The following pages are divided into two parts. The first part is a hybrid of my studies of various social subjects, aimed at constructing an overview on the characteristics of the modern social and economic systems. My presentation in this part follows mostly the qualitative style of neo-classical theory of political economics. Conclusions of this part are critically important for the development of the mathematical model introduced in the second part, by which we investigate the short and long term stability of the modern economic system.

PART I  SOCIAL AND ECONOMIC ORGANIZATION

1. Organizational principle of past human societies

Social and economic activities of human society of the past were organized under a hierarchic framework. At each level of this hierarchy, complicated rules were designed to arbitrate conflicting interest of different individuals and groups. To organize a well-functioned human society was to introduce a rigid, stable social and economic structure, in which the tasks of every member of the society were well-defined to facilitate the employment of a few time-tested means of material production for subsistence.

Life is a game of self-preservation in which individuals must constantly acquire food and shelter to maintain a physical existence. By natural design, the first principle of individual human behavior is the principle of self-interest. Individuals, however, are short-lived. Life in the long run is maintained through biological reproduction, in which two adults of opposite sex not only have to be physical united to give birth, but also have to work together to support their children; to teach them the skills they must acquire for their future survival in a brutal world. This is an awfully long process, in which the self-interest of all parties clashes frequently. Therefore, in addition to the principle of self-interest, a second principle, which we name as the principle of parental mandate, was introduced to confuse in adults their own self-interest to that of their children. For humankind, parental mandate is imposed on both sexes, perhaps more so charged to the female sex than to the male sex. All selfish and predatory human behavior are rooted in the principle of self-interest, and all unselfish and altruistic human behavior are secondary extensions of the principle of parental mandate. These two principles are also implemented in other animals of the world.

What has separated the fate of mankind from the fate of the others is human intelligence. Primitive intelligence, as an individual mental ability to attribute effect
to a cause in simple circumstances and to use such previously acquired knowledge to prompt instinctive reactions to occurring events, are undoubtedly embedded to various degree in other animals as well. Human intelligence, however, is unique in the sense that it is highly able in inventing abstract mental object, and to store these mental object in mind by using mental structures framed on the a priori of time, space and logic. Individual human intelligence actively looks around to collect supporting evidences for the mental structures it devised, and is able to modify the established mental structures to adopt unfitting facts. The high ability of human intelligence was also greatly facilitated by the use of language. Sophisticated language not only sharpened individual thought process but also allowed human intelligence to work collectively and to accumulate whatever it deemed most useful of its creations over the time.

The physical purpose of human life is objectively to live on and the spiritual purpose of human life is subjectively whatever conceived either collectively or individually by human intelligence. No more and no less.

In the midst of the long course of evolution, human intelligence picked up a few exceedingly useful tricks in material production that have granted mankind somewhat an edge in their daily struggle against their nature environment. Improved material production induced high population density. They multiplied quickly, and were forced to confront one another. Soon enough, their perpetual struggle against the ever menacing power of nature for subsistence was over-shadowed by the conflicting self-interests among the co-existing fellow humans. They were then compelled to deal not only with the issue of material production in a struggle against their nature environment, but also with the issue of distribution of the produced in a struggle against each other. Human societies were formed, not so much for the purpose of working together to collectively improve the existing conditions for subsistence but to overcome the evil of self-centered predatory human instinct to make a feasible arrangement for coexistence. Primitive civilizations were induced into existence, and the fate of mankind was forever diverged from that of all others.

Many have argued that the evolution of various human societies can be divided generically into a list of distinctive stages. At each of these stages, some argued, that various human societies were compelled to apply similar organizational principles, uniquely attached to the adopted means of material production. The size of independent human societies, in any case, has been growing from small to large with internal structures of ever increased complexity and interconnectedness, and for past human societies advanced beyond certain primitive stage, the following appear to be the case

- Social and economic activities were organized under a hierarchic structure, which took family, mostly comprised of one adult male, one adult female and their direct offspring, as the basic functioning unit.

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1To deny the critical influence of the restraints imposed by the adopted means of material production to the organizations of a human society would be foolish. But to deny the active role human intelligence played and the different options it had faced to argue for the kind of deterministic path of social development proposed by Karl Marx was equally unfortunate.
At each level of this hierarchy, complicated rules were designed to arbitrate conflicting self-interest of different individuals and groups. These rules were universally accepted either by voluntary submission through the persuasive force of religion, education and tradition or by coercive submission through the administrative force of the government.

At each of the stages of social development, the adopted means of material production for subsistence appeared to consist a set of well-defined, time-tested tricks. To organize a well-functioned human society was therefore to introduce a rigid, stable social and economic structure, in which the task of every member of the society were well-defined to facilitate the employment of these time-tested means of material production for subsistence.

Historically, humanity had been ultimately humbled by the overwhelming force of nature. Though it nonetheless suggested that the world was somewhat designed to revolve around the activities of the mankind, an anthropocentric thinking rooted in the principle of self-interest, humanity everywhere was convinced that the force of nature was beyond the comprehension of human intelligence. It often assumed that nature was under the influence of some super powers, to whose will the only option for mankind was to appease and to obey. The main theme of humanity was, therefore, not to conquest the natural but to conquest each other. Though social stratification created leisure class; inquisitive souls liberated from vulgar physical labor started to invent mental frameworks to try to comprehend the unfolding processes of nature, what was tried were not systematic and results acquired far from certain. The realization that there is in fact a feasible way for human intelligence to systematically comprehend the force of nature, and more importantly, the acquired comprehension could be applied persistently in grand scale to improve human material production was utterly liberating. When Francis Beacon stated that knowledge is power, his statement represented a major change in human attitude. This mentality shift was truly revolutionary and it was only made possible by the exciting achievement of modern science at the time.

Here we then meet a main difference in the organizations of human societies in the past time and in the modern time. For societies of the past, the focus of social and economic organization was to create a stable hierarchic structure in which different tasks associated with a fixed set of means of material production for subsistence were explicitly assigned to different members of the society. Detailed social and economic rules were imposed on people who perform different social and economic functions to settle conflicting self-interest. Division of labor led to stratification of people into different social classes. Farmers were tied to the soil they till, and so were their descendants. Artisans were organized into guild, and their skills and their way of life were inherited by their children. Rules of social behavior were well formulated and they were either adopted through voluntary submission or imposed by brute force. For every member of the society, all conflicting self-interest were settled by a set of well-defined courses, often ostensibly more favorable to some and less favorable to others. To justify social and economic structures in existence and to maintain internal stability, human intelligence invented religion, ethics, honor and legal code,
and government, a whole collection of super structures to induce either voluntary or coercive submission of all to the designed settlement.

Because past human societies were preoccupied with imposing rigidly designed social and economic structures based on the adoption of a fixed set of means of material production for subsistence, they would all instinctively regard creative intellectual and economical human activities as potential threat to the existing order. Modest and gradual improvements might very well be adopted but whoever proposes radical changes incompatible to the established way of thinking and to the adopted means of human material production would be treated as a grave threat. Free thinking, free trade and science were therefore universally suppressed all along in history.\(^2\) That modern science was not choked to oblivion in its infancy was a pure stroke of luck, probably due to the fact that the power of secular administration and the power of religion were not consolidated completely into one in European societies of the middle age. In the strives of the two, modern astronomy helped secular power to discredit the authority of catholic church. After science established itself, however, it became apparent that human intelligence could do much better than previously thought possible in harnessing the power of nature. Exploded human knowledge led to a constant flux of technological innovations.

The newly found power of material production, unfortunately, could not be adopted into previously established social and economical frameworks, for what was built based on a fixed set of means of material production could not handle the constantly improving, therefore constantly changing, method of material production introduced by the advancement of science and technology. New principles of social and economical organization that are flexible enough to adopt the incessant flux of proposed changes, quite radical in times, in human material production are called for. But where are these new principles and how do we implement them?

2. Organization of Free Capitalism

*Humanity of modern time has found in private property right, free market exchange and money, a new organizational scheme that is flexible enough to adopt the constant flux of innovations in material production introduced by the incessant advancement of science and modern technology. This new scheme treats the organization of economic activities of a human society as that of a non-collaborating game. The primary tool of this economic game is money and the fundamental rule is an absolute respect to the right of private property and the principle of voluntary exchange.*

As division of labor gradually dominated the process of human material production, voluntary exchange respecting private ownership became a critically important part of the daily arrangement of a human society. Free trade and private property right, however, were only regarded by humanity as technical necessities, never acquired the kind of sacred status as guiding principles of social and economic organizations, as they are now in modern time. Predatory human nature was with a propensity,

\(^2\)For an instance, Socrates was sentenced to death in Athenia at a time that was most favorable to free thinking and his crime was that “he is an evil doer and a curious person, searching into things under the earth and above the heaven; and making the worse appear the better cause and teach all this to other”.
clearly rooted in the first principle of self-interest, to dis-regard private property right of others whenever circumstances allowed it. To people in possession of power over others, to take away whatever from the ones who were subjected to their power appeared *de facto*.

