Consumption Oriented Free Capitalism

Qiudong Wang

An economic system is a framework, under which people are organized to produce consumption-goods and to consume the produced. Concerning economic activity of a human society, the two fundamental issues are the issue of production and the issue of distribution. A free capitalist system uses money (the capital part) as a primary media and employs the principle of property right and voluntary exchange (the free part) to address both the issues of production and distribution through the invisible hand of supply and demand. It worked well initially, but as productivity of the system are substantially improved, the principle of free competition became insufficient. Absolute unevenness of resulted formation of distribution imposed serious constraint on the collective ability of consumption, which in turn not only hindered further improvement of overall productivity, but also threatened to destabilize the system. To alleviate such threat it is necessary to redistribute so that the formation of distribution does not hinder the overall ability of the population to consume the produced. Since re-distribution is by definition against laissez faire, we must also insist on making it secondary in the sense that we do no more of re-distribution than the minimum that is necessary. Ours is therefore a consumption oriented free capitalist system where redistribution is the consumption oriented part.

For a consumption oriented free capitalist system, the importance of money can never be over-stressed. Money, however, is a technical tool. To study the dynamics of an economic system at macro-level, not only we need to chase the flow of money but also we need to carefully recognize different nature of things money represents in different places of the economic system. Taking the term investment as an example. If an individual uses a sum of money currently in his possession to manipulate resources to produce goods or services he is an entrepreneur investor. If he uses this money to buy government or corporate bond he is a speculative investor. An entrepreneur investor wields the power of his capital to produce and his activity is mainly in the domain of production. A speculative investor negotiates a deal with an entrepreneur investor to exchange the money he is holding, which represents the right to consume a share of consumption-goods at present time, for a promise of a hopefully larger share of consumption-goods (not necessarily larger in terms of absolute quantity of goods and services) in future, and his activity is in the domain of distribution. Not to make a clear distinction between the two would cause confusion in practice and in theory.

In this paper we construct a new macro-level model to investigate the work of forces that are fundamental in a consumption oriented free capitalist economy. As a first step, our objective is not to propose exogenous actions to fight against the tide created by these underlie forces, but to identify them and to understand the ups and downs they work to create. Without a clear understanding of the work of these forces, we would be forever trapped in a perpetuate fight to delay the coming of things that are inevitable, only been punished later by a fallout resulted from the accumulative effect of our delaying strategy.
1. Five Fundamental Elements

To model a consumption oriented free capitalist system, we first introduce five fundamental elements and they are (1) a component of production, (2) a human population (acting as investor/laborer/consumer), (3) consumption-goods, (4) a currency (money), and (5) a monetary authority. Investor/laborer/consumer are from the same human population. Laborers are employed by the component of production to produce consumption-goods, and consumption-goods are consumed by the population. Processes of production and consumption are facilitated by money, and the responsibility of monetary authority is to guarantee the integrity of money that are circulating in the system.

Component of production consists of a collection of economic unit, each of which is invested into by a group of people who are its investors. The product of an economic unit can be put roughly into two categories: consumption-goods to be consumed by the population and production-goods to be used as raw materials for other economic units. Both consumption-goods and production-goods can be either physically tangible objects such as an apple or services such as a business advise. A computer is a production-goods if it is bought by an economic unit for the use of its employees at work, but a consumption-goods if it is bought by a consumer. An economic unit employs laborers and acquires production-goods from other economic units to produce its own product. If one views the component of production as a black box, then production-goods are hidden inside and only consumption-goods come out of it. As a general rule, production-goods an economic unit uses as raw material are different from the products it produces. All consumption-goods, that is the total output of the component of production, are then acquired and consumed by the population.

An action of acquiring and to be acquired is a trade, and to trade we come to the market place of Adam Smith, where the ruling principle is that all trades are voluntary. The design of voluntary trading principle is based on the understanding that everyone involved, economic units and investor/laborer/consumer alike, would act on self-interest. This is to say that all will try to acquire as much as possible from others by giving up as little as possible of one’s own possession. Because bartering is impossibly complicated, money is introduced to simplify and to facilitate the process of trading.

With money, trading practices of an economic unit are decomposed into sell and buy. It sells its products to acquire certain amount of money and uses the money acquired to buy labor and production-goods from others. The money an economic unit acquired from selling its own products is its revenue and the money it pays to laborers it employed (wage), to other unit for production-goods (material cost), and to individual investors (finance cost) are its cost. Let \( R_c \) be the revenue of an economic unit generated by selling consumption-goods it produced and \( R_p \) be the revenue generated by selling production-goods it produced. Then \( R = R_c + R_p \) is its revenue. Let \( S_w \) be the wage paid to laborers it employed, \( S_m \) be its material cost and \( S_f \) be its finance cost. Then \( P = R - S_w - S_m - S_f \) is its profit. We assume that all economic units strive to maximize their own profit \( P \). Using \( \sum \) to represent the action of adding a quantity over all economic units, we then define \( \sum R_c \) as aggregate
revenue of the component of production and $\sum (S_w + S_f)$ aggregate personal income of the population.

