}{5}\frac{s}{s^2+9}$

So: $y(t) = \mathcal{L}^{-1}(\mathcal{L}(y)) = \frac{6}{5}\cos(2t) - \frac{1}{5}\cos(3t)$.