Since barter was impossibly complicated, money was invented as a technical tool to facilitate voluntary exchange. With the use of money then there came into existence also the traders and the debtors who took the advantage of the convenience of money to amass wealth, and it was ostensibly clear that the wealth they amassed represented also a social and economic power of formidable potential. The manipulative power of money based on free trade, however, was not directly compulsive by definition so it did not offer the promise of an imposing power needed to suppress the animal instinct of predatory human nature for the purpose of creating a social and economic order in which individuals could not only live side by side in peace but also be able to collaborate to benefit each other. To people who struggled to impose social and economic order, the whole idea of money right was troublesome, contradicting directly to the absolute power of government they were trying to implement. Money and trade appeared to be a necessary evil, a technicality one could not possibly do without but an annoying discord and a potential threat. Therefore, human society, though must accept the use of money as a technical facilitator, had tried to prevent money men from gaining real social and economic status through money schemes. They were therefore discriminated against, their ways of amass wealth were portrayed mostly as immoral, their potential gains were closely regulated and the manipulative power of money were seriously restricted.\(^3\)

Humanity of modern time, however, has found in private property right, voluntary exchange and money a new organizational scheme that is flexible enough to adopt the constant flux of innovations in material production introduced by the incessant advancement of science and modern technology. This new scheme treats the organization of economic activities of a human society as that of a non-collaborating game. The primary tool of this economic game is money and the fundamental rule is an absolute respect to the right of private property and the principle of voluntary exchange. In this non-collaborating game, all individuals are given a free hand to pursue their own self-interest as long as the rules of the game are respected. Modern society openly acknowledge individual self-interest as the ultimate motivation for human activities, in which it found a non-exhaustible drive for an ever improving human material productivity. In this new economic system money is the designated media of access to all social and economic resources. To acquire materials of subsistence, food, shelter, anything, and everything that are available in a human society, an individual must

\(^3\)With respect to this generality the ancient Greece, where humanity found a fertile ground for various social and economic experiences in the unique setting of city states, was perhaps a partial exception. A substantial part of Hellenic civilization was originated in Ionian cities and in Athens, where commerce was respected. Benefited from this rare exemption, Greeks invented deductive reasoning, philosophy and science but they unfortunately failed to convert such human ingenuity into practical power of material production. Sparta beaten Athens, and great social experiences of Hellenic world ended by first the abrupt military conquest of Alexander the Great, then that of Rome. Nevertheless, in Greek civilization, human intelligence not only had planted the seed of science but also illustrated the promise of its blossom.
first acquire money, and to acquire money he must voluntarily work for someone who is willing to pay for his labor. In this non-collaborating economic game, all social and economic resources of a human society are manipulated by money on voluntary basis. All economic exchanges are simplified to actions of buy and sell. The name of this game is either to work for somebody to earn a living as a laborer, or to use the money one possesses to manipulate labor to produce and to sell the produced to make a profit as an industrial entrepreneur. Laborers sell their labor to entrepreneurs to acquire money, then use the money acquired to buy whatever the consumption goods they would like to buy from the free market. Industry entrepreneurs use money in their possession to buy the labor from laborers to produce for them whatever the consumption goods they intend to produce, then sell the produced on a free market to whoever is willing to pay. In this system, whoever wields the power of money organizes material production, and it is on the best of his self-interest to produce the things people of the society at large are willing to pay to possess. It is also on the best of his interest to explore and to adopt the most efficient ways of material production, reducing cost and improving productivity. This system compels laborer to work hard as well, for if he does not then boss would find someone else.

The major hurdle in implementing this new scheme is again the predatory human nature. Voluntary exchange is not a propensity of human nature. It is in fact the opposite, rooted in the principle of self-interest. A primitive rule such as you shall not steal needed to register the help of the image of an all mighty god to implement in a human society of modest size. He steals if he could get away and he kills to take possession of what he desires from others. The fact that, in an orderly human society, he does not so behave is only because he is more or less convinced that who kills would get killed, and who steals would almost surely loss much more; and the only way to convince him is to impose such as reality by a brute physical force. To form a coherent human society, forces of imposing power must be assembled to suppress predatory human behave.

But how and by whom? Whoever wielded the power of this imposing force would be in a position to define the rules of the society and they would, following the first principle of self-interest, introduce and implement social and economic rules in such a way that their own interest were best served. Members of human society, therefore, were divided into a ruling class, comprised of rulers and their lieutenants, who were with all social and economic prestige of the society, and a lowly class, who were compelled to serve the ruling class. Recorded human history everywhere had been a long string of the deeds of great rulers of the past, who were in fact universally admired, and were perhaps justly so, for ruling forces were hard to assemble and even harder to harness, and to be able to formulate and to impose sustainable rules that held a human society together was an incredible deed for human intelligence. In any case, once bonded together to collaborate, the majority of the society, including most of the lower classes, found that, despite being badly exploited, they were still far better off under a wise ruler than in anarchy.

In order to organize a society according to the new scheme based on private property right and voluntary exchange, however, there could be no place for rulers, for whoever hold the power to impose a settlement would not have to, therefore for sure would
not bother to, respect voluntary agreement and private property right of others. On the other hand, the designated economic game is now in more desperate need for umpires with imposing power, for in a game based on self-interest, people would be allured more than ever by their predatory nature to break the rules to gain. The dilemma, therefore, is how to allocate the formidable governing force to some but to prevent them from becoming rulers of the past. A solution to this dilemma is representative democracy based on the organizational principle of check and balance, an implementation of which is best put forth in the constitution of the United States. The role of the government, in so far as internal affairs are concerned, consists the following

1. To maintain a social order based on equal political right and private property right that makes the desired social and economic collaborations possible.
2. To impose an economic order based on the principle of free market exchange.
3. To serve as a guardian of good-intention to ensure the overall stability of the economic system.

In a modern society, a political system of representative democracy and an economic system of non-collaborating competition stand side by side, each is leaning on the support of the other; but there exist no rulers in neither of the systems. Like in all humans societies of the past, in modern society there is no base to argue against and to disobey the authority of government using reasons such as personal freedom and individual right, for the rule of government is necessarily mandatory and it is, by definition, against voluntary principle and personal freedom. Modern representative democracy, however, is so designed that it is against the self-interest of the people who are holding the governing power to directly participate in the non-collaborating economic game to seek monetary profit for either the government or themselves. Representative political system goes to extreme to prevent the formation of a ruling class. It favors mediocrity over extraordinary talent because rulers are almost without exception people of extraordinary talent. It only adopt actions decided by a representative majority, or in matters of fundamental importance even a super majority, so to make sure that no foresightedness of a few, even with all good-intention and best possible wisdom, can manipulate the imposing power of the society into action. Extraordinarily good deed based on foresightedness would inevitably increase social prestige of the foresighted few, paving the way for some to eventually become rulers, a danger modern society are not willing to risk.

Money is a tremendous manipulative power in modern society but the power of check book is handicapped by the rules of the economic game. Since the rules are made by a political system based on majority, or super majority, decision, people with check book would be at odds with the political system if they try to benefit themselves through actions that would hurt the majority of the public. Whatever they do to benefit themselves must also benefit the general public. In modern society, consequently, big money had long forsaken the hope of controlling rule-making.

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4Here goes your complaint about the short-sightedness, ignorance and stupidity of your representatives in congress and your president. Clinton was impeached by the Congress, Nixon was forced to resign but Bush Jr. enjoyed two full terms with a republican majority in both the House and the Senator.
power of the political system. All they are trying with their check book now are to establish a political influence that is strong enough to stop the government from further weakening their control over their own check book. In this sense, they are the vanguard, and the ultimate defense of the fundamental principles of free capitalism. They have been successful in normal circumstances so far, for individually, all members of the society, therefore the public opinion, are naturally prone to the spells of the hypnotizing power of money.

But we all know that the aim of money men is not to protect the status quo to benefit the public. As far as the rules are fixed, they would be able to find loopholes and use their manipulative influence to introduce deceptive schemes with extreme intricacy to hurt the public to enlarge their own profit. When these schemes are pushed to extreme, the economic system would be also pushed to the verge of collapse. When this happens, an abnormal situation arise, in which the invisible hand of supply and demand would introduce corrections in such a violent way so to cause substantial structure damage. The public, awaken by the forming thunders of a destructive storm, will act later, manipulating the governing power of the political system to modify the rules of the economic game to prevent the future use of similar guilty schemes. But when storm comes emergency measures must be in place to stabilize the situation and to control the ensued damage. This is why item (3) is added as a major responsibility of the government.

3. Consumption oriented free capitalism

Limited wealth re-distribution, uniformly imposed by the government, is a unique way for a society to extend the demanding side of the economic coin without hindering free market competition, therefore a way that would eventually benefit all. With social reform and wealth re-distribution, free capitalism evolved into consumption oriented free capitalism, where the consumption oriented part is represented by the rules of wealth re-distribution.