Fig. 1 Production, Income and Consumption

The way money are circulated in between the module of consumption-goods, the component of production and the module of population are schematically illustrated in Fig. 1. We denote total output of component of production as $\sum Q_c$. Component of production delivers $\sum Q_c$ as consumption-goods. It also delivers aggregate personal income $\sum (S_w + S_f)$ to the population. Population then buy consumption-goods using part of aggregate personal income, and aggregate revenue $\sum R_c$ is circulated back into component of production through sales of consumption-goods.

Fig. 2 Personal Investment
We add to Fig. 1 the effect of time, through which the concept of investment and the quantity $S_f$, originated from personal investment, are unambiguously defined. To include time we make infinitely many copies of Fig. 1 and put them linearly in one raw from left to right as a bi-infinite sequence. The one in the middle represent the present, the ones on the left the past, and the ones on the right the future. Our focus here is aggregate personal income $\sum (S_w + S_f)$ at a given time, which is divided into two parts. The first part is the cost of consumption, which generates aggregate revenue $\sum R_c$. The second part is the difference between aggregate personal income and aggregate revenue, that is $\sum (S_w + S_f) - \sum R_c$, aggregate personal saving at the time. The economic system recycles aggregate personal saving back into the component of production, turning personal saving into personal investment. This investment will have a payback, which is $S_f$ part (finance cost) of the aggregate personal income. Current finance cost $S_f$ is a payback to past personal investment. See Fig. 2. For notation we will attach an index $i$ to various quantities introduced above to indicate the time. This is to say that $\sum R_c(i)$ represents aggregate revenue at time $i$ and $\sum R_c(i + 1)$ that at time $i + 1$, and so on.

Let $M(i)$ be the aggregated amount of money distributed to all economic units in component of production at the beginning of time period $i$. $M(i)$ is also all money that is circulated in economic system in time period $i$. Part of $M(i)$ is transferred to become aggregate personal income $\sum (S_w + S_f)$ of period $i$ and part is retained inside of the component of production. Let us denote aggregate personal income as $M_r(i) = \sum (S_w + S_f)(i)$ and the money remained inside of the component of production as $M_c(i)$. The retain-income ratio is the ratio of $M_r(i)$ over $M_c(i)$, which we denote as $r_{rI}(i)$. This is to say that

$$r_{rI}(i) = \frac{M_r(i)}{M_c(i)},$$

and we have

$$M(i) = (1 + r_{rI}(i))M_c(i) = (1 + r_{rI}(i))\sum (S_w + S_f)(i).$$

Recall that aggregate personal income is in turn divided into cost of consumption and personal saving. Denote aggregate cost of consumption as $M_{sc}(i) = \sum R_c(i)$ and total personal saving as $M_{ts}(i)$. The saving-consumption ratio is the ratio of $M_{ts}(i)$ over $M_{sc}(i)$, which we denote as $r_{sc}(i)$. This is to say that

$$r_{sc}(i) = \frac{M_{ts}(i)}{M_{sc}(i)},$$

and we have

$$M(i) = (1 + r_{rI}(i))(1 + r_{sc}(i))\sum R_c(i).$$

Let $L(i)$ be the list of all types of consumption-goods produced at time $i$ and, for $g \in L(i)$, let $O_g(i)$ be the total of product $g$ produced and consumed by the population. The aggregate output of the component of production is

$$O(i) = \cup_{g \in L(i)} O_g(i).$$
We now need a single numerical measurement on the size of \( O(i) \) so we could quantitatively compare \( O(i) \) to \( O(i + 1) \).

If there is only one type of product, say grain, in \( L(i) \), then this quantity would be easily the total pounds of grain produced. Since at any given time, the economic system produces all kind of consumption-goods, and the list of products changes constantly as time varies, this numerical measurement becomes tricky to define. We caution that aggregate revenue \( \sum R_c(i) \) can not be used here for the intended numerical measurement because the value of currency varies over the time. This desired quantity should be an objective, abstract measurement on the aggregate physical output at time \( i \). We postpone the details of this technical definition momentarily and for the purpose of now let us assume that for every \( g \in L(i) \), an abstract numerical measure \( Q_g(i) \) is properly assigned to \( O_g(i) \) and we can use \( Q_g(i) + Q_g'(i) \) to represent the size of \( O_g(i) \cup O_g'(i) \) for all \( g, g' \in L(i) \). The size of production at time \( i \) is then \( Q(i) = \sum_{g \in L(i)} Q_g(i) \). Let

\[
P(i) = \frac{\sum R_c(i)}{Q(i)}.
\]

We call \( P(i) \) the abstract unit price of consumption-goods at time \( i \). To use the same oversimplified example of one item list of product again, \( P(i) \) would be the money you pay to buy one pound of grain at time \( i \) and \( P(i + 1) \) would be that at time \( i + 1 \).

Let us define consumer price index (CPI) as the ratio of \( P(i + 1) \) over \( P(i) \). This is to say that

\[
CPI(i + 1) = \frac{P(i + 1)}{P(i)}.
\]

From (2) and (4), we have

\[
CPI(i + 1) = \frac{M(i + 1)}{M(i)} \cdot \frac{Q(i)}{Q(i + 1)} \cdot \frac{(1 + r_{rI}(i))}{(1 + r_{rI}(i + 1))} \cdot \frac{(1 + r_{sc}(i))}{(1 + r_{sc}(i + 1))}.
\]

We say that there is an inflation at time \( i + 1 \) if CPI(i+1) > 1 and there is a deflation at time \( i + 1 \) if CPI(i+1) < 1. To maintain the integrity of money over the stretch of time, it is imperative to make CPI as close to one as possible. Letting CPI = 1 in equation (7), we obtain

\[
M(i + 1) = M(i) \cdot \frac{Q(i + 1)}{Q(i)} \cdot \frac{(1 + r_{rI}(i))}{(1 + r_{rI}(i + 1))} \cdot \frac{(1 + r_{sc}(i + 1))}{(1 + r_{sc}(i))}.
\]

Therefore in order to maintain the integrity of the currency in use, it is necessary to inject money in or to take money out of the economic system from time to time. This responsibility is assigned to a monetary authority, the fifth fundamental element. In this model, the responsibility of monetary authority is to maintain the integrity of money by making CPI as close to one as possible at all time.

We now precisely define \( Q(i) \) in (7) and (8). Observe that, instead of defining \( Q(i) \), it suffices for us to define the ratio

\[
X(i : i + 1) = \frac{Q(i + 1)}{Q(i)}.
\]
\( X(i, i + 1) \) is defined as follows. Let \( L(i) \) be the list of all types of consumption-goods produced at time \( i \) and \( L(i + 1) \) be that at time \( i + 1 \). It is not unreasonable to assume that \( L(i) \) and \( L(i + 1) \) has a substantial overlap. Let \( B(i : i + 1) \) be a subset of \( L(i) \cap L(i + 1) \). We put \( g \in L(i) \cap L(i + 1) \) in \( B(i : i + 1) \) if and only if the total number of units of \( g \) physically produced and consumed by the population at time \( i \) is (roughly) the same as at time \( i + 1 \). For \( g \in L(i) \), let \( Q_g(i) \) be the quantity we intend to assign to product \( g \) at time \( i \). We must have

\[
Q_g(i) = Q_g(i + 1) \quad \forall g \in B(i : i + 1).
\]

We also have

\[
Q(i) = \sum_{g \in L(i)} Q_g(i) - \sum_{g \in B(i : i + 1)} Q_g(i) + \sum_{g \in B(i : i + 1)} Q_g(i)
\]

\[
= \left( \frac{\sum_{g \in L(i)} Q_g(i) - \sum_{g \in B(i : i + 1)} Q_g(i)}{\sum_{g \in B(i : i + 1)} Q_g(i)} + 1 \right) \sum_{g \in B(i : i + 1)} Q_g(i).
\]

Let \( R_{L \setminus B}(i) \) be revenue generated by all products that is in \( L(i) \setminus B(i : i + 1) \) at time \( i \) and \( R_B(i) \) be revenue generated by all products that is in \( B(i : i + 1) \) at time \( i \). We let

\[
\frac{\sum_{g \in L(i)} Q_g(i) - \sum_{g \in B(i : i + 1)} Q_g(i)}{\sum_{g \in B(i : i + 1)} Q_g(i)} = \frac{R_{L \setminus B}(i)}{R_B(i)}.
\]

Note that this replacement is not affected by changes of value of currency over the time. It is based on an assumption that the objective values of consumption-goods \textit{at a given time} are reflected in voluntary exchanges of money in trade. It then follows that

\[
Q(i) = \left( \frac{R_{L \setminus B}(i)}{R_B(i)} + 1 \right) \sum_{g \in B(i : i + 1)} Q_g(i).
\]

From (9), (10) and (11) it follows that

\[
X(i : i + 1) = \frac{R_{L \setminus B}(i + 1)}{R_B(i + 1)} + 1.
\]

We say that the economy grows at time \( i + 1 \) if \( X(i : i + 1) > 1 \) and it is in recession if \( X(i : i + 1) < 1 \).