For an animal who is in an endless struggle of seeking food to ease the persistent torment of hunger of its half-empty stomach, life is miserable but simple. Food is the good and predators are the evil, and to live is to chase the good and to run away from the evil, in which there are only a few courses of actions prompted by animal instinct to follow. For an individual living in a human society, life is much less miserable but it is also with great complications. The good of life appears much more than a full stomach and the evil of predators dwells in his fellow humans, from whom he could not possibly run away. The few courses of actions prompted by animal instinct are still there but these are exactly the ones prohibited by the civil rules of a society. He therefore has to rely on his intelligence, not his animal instinct, to find suitable courses of actions to best serve his self-interest.

It is impossible for individual intelligence to work independently to develop a systematic mental framework to guide his own behavior in a human society. But this is not a problem because a human society, in its long course of evolving into existence, has collectively and accumulatively evolved a systematic mental framework that could be fed directly into the mind of the youngsters as part of their training program in
becoming independent adults. These mental frameworks are designed also to collaborate and to reinforce the existing social and economic order and they are in fact an essential part of a coherent and stable human society. Physical forces are necessary in order to maintain a social and economic order but they are far from sufficient, for to tame the animal instinct of an individual is to also control his mind through brainwashing.

These past mental frameworks were all introduced behind the disguises of religion, ethics and tradition. They all tried to place some common objectives, which was portrayed as the Good, over individual interest. Humanity accepted its limitation in front of the powers of nature but fancied that these powers were under the control of one (or a group of) super human, to whose purpose, which was the Good, a human society ought to serve. This omnipotent power, in control of the fate of the world till eternity, was watching over us. He was constantly pleased or displeased of what we did and would reward or punish us according to our deed. To his will we ought to obey; and for his favor we ought to do specific rituals to beg. Individual interest for sure had to give way if it was in conflict with his purpose. To our good luck by the way, everyone was told, that the governing power of our society was somewhat intimately related to and was directly endorsed by this omnipotent power. Ten Commandment were given to Moses directly by the God; Quran was the verbatim word of Allah; and all Chinese emperors were son of the Heaven. Buddha was neither as powerful nor as directly related to the administrative power but his teaching was to accept whatever came to you, a message fitted the interest of the rulers the best.

Science, owing its birth to mainly the work of a few exceedingly inquisitive and ingenuous mind, had shaken the reign of this omnipotent, made-believe power. Modern astronomy placed the home of humanity on an insignificantly tiny rock in an unimaginably vast universe of material existence. God, even if existed, must have better things to attend than measuring the grease offered to him by humanity and listening to their prayers of petty request. Humanity was neither the center of the universe nor was it forever under the menacing shadow of an almighty. Modern astronomy and physics, at the same time, offered navigational potential to unprecedented distance and the ominous power of modern firearms to Europeans, with which they conquered the world, a deed of unconceivable magnitude for humanity of the past.

But gone with the omnipotent power of god were also the mental frameworks that had controlled individual mind and ensured social and economic stability of the European societies for more than one thousand years. Without the reign of the fear to god, barbarous human nature came out of Genie’s bottle so anarchy and war ruled the day. Practical men rode to war to grab secular power and men of gifted intelligence scrambled to conceive their own mental frameworks in the hope of helping to establish a new social and economic order. For humanity this was a titanic struggle and the ultimate output is free capitalism, with a political system of representative democracy based on the rule of majority decision and an economic system of a non-collaborating game based on inalienable private property right and free trade. In this new system there is no more Good standing above individual self-interest. The game

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5Copernicus, Kepler, Galileo and Newton are certainly among the few who are responsible for the creation of modern astronomy and modern physics.
of life is returned to become a game of self-interest again, in which all are compelled to work for money. Individuals, of course, have to obey the rules of the game, which are whatever set forth by a representative majority.

Aided by the rising tide of science and modern technology, free capitalism was gradually established and human material productivity was advanced in a stunning pace. Humanity, unfortunately, had not been on a pleasant ride of steady progresses but that of a roller-coaster. Repeatedly, economic progresses would bring mankind to triumphs of unprecedented height, then it would send it violently to a fast downward track, going straight towards the abyss of anarchy. Something was fundamentally wrong in the system and it was as the following:

The two fundamental issues of an economic system are the issue of production and the issue of consumption. Production and consumption are two side of the same economic coin. In order for an economic system to work properly, it must be able to not only produce but also to collectively consume the produced. In a free capitalist system, the objective of a capitalist is to maximize his profit, in doing so he would try to make his employees work as hard as possible to increase output and at the same time he would try to pay them as little as he could to cut cost. In both initially he met only feeble resistance, for capitalists held all economic resources hence all the manipulative power in the economic system. But an increment in output is an increment in supply and a reduction in labor payment is a reduction in consumption demand. This is to say that, under the pressure of free market competition, the capitalists are all trying to grow the supply side of the economic coin but at the same time they are all trying to cut the demanding side. Such effort is destined to fail, for the invisible hand of free market does not allow the size of these two sides to go independently in opposite directions. The farther the size of the two sides are differentiated, the stronger would be the tension created by the invisible hand of supply and demand, which would eventually snap, causing social and economic hardship for all.

What has saved the system is majority decision of the political system and the labor movement. Impossible to improve their conditions under the existing economic framework, labor class, which evolved gradually into a majority in number, found a common interest in seeking political power to re-distribute the wealth of the society. Capitalists instinctively resisted, but gradually came to the realization that limited wealth re-distribution, uniformly imposed by the government, was a unique way for a society to extend the demanding side of the economic coin without hindering free market competition, therefore a way that would eventually benefit all including themselves. Free capitalism then entered an epoch of social reform and wealth re-distribution. Working hours were cut shorter; minimum wage were imposed; progressive taxation, estate tax and social security system were established. With these reforms free capitalism evolved into consumption oriented free capitalism, where the consumption oriented part is represented by the rules of wealth re-distribution.

This is, I believe, the gist of the doomsday prediction of free capitalism of Karl Marx.

The surplus value theory of Das Kapital was then invalidated by these social reforms.
It is worth noting that, free capitalism, in its long struggle against the social organization of the past, had developed an instinctive suspension on government interventions of economic activities. To promote the principle of laissez faire, heralds of free capitalism have construed, in the place of God and his purpose, a new doctrine on the sacredness of individual freedom. Nobody, of course, dared to argue that individual freedom is absolute but they all argued that any restrictions on individual freedom, in particular, governmental rules that alienates private property right, should be abundantly justified, implicating that any such rule was with a bad smell of evil against good. The principle of laissez faire, unfortunately, was in direct conflict with the principle of wealth re-distribution, the supporters of which were soon gathered under the banner of social and economic justice. There then came the clashes of the right and the left wing ideologies, in which people passionately vested in one ideology or the other nurtured hatred against each other. This was not a healthy trend, for like all ethic rules and religious believes of the past, ideological doctrines were no more than subjective and useful creations of human mind and should be treated merely as such.\textsuperscript{8} For the consumption oriented free capitalism, the principle of laissez faire and the principle of wealth re-distribution must strike a balance, and to make this balance, one principle must not stampede the other. Since the ultimate power of our society is vested in majority decision, to advocate to the public that one is good and the other is evil is utterly irresponsible.

4. Modern economic system as a Ponzzi game

Modern economic system offers anyone who wishes for a future return of his money an opportunity, but to take the advantage of this opportunity, one must invest, that is, to first surrender his money to the system. The system then gives the money one just invested, through a rather complicated delivery system, to people who invested in the past. At any given time it takes in more than it delivers back, for it is a game of negative aggregate profit.

To model a consumption oriented free capitalist system, we first introduce four fundamental elements and they are (1) a component of production, (2) a human population (as investor/laborer/consumer), (3) consumption-goods, (4) money. Investor/laborer/consumer are from the same human population. Component of production employs laborers to produce consumption-goods, which are then consumed by the population. The process of production and consumption are facilitated by the use of money, which are supplied on demand by a monetary authority.

Component of production consists of a set of economic unit, each of which is controlled by a group of people who are its owners. Economic units produce either consumption-goods to be consumed by the population or production-goods to be used as raw materials for other economic units. They can be either physically tangible goods such as a computer or intangible services such as a business advise. When a computer is bought by an economic unit for the use of its employees at work it is production-goods but if it is bought by a consumer for personal use it is a

\textsuperscript{8}It is somehow regrettable that the leading social and economic minds of our time are still arguing based on such subjective nonsense.
consumption-goods. An economic unit employs laborers and acquires production-goods from other economic units to produce its own product. If one views the component of production as a black box, then all production-goods are hidden inside and only consumption-goods come out.

With money as a facilitator, exchange of materials are reduced into the two actions of sell and buy. One sells labor and materials in possession to others to acquire money and uses the money acquired to buy labor and material from others. For an economic unit, the money it receives from the sales of the consumption-goods and the production-goods it produced is its total revenue and the money it pays directly to the population and to other economic units are its total cost. Let $R_c$ be the revenue generated from the sales of consumption-goods and $R_p$ be the revenue generated from the sales of production-goods. Then $R = R_c + R_p$ is the total revenue. Let $S_c$ be the direct payment to the population and $S_m$ be the payment to other economic units. Then $S = S_c + S_m$ is the total cost. All economic units try to maximize the profit $P := R - S$. Using $\sum$ to represent the action of adding a quantity over all economic units, we then define the aggregate revenue of the component of production as $\sum R_c$, and the aggregate personal income of the population as $\sum S_c$.