We finish this section with a brief summary on the circulation of money. At the beginning of time period \( i \), \( M(i) \) is distributed as capital investments among all economic units in the component of production. This capital is split into two parts. One is \( M_f(i) \), retained inside of the component of production. The other is \( M_f(i) \), delivered to the population as personal income. The aggregate personal income \( M_f(i) \) is in turn split into aggregate revenue \( M_f(i) = \sum R_c \) and aggregate saving \( M_f(i) \), the formal is recirculated back into the component of production through sales of consumption-goods and the latter in the form of new investment. At the beginning of time period \( i + 1 \), the monetary authority also adjusts the money supply by an amount,
which we denote as $IM(i)$, in an effort of maintaining the value of the circulating currency. See Fig. 3. We have

\begin{equation}
M(i) = M_r(i) + M_I(i) = M_r(i) + M_{Is}(i) + M_{Ic}(i)
\end{equation}

and

\begin{equation}
M(i + 1) = M(i) + IM(i).
\end{equation}

Fig. 3 Circulation of money

2. EXPLANATORY REMARKS

A. Financial Institutions  Part of $M(i)$ is retained inside of the component of production as $M_r$ and part is passed to the population as aggregate personal income $M_I(i)$. Part of $M_I(i)$ is recycled back into component of production through sales of consumptions-goods as $M_{Ic}(i)$ and $M_{Is}(i) = M_I(i) - M_{Ic}(i) > 0$ is the total personal saving of this time period. Businesses making positive profit would also like to save part of their profit for future use, motivated by various reasons including the need of dealing with unforeseeable emergencies in the future.

Businesses at the initial stage of investment usually lose money so their revenue is often less than what is needed to maintain the current level of production in the next production cycle. These businesses, on the other hand, have to increase their level of production to become profitable. Facing a deficit in between the money possessed and the money needed to move forward, they need to convert savers to investors. For the intended conversion, however, the issue of liquidity is a major obstacle. After a money is invested, there will have to be at least a period of time in which the invested money is no longer available to be used for any other purpose. Consequently the idea of investment is intrinsically in conflict with the idea of saving for raining days. To solve this problem, financial institutions are created as an interface between businesses in need and money available.

Financial institutions offer a place for population and businesses to deposit their savings and promise complete liquidity for money that are deposited. Bank operation is based on the assumption that, though all are motivated to save for an unforeseeable future, there is only a very small fraction who would actually face an emergency at
any given time. Financial institutions would only need to use new deposit, or a very small portion out of all that are deposited, to maintain their promise of complete liquidity. Almost all money deposited can be invested back into the component of production. To further motivate people to deposit their money, banks also offer an extra return in the form of interest to share the profit they made by investing the money entrusted in them. In between the choices of hoarding cash or putting them to a place that is not only safer but also with a future growth, few would refuse to go with the latter. The savings are therefore gathered and are ready to be recirculated back into the component of production.

For businesses in need of new investment, banks are now a place to go. Banks are willing lenders, for they are businesses making money through lending. We note that, in addition to banks, established business entities can also attract investment through stock market and bond market.

B. Negative Aggregate Profit Among the four components of $M(i + 1)$, which we denoted as $M_r(i), M_{IE}(i), M_{IS}$ and $IM(i)$ respectively, $M_r(i)$ is the aggregated amount of money retained by all economic units in component of production. $M_{IE}$ is redistributed through sales of consumption-goods. $M_{IS}$ is recycled back and distributed into component of production by using of various financial instruments through banks, stock market and bond market, again under the guidance of the principle of voluntary exchange: instead of exchanging money for consumption-goods at present time, for which $M_{IE}$ is used, individuals negotiate with economic units to exchange their share of $M_{IS}$ for a promise of future delivery of money in mutually agreed manners and schedules. $IM(i)$, the money injected by monetary authority, is similarly distributed to economic units through the banking system.

Since all savings are constantly recycled back into component of production and are used as part of current aggregate business cost, there is no money reserved accumulatively for the saving part of personal income over any given stretch of time. Consequently, in any given time period, the only money in our model that is available to the population to buy consumption goods is the current aggregate personal income $M_I(i)$. This implies

$$\sum P(i) = \sum (R_c(i) + R_p(i)) - \sum (S_w + S_f + S_m)(i)$$

$$= \sum R_c(i) - \sum (S_w + S_f)(i)$$

$$= -M_{IS}(i) < 0,$$

where the second equality is obtained by using $\sum R_p = \sum S_m$. This is to say that, for a consumption oriented economy, *Aggregate profit of the component of production must be negative at any given time.*

C. Investment and Capital Gain People participate in the process of production in three ways. Most are employed by businesses to work to earn their wages and they are the laborers. They put the saving part of their income back into the component of production, negotiating a contract in the hope of getting a larger payback in the future and these are *speculative investors.* There are also people who uses existing capitals to organize the production of certain production-goods or consumption-goods in the hope of making a profit from the sales of whatever they would be able to produce and
they are the entrepreneur investors. The main motivation for entrepreneur investors to set economic unit to produce is to seek a positive profit but (15) imposes a very strict constraint. As far as profit is concerned, businesses in component of production are playing a game of negative sum. The aggregate business loss must be larger than aggregate business gain at any given time.

For an entrepreneur who is running a business of positive profit, (15) is not a bothering, for his purpose is to make a profit and as far as a profit is coming in, he is in good shape. The fact that his profit must come from a loss of other businesses is a no matter for him. Neither is (15) a real bothering for an entrepreneur who is currently running a business of negative profit because he understands that all business must start with a loss, and the gain he strive to acquire is not at present time, but in future. What will work at his advantage, at least from his own perspective, is his foresightedness and his understanding of the ability of the economic unit, which he is current setting up, in future production. To trade a current loss for a greater return in future is perhaps the greatest and the most successful practical trick ever learned by human intelligence.

As to what economic unit to set up, an entrepreneur investor usually has two choices. The first is to do what is tried and worked. This is to imitate the success of a currently established business that is receiving a relatively large return. Such investment we name as copy-cat investment. The second is to venture into something new and such investment we name as entrepreneur-investment. Among the two, entrepreneur-investment is obviously the driving force for economic growth. It not only increases the volume of total production but also extends the horizon for human consumption therefore raises the upper limit on the potentiality of economic growth.

What (15) represents is a reality that, in order for this modeled economic system to work well, there must be entrepreneur-investors constantly working all the time. They put a substantial part of the money they are going to make in future into the present system, passing that money to laborers/consumers, so that other established economic units can take in a positive profit. For a currently established business, an economic environment in which entrepreneur investors are in abundance would be a friendly environment that is easier to make a positive profit. If nobody is willing to start new business, currently established businesses would inevitably experience hardship in turning in a positive profit, some would have to lose money because of (15).

Since copy-cat investments are derived from entrepreneur-investments, it is highly desirable to maintain a high level of entrepreneur-investment at all time. To maintain a high level of entrepreneur spirit there must be new opportunities feasible for entrepreneur investors to venture into and to make a profit in the future. An investment such as to first hire a group of people to dig holes to hide money, then to hire another group to dig the money hidden affects only the formation of distribution therefore is not an entrepreneur investment, though it might be a thing that is necessary for government to do in order to hold the system together in an economy of high unemployment rate. New opportunities for entrepreneurs can only be created by the advancement of science and technology.
Among all forces acting upon a consumption oriented free capitalist economic system, there are two that are most fundamental. The first is the system restraint imposed by (15). This system restraint creates constantly an intrinsic downward tension. The second is the force of modern science and technology. The advancement of modern science and technology is the only force that is powerful enough to check the downward tension created by (15) and the opportunity and growth potential it created for industry entrepreneurs have been sufficient to keep the system moving forward thus far.

D. Income Distribution  Concerning the two fundamental issues an economic system must resolve: the issue of production and the issue of distribution of the produced for consumption, answer to the former is represented by the total output of consumption-goods \( O(i) = \bigcup_{g \in L(i)} O_g(i) \) of the component of production and answer to the latter is represented by the way personal income are delivered from the component of production to the population. So far we have appeared to assume that the method to resolve these two issues are through the invisible hand of supply and demand, which works through implementation of principle of voluntary exchange.

This is in fact not the case because in our model we have also tacitly assumed, concerning the distribution of income, something that does not necessarily follow from the principle of voluntary exchange. We have assumed that, distribution of personal income, pushed down to the population through the production of \( O(i) \), is such that the population is able to consume \( O(i) \). Therefore in implementation of this proposed economic model, there are in fact two major principles, one is the rule of voluntary exchange, representing the free part of the capitalism; and the other is a new principle on distribution of income, representing the consumption oriented part of the economic system.

Since there is, unfortunately, no intrinsic human trait that compels individuals to obey neither, both principles have to be imposed on the economic system through an exogenous force. This exogenous force is government, which is the product of a political system. The government, with tremendous imposing power, is a necessary evil to a free economic system. Without it economic system can not function properly but the imposing power that must be assigned to a group of individuals is easily subjected to abuse. This problem, however, has be resolved through the design of modern democratic political system to the degree that we might assume, with a high degree of confidence, that government today would do its best to play the role of a guardian of the economic system with good intention.