The way money is circulated around the module of consumption-goods, the component of production and the module of population are schematically illustrated in Fig. 1. Component of production delivers consumption-goods, which we denote as $\cup Q_c$. It also delivers aggregate personal income $\sum S_c$ to the population. Population then buy consumption-goods using part of the aggregate personal income. The aggregate revenue $\sum R_c$ is then circulated back into component of production through the sales of $\cup Q_c$.

![Fig. 1 Production, Income and Consumption](image)

We add the dimension of time by making infinitely many copies of Fig. 1 and line them up from left to right as a bi-infinite sequence, representing the endless flowing of consecutive time period. For a given period of time, aggregate personal income $\sum S_c$ is divided into aggregate consumer spending, which equals to aggregate revenue
\( \Sigma R_c \) of the component of production, and aggregate personal saving, which equals to \( \Sigma S_c - \Sigma R_c \). The economic system recycles personal saving back into the component of production, and converting personal savings into investment. See Fig. 2.

![Fig. 2 Personal Savings and Investment](image)

Let \( M(i) \) be the total stock of money circulating inside the economic system in the \( i \)-th period of time. \( M(i) \) is initially distributed to all economic units. Then part of \( M(i) \) is delivered to the population as aggregate personal income \( \sum S_c \) and part is retained inside of the component of production. Let us denote aggregate personal income as \( M_I(i) = \sum S_c \) and the money remained inside of the component of production as \( M_r(i) \). \( M_I(i) \) is then divided into cost of consumption, which we denote as \( M_{Ic}(i) = \sum R_c(i) \) and personal saving, which we denote as \( M_{Is}(i) = \sum S_c(i) - \sum R_c(i) \). At the beginning of the \( (i + 1) \)-th period of time, the monetary authority also elevate the total money stock by an amount, which we denote as \( IM(i) \), upon the request of needy economic units. See Fig. 3. We have

\[ M(i) = M_r(i) + M_I(i) = M_r(i) + M_{Is}(i) + M_{Ic}(i) \]

and

\[ M(i + 1) = M(i) + IM(i). \]

Among the four components of \( M(i + 1) \), which we denoted as \( M_r(i), M_{Ic}(i), M_{Is}(i) \) and \( IM(i) \) respectively, \( M_r(i) \) is retained in the respective economic units in the component of production. \( M_{Ic} \) is recycled back to the component of production through the sales of the consumption goods and are re-distributed to all economic units as their respective revenue. \( M_{Is} \) is recycled back and distributed into component of production through banks, stock market and bond market, again under the guidance of the principle of voluntary exchange: instead of exchanging money for consumption-goods at present time, for which \( M_{Ic} \) is used, individuals negotiate with economic units to exchange their share of \( M_{Is} \) for a promise of future delivery of money in mutually agreed manners and schedules. \( IM(i) \), the money supplied by monetary authority, is similarly distributed to economic units through the banking system.
Since all personal savings are constantly recycled back into component of production and are invested into various economic units, economic system does not reserve accumulatively money for the saving part of the personal income over any given stretch of time. *Savings from the past* is therefore illusionary in the sense that all savings are no more than a current promise of an advantage in future distribution of consumption-goods. When a person is tapping into his saving from the past for current consumption, he is merely adding to his current personal income something that was promised to him by someone else in the system. In any given time period, the only money in our model that is available to the population to buy consumption goods is from the current aggregate personal income $M_I(i) = \sum S_c$. This implies

$$\sum P(i) = \sum (R_c(i) + R_p(i)) - \sum (S_c + S_m)(i)$$

$$= \sum R_c(i) - \sum S_c(i)$$

$$= -M_{Is}(i) < 0,$$

where the second equality is obtained by using $\sum R_p = \sum S_m$. This is to say that, for a consumption oriented economy, *aggregate profit of the component of production must be negative at any given time*.

**Economic System As A Ponzi Game**

A. *The Economic Ponzi Game* Modern economic system offers anyone who wishes for a future return of his money an opportunity, but to take the advantage of this opportunity, one must invest, that is, to first give his money to the system. The system then takes the money one just invested in, give it to the people invested in the past through a rather elaborated delivery system. At any given time, it would take in more than it gives back, for this is a game of negative aggregate profit. The money one invested is ones *capital* and the extra return one expects from the invested capital is its *profit*. This economic game, based on voluntary participation, must constantly

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9The real material contents of such promises could vary substantially over even not so long a stretch of time. This must be apparent to anyone who was hit by a stock bubble or a super-inflation.
deliver a profit to the invested capital. Otherwise the investors and entrepreneurs would stop coming to participate and the system would collapse.

The economic game of the modern society, therefore, is a Ponzi game in which, at all time, current profit for past investment is delivered by current investment that expects in turn a future profit. This game, however, is not a deceptive scheme, as far as the rate of improvement of material productivity of the economic system matches the rate of return it promised. The positive improvement, as correctly identified by Joseph Schumpeter, is created by the effort of industry entrepreneurs, whose successes are rooted in the advancement of modern science and technology.

**B. Economic Cycles** The pace of innovations of science and technology is, however, uneven. Progresses could be rather sluggish for a while. Lacking real growth potential in immediate future, economic system would have no choice but to default its promised returns to at least some of the investment of the past. As far as this is done gradually, and the default is not suddenly widespread, there will be only mild overall reduction of economic activity. But as the productivity shrinks, room for future growth would recur, so default would gradually stop and economic activity would pick up. Such time would be one of a general stagnation, and the economy would oscillate between mild recessions and weak recoveries. In reality, new technological innovations will eventually set in, and the upper limit for productivity would be raised. So in addition to regular cycles, economy system would also be in an upward trend in the long run. Most of economic cycles are this normal type.\(^{10}\)

We now turn to the reasons for near collapsing recessions. Once in a while there would come a strong wave of new technological innovations that can be gradually but persistently turned into economic growth. Then the Ponzi game would work at ease and an economic boom would last for a relatively long period of time. Total production of consumption good would grow at a relatively fast pace and so would be the total saving and the total social wealth. By the time this technological revolution finally ran out of steam, however, growth potential would diminish. However, the population at large has gotten used to the happy time and would, in an effort to hold on to a growth rate that is no longer sustainable, turn the existing economic game from a Ponzi game into a grand, deceptive Ponzi scheme, which would last for a while but will have to bust eventually. Default would then be wild spread, causing vicious cycles of reduced investment and productivity that can not be easily corrected by the new room of growth created by recession. When unemployment runs sufficiently high, social instability would ensue, and the social and economic system would face the danger of a total collapse. Strong government intervention would then become inevitable.

\(^{10}\)Various ways of stimulation, ranging from tax rebate, to government stimulates, to monetary ease, are all efforts to turn the economic Ponzi game into a deceptive Ponzi scheme. They could all temporarily ease the existing social and economic tension of a recessing economy by generating fictitious growth. However, unless government is determined to risk a near collapsing recession to run a full scaled deceptive Ponzi scheme, such effort are not sustainable, and would have to eventually give way to the reign of regular, probably slightly worse, economic cycles, unless the stagnation is short-lived and new innovation happens to set in on time.
C. Income Re-distribution  Concerning the two fundamental issues an economic system must resolve: the issue of production and the issue of distribution of the produced for consumption, answer to the former is represented by the total output of consumption-goods $\cup Q_c(i)$ of the component of production and answer to the latter is represented by the way personal income are delivered from the component of production to the population. So far we have appeared to assume that these two issues are resolved by applying the free market principle through the invisible hand of supply and demand. This is in fact not the case because in our model we have tacitly assumed that, distribution of personal income, directly pushed down to the population, is such that the population is able to consume all that is in $\cup Q_c$. This is not necessarily the case if the distribution of income handicaps the consumption ability of the population.
5. Micro Dynamics of Production, Cost and Profit

In this section we develop a micro-level dynamics model on production, cost and profit, which we will use as a starting point to build a new model for the modern system of production in later sections. Since the existence of a unique equilibrium state in this micro-level model is a matter of triviality, the main focus of our consideration is on how to adjust the quantity of production to approach the existing equilibrium state.

Consider an economic unit that produces one product, which we denote as $g$. We assume that the cost of production of $g$ and the ensuing revenue depend exclusively on the produced quantity of $g$, which we denote by using a real variable $q_g$. Everything else is fixed. Hence we start with two twice differentiable functions $S_g(q_g)$ and $R_g(q_g)$ of $q_g$: $S_g(q_g)$ is the cost schedule of $g$ and $R_g(q_g)$ is the revenue schedule. For $q_g \in [0, \infty)$, $S_g(q_g)$ is the would-be cost to produce $q_g$ units of $g$ and $R_g(q_g)$ is the would-be revenue. Standard assumptions on $S_g(q_g)$, $R_g(q_g)$, $q_g \in [0, \infty)$ are

$$(4) \quad S'_g(q_g) > 0, \quad S''_g(q_g) > 0; \quad R'_g(q_g) > 0, \quad R''_g(q_g) < 0$$

where each prime represents one derivative with respect to $q_g$. For a product $g$ to have a chance to establish itself, there must be an interval $[q^1_g, q^2_g]$ of $q_g$ on which the profit function

$$P_g(q_g) := R_g(q_g) - S_g(q_g) > 0.$$ 

See Fig 4(a). The otherwise situation is depicted in Fig. 4(b), in which no positive profit is possible. Product $g$ of Fig. 4(b) represents an entrepreneur venture that is destined to fail.

![Fig. 4. Cost and Revenue Schedule](image-url)
such that $P'(q) > 0$ for all $q < q_g^m$ and $P'(q) < 0$ for all $q > q_g^m$. The profit function $P_g(q_g) = R_g(q_g) - S_g(q_g)$ reaches a global maximum at $q_g = q_g^m$ and the quantity of production $q_g$ is naturally gravitated towards $q_g^m$. General economics theory has focused on issues related to the existence and uniqueness of equilibria such as $q_g^m$. Here we are more interested in modeling how the quantity of production is adjusted to approach $q_g^m$. Let $t \in \mathbb{R}$ be the time, and $q_g(t)$ be the quantity of $g$ that is produced at time $t$. We assume that the entrepreneurs monitor the marginal profit $P'(q_g(t))$ to constantly adjust $q_g(t)$. They would increase production if $P'(q_g(t)) > 0$, and reduce production if $P'(q_g(t)) < 0$. The adopted adjustment of production is proportional to marginal profit and to the current quantity of production as well at all time. Such proposed practice is modeled by a first order differential equation in the form of

\begin{equation}
\dot{q}_g = kP'(q_g)q_g
\end{equation}

where $k > 0$ is a constant and the dot on top denotes derivative with respect to time. By using (4), it is straightforward to confirm that $q = q_g^m$ is an asymptotically stable equilibrium solution of equation (5).

Equation (5), however, fails to distinguish Fig. 4(a) from Fig. 4(b). All solutions of equation (5) approach to $q_g^m$ hence this equation suggests that, in the case of Fig. 4(b), entrepreneurs would keep the production of $g$ going indefinitely even if there is no possibility ever to make a positive profit, an obviously absurd assertion. To be able to recognize a bad investment situation, entrepreneurs ought to also monitor the profitability of $g$ so they would not be forever trapped in a case of Fig. 4(b). The object they ought to monitor would naturally be the profit history of $g$, which we propose to write into one quantity as

\begin{equation}
\xi_g(t) = \int_0^t e^{\theta_g(s-t)} \frac{P_g(q_g(s))}{S_g(q_g(s))} ds
\end{equation}

where $\theta_g > 0$ is a constant. With the factor $e^{\theta_g(s-t)}$ inside the integral we have assumed that the entrepreneurs weigh more on recent history. The term

\begin{equation*}
\frac{P_g(q_g(s))}{S_g(q_g(s))}
\end{equation*}

is profit per-cost.

We now consider the options available to us in including $\xi_g(t)$ into the modeling equation. There are of course infinitely many ways but only two would appear natural; the first is to add, and the second is to multiply $\xi_g(t)$ to the right hand side of equation (5). By to add we mean to propose a modeling equation in the form of

\begin{equation}
\dot{q}_g = (k_gP'(q_g(t)) + \gamma_g\xi_g(t))q_g(t),
\end{equation}

in which the adjusted rate of production is determined by a weighed average of marginal profit and profit history. This option, however, contradicts to the assumption that the entrepreneurs know that they would maximize their profit at a quantity of production of zero marginal profit. It would suggest that even after the entrepreneurs
hit a quantity of production with a negative marginal profit, they might still expand production because of a favorable profit history. This is absurd so we move to the second option of including $\xi_g(t)$ through multiplication. Our proposed modeling equation in this case would be in the form of

$$\dot{q}_g(t) = k_g(\gamma_g + \xi_g(t))P_g'(q_g(t)) \cdot q_g(t)$$

where $\gamma_g > 0$ is a constant. To equation (8) we would impose an additional constraint: in order for the production of $g$ to continue, it is necessary for us to have

$$\xi_g(t) \geq -\gamma_g.$$  

This is to say that, while acknowledging that negative profit is sometimes inevitable in the process of production, there is a level of loss in terms of profit history, which we denote as $\gamma_g$, beyond which our entrepreneurs would be unwilling to tolerate. Entrepreneurs would stop producing $g$ if the level of loss exceeds $\gamma_g$.

Let $x = q_g$, $y = \gamma_g + \xi_g$. We re-write the modeling equation (8) and the associated constraint as

$$\dot{x} = k_g y P_g'(x) x$$

$$\dot{y} = \frac{P_g(x)}{S_g(x)} - \theta_g (y - \gamma_g)$$

where the second equation is obtained by differentiating (6). Equation (10) is defined on

$$D = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0\}.$$  

It is straightforward to verify that

$$x = q_m^g, \quad y = \gamma_g + \frac{1}{\theta_g} \frac{P_g(q_m^g)}{S_g(q_m^g)}$$

is an equilibrium solution of equation (10) and this equilibrium is a stable node. It is also worth noting that if $\gamma_g$, the tolerance level on losses, is low then production of $g$ could be terminated long before it reaches a point of positive profit.

### 6. A Dynamics Model for The System of Production

#### A. Short Term Economic Environment

A production unit that produces a fixed quantity of the same product forever and ever belongs only to the economic system of the medieval age, not that of the modern time. The fact that all solutions of equation (10) quickly approach to a stable equilibrium is therefore an indication that equation (10) is rather unsatisfactory even in qualitative sense in modeling the long term behavior of a production unit in modern economic system. Our problem obviously resides in the assumption that everything else is fixed in the economic environment. For a non-monopoly economic unit in modern times, it is not even reasonable to assume equation (10) for a relatively short period of time, for the competitive economic environment a production unit is subjected to is full of surprises. The modern economic system, as we have discussed earlier, was not designed to reach and thereafter to maintain an equilibrium, but to handle the endless stream of technological inventions
that constantly try to creatively destroy the existing equilibrium states in material production.

But before getting into the construction of a complicated mathematical model for an economic system in which products and relations are ever changing, we would like, as the first step, to introduce something that is simpler but more definite that we could hold temporarily as a lunching pad. Let \( t \) be the time and \( L(t) \) be the list of all consumption-goods produced and consumed in a consumption oriented free capitalist society at time \( t \). Let \( q_g \) be a real variable representing the quantity of production of \( g \) for \( g \in L(t) \) and \( q_{L(t)} \) be a vector that enlists all \( q_g, \ g \in L(t) \). We also use \( L(t) \setminus g \) to denote the set of all products in \( L(t) \) excluding \( g \). In this section we work on a time interval \( I = [a, b] \) and assume that

(a) \( L(t) \) does not change as \( t \) varies on \( I \).

(b) The natural and the human environment and the process of production of all goods, consumption-goods and production-goods alike, do not change as time varies over \( I \).

This is to assume a steady overall economic environment in which there is no nature or man-made shocks to the system of production. Denote \( L = L(t) \). Our modeling variables on \( I \) are the quantity of production \( q_L \) of various products in \( L \), which we would assume to be a function of time. This is to say that \( q_L = q_L(t), \ t \in I \).

We introduce a set of differential equations to model the functions \( q_L(t) \) on \( I \) under the framework introduced in the previous sections. This is therefore a short term model of the system of production, a first step in which we take the difficulties of creative destruction temporarily out of the picture of our consideration. As we will see momentarily, the reasons that helped us to get to equation (10) in the previous section are more than useful in this step, in which we would incorporate the free market principle of individual profit maximization and constraints of negative aggregate profit (See (3)).

B. Schedule of Revenue and Cost

We start with the revenue schedule for \( g \in L \), which we denote as \( R_g \). Since supply and demand for various consumption-goods are inter-dependent, revenue generated from sales of \( q_g \) units of \( g \) depends also on the quantity of other consumption-goods produced. Therefore we could only determine the would-be revenue from the sales of \( q_g \) units of \( g \) if we know the quantities at which other products in \( L \setminus g \) are on sale. We can then write

\[
R_g = R_g(q_L) = R_g(q_g, q_{L \setminus g}).
\]

We now turn to cost schedule, which incurs to all economic units involved in production of \( g \) as consisting of two parts. The first is the material cost \( S_m \), which is illusionary in the sense that they represent a way revenue is distributed among all economic units involved in the production of \( g \). From bird’s view there is no real cost that is associated to \( S_m \). The second is the direct payment of all economic units involved in the production of \( q_g \) unit of \( g \) to the population. This cost is real and is denoted as

\[
S_g = S_g(q_L) = S_g(q_g, q_{L \setminus g}).
\]
Let

\[ P_g(q_L) = P_g(q_g, q_{L\setminus g}) := R_g(q_g, q_{L\setminus g}) - S_g(q_g, q_{L\setminus g}) \]

be the profit schedule. It is the sum of profit that would be made by all economic units involved in producing \( q_g \) units of \( g \).