But to a human society, the roles of government are beyond a well-intended guardian of its economic system. Even if this is the only role, there is still an intrinsic difficulty for the government because the two guiding principles of a consumption oriented free capitalist economic system is by definition conflicting to each other. A society must strike a proper balance in implementing the principle of laissez faire and in addressing the issue of redistribution. It is not feasible to abandon one to implement the other. Exclusive implementation of the free part would lead to uneven formation of distribution that will destabilize the system. This had been illustrated more than convincingly by the history of the first half of the 20th century, which was marked by one great depression and two world wars. On the other hand, to abandon the free part
completely would lead to a general stagnation, as empirically illustrated by the practices of communists in the former Soviet Union. The current system implemented, in the United Stated and in other developed countries, is a mix and a compromise of the two, resulted from trial and error. Reflected in modern macro-economic theories, the Keynesians has placed more emphasis on the necessity of re-distribution and the Monetarists have placed more emphasis on laissez faire. For our model we would assume that the personal income is so distributed that it does not hinder the overall ability of the population to consume the produced. Since re-distribution is by definition against laissez faire, we must also insist on making it secondary in the sense that we do no more of re-distribution than the minimum amount that is deemed necessary.

E. Economic Cycles  Capitals do not carry an intrinsic net productivity. What makes capital appear to carry a net productivity is the feasibility for an entrepreneur investor to create new capital formation that is more productive. New advancement of science and technology would make it feasible for an entrepreneur to deploy the same amount of resources over a stretch of time to produce more than in the past. Feasibility of improvement is dictated by the advancement of science and technology.

When an entrepreneur investors transforms a new possibility into reality, part of the money he acquires from the sales of his additional products he delivers to speculative investors to fulfill his promise to the capitals entrusted in him. If an entrepreneur fails to improve the productivity of the specific thing he tries, he is still obligated to fulfill his promise to his speculative investors but the net productivity realized by the capital entrusted in him would either be zero or negative. In a bird’s view, most success and failure would cancel each other, and the surplus products entrepreneurs shared with speculative investors is the nominal interest of the capitals employed.

Science and technology, unfortunately, do not advance in a continuous and constant pace. When potentiality for improvement is exhausted, this is to say that when the total amount of consumption-goods we are able to produce today is more or less intrinsically the same as that of yesterday, entrepreneurs would fail inevitably in gaining a net improvement on productivity. However, they have to fulfill their promise to pay speculative investors according to a previously negotiated schedule and the only way to do so would be to give up part of the capital they reserved for the purpose of keeping up the current level of production. Total production of consumption-goods would go down and unemployment would go up. What speculative investors received in this case is essentially a larger share in a reduced pile of consumption-goods in the game of distribution against wage earners. This process, however, is self-rectifying since recess of production creates potential for future growth. Such is the coming and going of the regular business cycles.

F. Assets Bubble and Depression  Once in a while there comes a wave of new innovation of science and technology that substantially extends the upper limit of human productivity. Total output of the economic system is then allowed to grow in a steady pace without the need of shrinking back for a long period of time, and such are the good times everyone would long to last forever.

But sooner or later, the potential for expansion would run out of steam and there would be again the need to create breath room for future growth through recession. But the society as a whole is not willing to acknowledge and accept such unwelcoming
reality. One of the main characters of human psychology is the sense of entitlement. When a man provided unexpected financial assistance to another of needy, his kindness is often greatly appreciated but if he delivers his good-will regularly, then the appreciation toward his kindness would diminish and good-will becomes an entitlement. He would then have to face a great deal of resentment if he stops the giving out. When an economy has expanded for, say a decade at a rather impressive pace, the abnormal rate of growth would become an entitlement to the population of the society. If total output of consumption-goods could grow no more, the society will turn to fictitious growth through the play of a grand Ponzi scheme.

Our reader might have noticed that, up to this point, we have not yet discussed the value of assets, that is the cash value of existing capital formation of the component of production. In particular, capital formation is excluded from our definition of aggregate personal income. To evaluate the worth of a given economic unit, it is a must to consider both its current level of production and its future potential. But to count the change of potential as a part of current income would be a subtle double counting, since the improved future productivity is only a potential, the value of which is included entirely in the future output of consumption-goods. To count a future potential as part of current income is to provide fertile soil for a Ponzi scheme. All existing economic text book, unfortunately, has insisted on this practice of double counting.

With the change of assets value be counted in as part of current income, we appear to have an alternative way of gaining new income without increasing the total output of consumption-goods. To increase income it suffices for us to exaggerate the future potential of production by artificially raise cash value of the existing economic assets, and to do so we come to Wall Street, where the principle of voluntary exchange is also implemented. The implications of the principle of voluntary exchange for the Main Street and the Wall Street, however, are radically different. Consumption-goods and business assets are economic objects of entirely different nature. For the Main Street, the impact of a business transaction is local in the sense that a trade defines only the currency value of the consumption-goods that are physically involved in a particular business transaction. This is completely not so for the Wall Street, where the impact of a current trade is global in the sense that it over-writes all deals made in the past. It does not take much to move up the current price of an asset, but counting the total cash value created by the global impact of the last transaction it is clear that there could easily be a magnifying money multiplier on assets. This magnifying multiplier, in normal circumstance, does not have an impact because the balance of long and short trade of speculative investors. But for a population in which the prevailing sentiment is for long, betting on short would be financially suicidal unless a speculator holds unlimited amount of resource. To further confuse the matter at hand, complete liquidity of asset at stock market gives a nominal impression that fictitious assets value created by this money multiplier is real income instead of a mere promise on future distribution. Assets bubble would grow. In the same way we can also blow a house bubble by relaxing the lending standard for mortgage loan to improve the liquidity of houses. In a Ponzi scheme one always uses a promise of the future to pay for obligation of the present time. Without the back of real growth
of productivity, the scheme would have to collapse eventually, causing investors to
default, which would lead to a complete melt down of the economic system. Instead of
a moderate recession, we would have a \textit{collapse}, the damage of which can not possibly
be repaired by self-rectifying ability of the system.

Ironically, there is not much anybody or any institution can do to prevent this from
happening. First, facing a world that hopes, and to a large degree turned a hope to
a belief that the good time could last forever, to forcefully convey the bad news that
good time has come to an end is politically suicidal. The instant human reaction
in facing a bad news is always to shoot the messenger. Second, simple cause and
effect psychology would directly portray whoever burst the bubbles as the villain who
ended a happy time (or worse, the villain who caused all the miseries associated to
the busted Ponzi scheme). Serving as a warning example, let us be aware that, even
up to this date, the Federal Reserve of the time of great depression of 1930’s are still
blamed almost universally (justly or unjustly) for letting the Ponzi scheme of that
time to more or less follow its natural course.

Facing a collapsing recession, government must step in. One can either use admin-
istrative power or economic power of the government to avoid anarchy. In the great
depression of 1930’s the former is the Fascism and the latter is the New Deal. In 2008
we did much better thanks to many protective schemes implemented after the great
depression. Still, it took a temporary government take over of the entire financial
system to avoid a general melt-down.

3. A Dynamics Model on Production, Cost and Profit

In this section we assume that monetary authority works in perfection. This is to
say that there is neither inflation nor deflation in the system we model. We introduce
a set of second order differential equations to model the quantity, cost and profit
of the production of consumption-goods under the framework of the previous two
sections. Activities of entrepreneurs, process of creative destruction and constraints
of aggregate negative profit represented by (15) are all incorporated. We use this
model to identify sources for economic growth and to illustrate a cyclic tendency of
ups and downs on the aggregate production of the economic system.

A. Cost and Revenue Schedule Let $L(t)$ be the list of consumption-goods at time
t, and $g = g(t_0) \in L(t)$ be an item in $L(t)$ initiated at $t_0 < t$. Let $q_g$ be a real variable
representing the quantity of $g$. We start with two functions of $q_g$, $0 \leq q_g < \infty$. The
first is $S_g(q_g)$, the \textit{cost schedule} of $g$ and the second is $R_g(q_g)$, the \textit{revenue schedule}
of $g$. For $q_g \in [0, \infty)$, $S_g(q_g)$ is the would-be cost to produce $q_g$ units of $g$ and $R_g(q_g)$
is the would-be revenue. Standard assumptions for $S_g(q_g)$, $R_g(q_g)$, $q_g \in [0, \infty)$ are

$$S_g'(q_g) > 0, \quad S_g''(q_g) > 0; \quad R_g'(q_g) > 0, \quad R_g''(q_g) < 0$$

where each prime represents one derivative with respect to $q_g$. For an item $g$ to have
a chance to establish itself, there must be an interval $[q_g(a), q_g(b)]$ of $q_g$ on which the
profit function

$$P_g(q_g) := R_g(q_g) - S_g(q_g) > 0.$$
See Fig 4(a). The otherwise situation is depicted in Fig. 4(b), for which no positive profit is possible. Product $g$ of Fig. 4(b) usually represents an entrepreneur venture that is destined to fail.