We again assume that for all \( g \in L \),

\[ \partial q_g S_g > 0, \quad \partial^2 q_g q_g S_g > 0; \quad \partial q_g R_g > 0, \quad \partial^2 q_g q_g R_g < 0. \]

This is to say that, assuming the quantities of production of all other items in \( L \) are fixed, (1) both cost and revenue increase as the quantity of production of \( g \) is increased; and (2) marginal cost increases as the quantity of production of \( g \) increases but marginal revenue would decline according to the law of diminishing revenue. Not much more could be said about the other derivatives. For instance, for \( \tilde{g} \in L \setminus g \), the sign of \( \partial \tilde{g} R_g \) can go either way. If \( \tilde{g} \) and \( g \) are competing, then the increase of the production of \( \tilde{g} \) would reduce the demand for \( g \). To sale the same quantity of \( g \), the producers would have to cut the price of \( g \), causing revenue for \( g \) to decline. Consequently we have

\[ \partial \tilde{g} R_g < 0. \]

On the other hand, if \( \tilde{g} \) is complementary to \( g \), then an increase in the production of \( \tilde{g} \) would raise the demand for \( g \). With a fixed quantity of \( g \), the producers would ration consumers by raising the price of \( g \), causing revenue for \( g \) to increase. Consequently we have

\[ \partial \tilde{g} R_g > 0. \]

Hence the sign of \( \partial \tilde{g} R_g \) can go either way. Reasoning for the signs of \( \partial \tilde{g} S_g \) are similar.

There is, however, a constraint we need to impose on the aggregate profit according to (3). For the consumption oriented free capitalist system of Section 4, we must have

\[ \sum_{g \in L} P_g(q_L) < 0 \]

for all values of \( q_L \). This is to say that, for all feasible configurations of production, aggregate profit must be negative. Constraint (16) is equivalent to

\[ \sum_{g \in L} R_g(q_L) < \sum_{g \in L} S_g(q_L). \]

So for all feasible configuration of production, aggregate revenue must always be smaller than aggregate cost.

### C. Dynamics of the System of Production

Let \( q_L(t) = \{q_g(t), \ g \in L\} \) be the configuration of actual production of \( g \) at time \( t \in I \). Then \( S_g(q_L(t)) \) is the actual cost of production of \( g \) and \( R_g(q_L(t)) \) is the corresponding revenue induced. We introduce a set of modeling equations for \( q_L(t) \) using the micro-model introduced in Section 5 as foundation. Reasoning along the same line, we would first write equations corresponding to equation (5) for the production of \( g \) for all \( g \in L \) in the form of

\[ \dot{q}_g(t) = k_g \partial q_g [R_g(q_L(t); q_{L\setminus g}(t)) - S_g(q_L(t); q_{L\setminus g}(t))] \cdot q_g. \]

Like in micro-level models, this is to assume that entrepreneurs monitor the marginal profit of their own product constantly and use it to adjust the momentum of
production. Equation (18) is a direct implementation of the free market principle, in which all entrepreneurs seek to maximize their own business profit. In this equation there resides the fundamental assumption that modern economic system is a non-collaborating game.

To obtain equilibrium solutions of equation (18) we solve the equations

\[ \partial_{q_g}[R_g(q_g; q_{L\setminus g}) - S_g(q_g; q_{L\setminus g})] = 0, \]

for all \( g \in L \). Assume that all \( q_g \) are uniformly bounded, which they must be due to the finite natural of human activity and limitations of natural resources, then the existence of solutions of equation (19) follows from (15) by a fixed point argument originally proposed by Nash. Nash equilibriums, however, are forced into existence by the uniform boundedness of \( q_g \). They are in general not unique and the impact of these equilibriums on generic behavior of solutions of equation (18) might very well be negligible and in any case are almost impossible to deduce mathematically. One might argue that the reason for lacking of assurance of a unique equilibrium is due to our lacking of more detailed knowledge of cost and revenue schedules. But this is very unlikely and it matters little anyway, for so far as the existence of a unique stable equilibrium that is globally attracting is not confirmed, there would be no base for us to assume that the entrepreneurs would act according to an equilibrium theory, as they were forced to in the models of Section 5.

Entrepreneurs would also add profit history of their own product into consideration. Let

\[ \dot{\xi}_g(t) = \int_{t_0(g)}^{t} e^{\theta_g(s-t)} \frac{P_g(q_{L}(s))}{S_g(q_{L}(s))} ds \]

where \( \theta_g > 0 \) is a constant. We modify equation (18) to include \( \xi_g(t) \) to the right hand. In between the two ways, to add or to multiply, represented respectively by equations (7) and (8) in micro models, there is no longer a compelling reason to reject adding for multiplying, for zero marginal profit does not necessarily imply absolutely maximized profit in current situation. To be consistent, however, let us stick to (8) to incorporate the consideration of profit history by the equation in the form of

\[ \dot{q}_g = k_g(\gamma_g + \xi_g)\partial_{q_g}(R_g - S_g) \cdot q_g \]

where \( -\gamma_g < 0 \) is the maximal level of losses producers of \( g \) would tolerate, and we assume \( \gamma_g + \xi_g(t) > 0 \) for all \( t \). It would make no difference for the main conclusions of our study to hold if one insists on using the correspondence of (7), that is, the equation in the form of

\[ \dot{q}_g = (k_g\partial_{q_g}(R_g - S_g) + \gamma \xi_g) \cdot q_g. \]

From this point on we will use (21). The momentum of production of a given product \( g \) is adjusted in proportion to marginal profit and profit history of this product.

We now turn to the measurement we are going to use to represent the overall economic climate. Let

\[ r(t) = \frac{R(t)}{S(t)} \]
where

\[ R(t) = \sum_{g \in L} R_g(q_L(t)), \quad S(t) = \sum_{g \in L} S_g(q_L(t)) \]

are the aggregate revenue and aggregate cost of production. \( r \) is the overall revenue per cost of production. Let \( r_g = R_g/S_g \) be the revenue per cost of production for \( g \). First we note that, \( r_g \) and \( r \) are things of very different natural in the calculation of the producers of \( g \). A reduction on the value of \( r_g \), as a direct indication of worsening efficiency of their own invested capital, would have a negative impact on the momentum of production of \( g \). This is what was adopted in equation (21). On the other hand, a reduction on the value of \( r \) is not in their immediate calculation as far as efficiency of investment is concerned. In the non-collaborating economic game of laissez faire, overall efficiency of the total invested capital does not influence the decision of individual producers the same way as that of their own \( r_g \). The impact of a reduction on the value of \( r_g \) and that of the value of \( r \) to the decisions of the producers of \( g \), as a matter of fact, are in opposite directions.

To consider the impact of \( r \) on industrial entrepreneurs, let us recall that \( S(t) \) is also the aggregate personal income of the population and \( R(t) \) is the aggregate consumption spending. Under the assumption that there is an ideal proportion of overall consumption spending and saving adopted by the entire population, a reduction on \( r \) would be a direct indication of overall underproduction of consumption-goods, and an increment on \( r \) would be a direct indication of overall overproduction of consumption-goods. Therefore a larger value of \( r \) would imply an economic environment that is tougher than usual for the sales of all products but a smaller value of \( r \) would imply an economic environment that is easier for the sales of all products.

In our model we would assume that entrepreneurs also adjust their own momentum of production in accordance to the history of \( r \), that is

\[(23) \quad \eta(t) = \int_{-\infty}^{t} e^{\Theta(s-t)} \frac{R(s)}{S(s)} ds \]

where \( \Theta > 0 \). The exponential term

\[ e^{\Theta(s-t)} \]

is again included to indicate that entrepreneurs pay more attention to recent economic climate. Our modeling equation on production would then take the form of

\[(24) \quad \dot{q}_g(t) = (k_g(\gamma_g - \xi_g(t))) \partial_{q_g} P_g(q_L(t)) + \beta_g(r_0 - \Theta \eta(t))) \cdot q_g \]

for all \( g \in L \) where \( \beta_g > 0 \) measures the sensitivity of the production of \( g \) in reacting to the overall economic climate. A copy of \( \Theta \) is multiplied to \( \eta \) in equation (24) because assuming \( r = r_0 \), we would have \( \eta = r_0 \Theta^{-1} \). With equation (24) we have assumed that there is an optimal expected value of overall revenue per cost \( r_0 \), representing an overall business climate all entrepreneurs would regard as neutral. Overall business climate is regarded as favorable if \( r < r_0 \), for the population is not in lack of money to buy the produced. Similarly, overall business climate is regarded as less favorable if \( r > r_0 \), for the money passed to consumers through entrepreneur investment is in
a general shortage for consumption spending. Equation (24) is equivalent to

\[ \dot{q}_g = (k_g(\gamma_g + \xi_g) \partial_{q_g} P_g(q_L) + \beta_g(r_0 - \Theta \eta)) \cdot q_g \]

(25)

\[ \dot{\xi}_g = \frac{P_g(q_L)}{S_g(q_L)} - \theta_g \xi_g \]

for all \( g \in L \), and we also have

(26)

\[ \dot{\eta} = \frac{\sum_{g \in L} R_g(q_L)}{\sum_{g \in L} S_g(q_L)} - \Theta \eta \]

by differentiating (23) with respect to time. Equations (25) and (26) are our short term model for the dynamics of the modern production system.

7. Dynamics of short term economic climate

In this section we use the model introduced in the last section to illustrate that the constraint of aggregate negative profit stabilizes short term economic climate. We also illustrate that irregular ups and downs of aggregate production in such relatively stable economic environment are partly due to responses of entrepreneurs to the change of the overall economic climate. Let

\[ r = \frac{\mathcal{R}}{\mathcal{S}} = \frac{\mathcal{P}}{\mathcal{S}} + 1. \]

We have

\[ \dot{r} = \frac{\dot{\mathcal{P}} \mathcal{S} - \dot{\mathcal{S}} \mathcal{P}}{\mathcal{S}^2} = \frac{\dot{\mathcal{P}}}{\mathcal{S}} - \frac{\dot{\mathcal{S}}}{\mathcal{S}} \cdot \frac{\mathcal{P}}{\mathcal{S}}. \]

Write

\[ \mathcal{P} = \sum_{g \in L} P_g(q_L), \quad \mathcal{S} = \sum_{g \in L} S_g(q_L), \]

we have

\[ \dot{\mathcal{P}} = \sum_{g \in L} \sum_{g' \in L} \partial_{q_g} P_{g'} \cdot \dot{q_g}, \quad \dot{\mathcal{S}} = \sum_{g \in L} \sum_{g' \in L} \partial_{q_g} S_{g'} \cdot \dot{q_g}. \]

By using equation (24), we obtain

\[ \dot{r} = \frac{1}{\mathcal{S}} \sum_{g \in L} \sum_{g' \in L} \left( \partial_{q_g} P_{g'} - \frac{\mathcal{P}}{\mathcal{S}} \cdot \partial_{q_g} S_{g'} \right) \dot{q_g} \]

\[ = \frac{1}{\mathcal{S}} \sum_{g \in L} \sum_{g' \in L} \left( \partial_{q_g} P_{g'} - \frac{\mathcal{P}}{\mathcal{S}} \cdot \partial_{q_g} S_{g'} \right) (k_g(\gamma_g + \xi_g) \partial_{q_g} P_g + \beta_g(r_0 - \Theta \eta)) \cdot q_g \]

\[ = \frac{1}{\mathcal{S}} \sum_{g \in L} \left( \partial_{q_g} P_g - \frac{\mathcal{P}}{\mathcal{S}} \cdot \partial_{q_g} S_g \right) (k_g(\gamma_g + \xi_g) \partial_{q_g} P_g + \beta_g(r_0 - \Theta \eta)) \cdot q_g \]

\[ + \frac{1}{\mathcal{S}} \sum_{g \neq g'} \left( \partial_{q_g} P_{g'} - \frac{\mathcal{P}}{\mathcal{S}} \cdot \partial_{q_g} S_{g'} \right) (k_g(\gamma_g + \xi_g) \partial_{q_g} P_{g'} + \beta_g(r_0 - \Theta \eta)) \cdot q_g. \]

Differentiating (26) with respect to time we then obtain

(27)

\[ \ddot{\eta} + \Theta \eta = \dot{r} = (I) + (II) \]
where

\[
(I) = \frac{1}{S} \sum_{g \in L} \left( \partial_{q_g} P_g - \frac{P}{S} \cdot \partial_{q_g} S_g \right) (k_g (\gamma_g + \xi_g) \partial_{q_g} P_g + \beta_g (r_0 - \Theta \eta)) \cdot q_g
\]

(28)

\[
(II) = \frac{1}{S} \sum_{g \neq g} \left( \partial_{q_g} P_g - \frac{P}{S} \cdot \partial_{q_g} S_g \right) (k_g (\gamma_g + \xi_g) \partial_{q_g} P_g + \beta_g (r_0 - \Theta \eta)) \cdot q_g.
\]

Item (I) is the direct effect of the actions of the entrepreneurs on \( \eta \) and item (II) is the indirect effect of the same action on \( \eta \) reflected in the interactions of different products. We further write (I) as

\[
(I) = A(t)(r_0 - \Theta \eta) + B(t)
\]

where

\[
A(t) = \frac{1}{S} \sum_{g \in L} \beta_g \partial_{q_g} P_g \cdot q_g - \frac{P}{S^2} \sum_{g \in L} \beta_g \partial_{q_g} S_g \cdot q_g
\]

(30)

\[
B(t) = -\frac{P}{S^2} \sum_{g \in L} k_g (\gamma_g + \xi_g) \partial_{q_g} S_g \partial_{q_g} P_g \cdot q_g + \frac{1}{S} \sum_{g \in L} k_g (\gamma_g + \xi_g) \partial_{q_g} P_g \partial_{q_g} P_g \cdot q_g.
\]

The equation for \( \eta \) is then in the form of

\[
\ddot{\eta} + \Theta \dot{\eta} = A(t)(r_0 - \Theta \eta) + B(t) + C(t)
\]

where \( A(t), B(t) \) are as in (30) and \( C(t) = (II) \) as in (28). We are obviously interested in the solution of equation (31) satisfying initial condition,

\[
\eta(0) = \int_{-\infty}^{0} e^{s} \frac{R(s)}{S(s)} ds, \quad \dot{\eta}(0) = \frac{R(0)}{S(0)} - \Theta \eta(0).
\]

Let us also remember that \( \eta \) by definition must be in the region where

\[
0 < \eta < \frac{1}{\Theta}.
\]

**Stability of Economic Climate and Business Cycles**

Even if \( A(t), B(t) \) and \( C(t) \) were given explicitly, which is obviously not the case here, equation (31) is in general not explicitly solvable. To study the properties of the solution of equation (31), we introduce, as a first approximation, an autonomous equation by averaging \( A(t), B(t) \) and \( C(t) \) over the time interval \( I \). This is to say that let \( I \) be the time interval on which equation (31) is defined, we first study the autonomous equation

\[
\ddot{\eta} + \Theta \dot{\eta} = A_0 (r_0 - \Theta \eta) + B_0 + C_0
\]

(34)

where

\[
A_0 = \frac{1}{|I|} \int_I A(t) dt, \quad B_0 = \frac{1}{|I|} \int_I B(t) dt, \quad C_0 = \frac{1}{|I|} \int_I C(t) dt.
\]

Equation (34) has an equilibrium solution at \( (\eta, \dot{\eta}) = (\eta_0, 0) \) where

\[
\eta_0 := \frac{1}{\Theta} \left( r_0 + \frac{B_0 + C_0}{A_0} \right).
\]
To argue for the overall stability of the short term economic climate, we now illustrate that

(P1) the value $\eta_0$ for the equilibrium solution is such that

$$0 < \eta_0 < \frac{1}{T};$$

(P2) the equilibrium solution $(\eta_0, 0)$ is stable.

Both (P1) and (P2), as we will see momentarily, follow from the constraint of negative aggregate profit (16).

**Argument for (P1)** As have been discussed earlier, the signs of $\partial q_g S_g$ and $\partial q_g R_g$ can go either way, depending on if $g$ and $\tilde{g}$ are competing or complementing in production and in sales. Since we are dealing with a summation of a huge number of terms, here we would assume that the overall effect of competing against complementing of all products tends to cancel through aggregation, making $C(t) = (I)$, hence $C_0$, more or less negligible.

For $B_0$ we consider the two sums of $B(t)$ in (30). The first is

$$-\frac{p}{s^2} \sum_{g \in L} k_g (\gamma_g + \xi_g) \partial q_g S_g \partial q_g P_g \cdot q_g.$$  

Since all businesses are aimed to stay close to $\partial q_g P_g = 0$ by trial and error, we would also assume that, at any given moment, the sign of $\partial q_g P_g$ are likely to be split in between positive and negative among all products. In aggregation the positive terms are likely to cancel the negative terms, end up diminishing the influence of this sum. The second sum

$$\frac{1}{s} \sum_{g \in L} k_g (\gamma + \xi) \partial q_g P_g \partial q_g P_g \cdot q_g$$

is of a quite different nature. Every term in this sum is positive, re-enforcing the influences of each other in aggregation. This is a major contributor to $B_0$.

We now move to $A_0$, which is the most critical among $A_0, B_0$ and $C_0$. There are also two sums in $A(t)$, the first is

$$\frac{1}{s} \sum_{g \in L} \beta_g \partial q_g P_g \cdot q_g,$$

and the splitting possibility of the signs of marginal profit again cause the terms to cancel each other, rendering the influence of this sum more likely to be negligible in aggregation. The second sum is

$$-\frac{p}{s^2} \sum_{g \in L} \beta_g \partial q_g S_g \cdot q_g.$$  

All terms involved in this sum maintain the same sign, so they re-enforce the influence of each other. We also observe that $P$ is always negative according to (16), and this sum does not involve marginal profit.
We have now reduced the argument for (P1) to

\[
- \frac{P}{S^2} \sum_{g \in L} \beta_g \partial_q \cdot g \gg \frac{1}{S} \sum_{g \in L} k_g (\gamma_g + \xi_g) \partial_q P_g \partial_q \cdot g
\]

where on the left we have the non-canceling sum for \( A(t) \) and on the right we have the non-canceling sum for \( B(t) \). This inequality is valid because (1) A non-trivial proportion of personal income is saved by the population, hence \( \frac{P}{S} \) is a fixed distance away from zero in average; and (2) entrepreneurs are more likely to make \( |\partial_q P_g| \) small.

**Argument for (P2)** Let

\[
\begin{align*}
\lambda_1 &= -\Theta + \sqrt{\Theta^2 - 4A_0 \Theta}, \\
\lambda_2 &= -\Theta - \sqrt{\Theta^2 - 4A_0 \Theta}
\end{align*}
\]

be the two eigenvalues of the equilibrium solution of equation (34). For this equilibrium solution to be stable we must have \( A_0 > 0 \), which we have already argued to hold because

\[
A(t) \approx - \frac{P}{S^2} \sum_{g \in L} \beta_g \partial_q \cdot g.
\]

Note that for the equilibrium solution to be stable we must have \( P < 0 \), which is exactly the constraint of aggregate negative profit (16). This finishes our justification of (P2).

That \((\eta_0, 0)\) is an asymptotically stable equilibrium solution of equation (34) follows from \( A_0 > 0 \). It is a node if

\[
\Theta - 4A_0 \geq 0,
\]

and it is a stable focus if

\[
\Theta - 4A_0 < 0.
\]

In the first case, all solutions approach directly to \((\eta_0, 0)\) along the direction of \((1, \lambda_1)\), which is the eigen-direction of \(\lambda_1\). In the second case all solutions oscillate around the equilibrium solution. The frequency of oscillation is \(\sqrt{\Theta|\Theta - 4A_0|}\) hence the period of oscillation is

\[
(36) \quad \frac{2\pi}{\sqrt{\Theta|\Theta - 4A_0|}}.
\]

Since the contraction rate is \(-\Theta\) in this case, the distance of the solution to the equilibrium in going one full round in phase space is reduced by a multiple of

\[
e^{-\Theta \sqrt{\Theta - 4A_0}} = e^{-2\pi \sqrt{\Theta - 4A_0}}.
\]

All these characteristic quantities are dependent of \(\Theta\) and \(A_0\).

We assess the possibilities for \(\Theta\) and \(A_0\). \(\Theta\) is a system constant representing the way industrial entrepreneurs respond to the overall economic climate. With a larger \(\Theta\), the entrepreneurs are more responsive to recent economic situation, looking back less into the long term history of the economic climate. With a smaller \(\Theta\), the entrepreneurs are less responsive to the current economic climate, taking more of the long term history of the overall economic climate into their respective consideration. With this interpretation, it is clear that \(\Theta\) can not be too small, for it does not make sense to assume that entrepreneurs would base their spontaneous reaction to the economic climate on a measurement that is averaged over a long stretch of time.
To gain a rough idea about the magnitude of $A_0$, let us first assume that we are in an exceptionally ideal case in which equation (19), that is, 
\[
\partial_{q_g} [R_g(q_g; q_L) - S_g(q_g; q_L)] = 0,
\]
has a solution $q_g = q_g^m$, $g \in L$, and this solution is in addition an asymptotically stable equilibrium solution of equation (18). To consider the behave of solutions of equations (25) and (26) close to this equilibrium solution, we would substitute $q_L = q_L^m$ to find $A_0$. First we observe that in this case, the correct value for $r_0$ would be 
\[
r_0 = \frac{R(q_L^m)}{S(q_L^m)},
\]
and we have 
\[
A_0 = (1 - r_0) \frac{1}{S(q_L^m)} \sum \beta_g \partial_{q_g} S_g(q_L^m) \cdot q_g^m.
\]
To do a rough estimate, let us assume $\beta_g = \beta$ for all $g \in L$. It then follows that 
\[
A_0 \approx (1 - r_0) \beta \frac{1}{S(q_L^m)} \sum \partial_{q_g} S_g(q_L^m) \cdot q_g^m > (1 - r_0) \beta \frac{R(q_L^m)}{S(q_L^m)}
\]
whereas for the last inequality we assumed that the sales price is larger than the marginal cost of production for all $g$, which must hold at equilibrium state. Therefore we have 
\[
A_0 > r_0 (1 - r_0) \beta,
\]
where $\beta$ is a measurement on the average the responsiveness of entrepreneurs to the overall economic climate. If the entrepreneurs are not very responsive to the overall economic climate then $\beta$ would be small and if the entrepreneurs are more responsive to the overall economic climate then $\beta$ would be large.

It is reasonable to assume that $\beta$, hence $A_0$, is neither too small nor too large, and we could easily be in either the case of $\Theta - 4A_0 > 0$ or the case of $\Theta - 4A_0 < 0$. We now present one concrete example for each.

\textit{Example 1: A Sink Case:} An extreme sink case is the case of $\Theta = \infty$. Observe that, as $\Theta \to \infty$, we have 
\[
\Theta \eta(t) \to r(t) = \frac{R(t)}{S(t)}.
\]
Hence in the place of equation (31) for $\eta$ we have 
\[
\dot{r} = A(t)(r_0 - r) + B(t) + C(t)
\]
and the averaged equation corresponding to (34) is 
\[
\dot{r} = A_0(r_0 - r) + B_0 + C_0.
\]
In this case 
\[
r = r_0 + \frac{B_0 + C_0}{A_0} < 1
\]
is a stable equilibrium.

\textit{Example 2: A Stable Focus} We assume in this case $\Theta = A_0$. See Fig. 5 for the phase portrait of equation (34).
In this case, the fixed point \((\eta_0, 0)\) is a stable focus with an oscillating period

\[ T = \frac{2\pi}{\sqrt{3}} A_0^{-1}. \]

and at the completion of every cycling period, the distance of a solution to the equilibrium state is reduced by a multiplier of

\[ e^{-\frac{2\pi}{\sqrt{3}}} \].

The overall economic climate swings more violently with a larger \(A_0\), and to make \(A_0\) large, \(\beta_g\) have to be large in average, implying an economic environment in which entrepreneurs are highly sensitive to the overall changes in economic climate.

**Business Cycles:** In relating the solutions of equation (34) to that of (31), we split functions \(A(t), B(t)\) and \(C(t)\) into two parts. The first part is \(A_0, B_0\) and \(C_0\), representing respectively the mean values; and the rest are \(A(t) - A_0, B(t) - B_0,\) and \(C(t) - C_0,\) representing respectively oscillatory behavior of the corresponding functions around their means. If equation (18) is with a stable equilibrium solution \(q_L^m\), then close to \(q_L^m,\) the appropriate value of \(r_0\) is \(\frac{R(q_L^m)}{S(q_L^m)},\) and the corresponding mean values of \(A(t), B(t)\) and \(C(t)\) can be approximated by using quantities that are explicitly written in terms of \(q_L^m.\) In this case the oscillatory parts do decline exponentially in magnitude as time goes, and the solution of equation (31) is well approximated by the solutions of equation (34).

Unfortunately, equation (18) is unlikely to admit a stable equilibrium solution. In a non-equilibrium case, the magnitude of the oscillations of \(A(t), B(t)\) and \(C(t)\) around their respective mean \(A_0, B_0\) and \(C_0\) do not necessarily decline in magnitude as time goes. Assume that the oscillations are not unreasonably large in magnitude in this case. Then corresponding to the equilibrium solution \((\eta_0, 0)\) of equation (34), there exists a solution of equation (31), to which all nearby solutions are attracted asymptotically. This solutions we denote as \((\eta^m(t), \dot{\eta}^m(t))\). The values of \(\eta^m(t)\) move up and down around \(\eta_0,\) and the magnitude of \(\eta^m(t) - \eta_0,\) in general, does not decline.
as time goes. $\eta^m(t)$ represents an irregular fluctuations of business climate in a non-equilibrium economy. We observe that, in general, the small fluctuations of $\eta^m(t)$ are not periodic in time. If we are in the case of $\Theta - 4A_0 > 0$, then $\eta^m(t) - \eta_0$ would be the main component that describes the changes in the overall economic climate.

If $\Theta - 4A_0 < 0$, then the irregular fluctuation of $\eta^m(t)$ is secondary to the systemic oscillations of solutions of equation (34) around $(\eta_0, 0)$. This oscillation bears an intrinsic period, which is determined by $\Theta$ and $A_0$ (see (36)). As time goes, however, the magnitude of this oscillation declines exponentially. Therefore in an economic environment in which $\Theta - 4A_0 < 0$, the ups and the downs of the overall economic climate, and hence the aggregate production of the system, first oscillates like a solution of equation (34) around a stable focus. This oscillatory behave, which is with an intrinsic period, would give way to the much irregular fluctuations of $\eta^m(t)$. 