We construct a time-continues model in this section. Let $t$ be the time, and $q_g(t)$ be the actual number of units of $g$ the system produces at time $t$. Then $S_g(q_g(t))$ is the total cost incurred in producing $q_g(t)$ of $g$, that is, the sum of the cost of natural resources and the aggregate total finance cost $S_f$ and labor cost $S_w$ incurred for the production of $q_g(t)$ of $g$ over all economic units participated. Let $P_g(q_g(t)) = R_g(q_g(t)) - S_g(q_g(t))$ be the profit made by all economic units involved in the production of $q_g(t)$ of $g$. To model the behave of the function $q_g(t)$, which is the quantity of $g$ produced and consumed at time $t$, we first assume that \emph{both the cost and the revenue schedule of $g$ are functions of $q_g$ independent of time} so we can use fixed schedule to track the cost and the induced revenue for the production of $g$ at any given time $t$. With this assumption we tacitly assume that there is neither inflation nor deflation in the modeled economic system since the production of the same quantity of $g$ at different times would incur the same cost and induce the same revenue.

![Cost and Revenue Schedule](image)

\textbf{Fig. 4. Cost and Revenue Schedule}

Though the use of cost and revenue schedules is common in economics theory, they are used mostly in a static environment for the analysis of production of a single product. To use $S_g(q_g)$ and $R_g(q_g)$ at macro-level in a dynamic economic environment might be somewhat problematic. Therefore it is necessary for us to look closer to see what we are implicitly assuming. First, to assume that the cost schedule $S_g(q_g)$ for $g$ is independent of all other items of $L(t)$ is to assume that there exists no other items in $L(t)$ that are directly engaged in competing resources against the production of $g$. Second, to assume that $S_g(q_g)$ is independent of time is to assume that the productivity of $g$, including that of any of the intermediate production-goods used for the production of $g$, does not improve as time goes. To reconcile the first point we would have to consider the competition of resources by assuming a cost schedule...
in the form of

\[ S_g = S_g(q_g; q_C(g)) \]

where \( C(g) \subset L(t) \) is the collection of all items in \( L(t) \) that directly compete either nature resource or production-goods against \( g \) in their own production, and \( q_C(g) = \bigcup_{q \in C(g)} q_g \) is used to represent the list of all items of \( C(g) \). To reconcile the second point we would have to regard \( g \) with improved productivity as a newly added product directly competing against the old \( g \). For the revenue schedule, we also need to count the influence of of all product in \( L(t) \) that is either directly complementary to or competing against \( g \) in consumption. Let \( \hat{C}(g) \) be the collection of all product in \( L(t) \), the consumption of which are either directly complementary to or directly competitive against \( g \). Instead of \( R_g = R_g(q_g) \), we would have to assume

\[ R_g = R_g(q_g; q_{\hat{C}(g)}). \]

For the cost schedule of (17) and the revenue schedule (18) the assumptions corresponding to (16) would be

\[ \partial_{q_g} S_g > 0, \quad \partial^2_{q_g q_g} S_g > 0; \quad \partial_{q_g} R_g > 0, \quad \partial^2_{q_g q_g} R_g < 0. \]

Let us also caution that, our assumption that \( S_g, R_g \) are time independent is subjected to the provision that, whenever a new item is added to or an old item is removed from \( C(g) \) or \( \hat{C}(g) \), all corresponding cost or revenue schedules would have to be reassigned.

**B. Maximizing Profit** Let us for the moment follow the logic of classical theory of equilibrium. We start with the static case in which \( S_g = S_g(q_g), R_g = R_g(q_g) \). Assuming (16), it is obvious that there is a unique value \( q^m_g \) of \( q_g \), at which the profit function \( P_g(q_g) = R_g(q_g) - S_g(q_g) \) is maximized. The value \( q^m_g \) is such that

\[ P_g'(q^m_g) = 0, \]

and in addition, we have \( P_g'(q) > 0 \) for all \( q < q^m_g \) and \( P_g'(q) < 0 \) for all \( q > q^m_g \). The quantity of production of \( g \) is gravitated naturally towards \( q^m_g \) because businesses seek to maximize their profit in market place. Industrial entrepreneurs, however, do not know explicitly the profit schedule of \( g \) before hand, so they have to move towards equilibrium through trial and error.

Classical economic theory has focused on equilibriums. The main themes are how to identify equilibriums and how to move equilibriums to desirable positions. Our focus here is somewhat different. We are more interested in modeling the ways in which entrepreneurs approach to an equilibrium. One model that would follow quite naturally from the classical economic theory is for the entrepreneurs to constantly monitor \( P'(q_g(t)) \), the marginal profit with respect to quantity of production at time \( t \), and to adjust directly the quantity of production in proportion to marginal profit. They would increase the quantity of production of \( g \) in proportion of \( |P'(q_g(t))| \) if \( P'(q_g(t)) > 0 \), and reduce the quantity of production in proportion of \( |P'(q_g(t))| \) if \( P'(q_g(t)) < 0 \). The corresponding model, according to such proposed practice, would be the first order differential equation

\[ \dot{q}_g(t) = kP'(q_g(t)) \]
where \( k > 0 \) is a constant and the dot on the top represents derivative with respect to time. By definition, \( q = q_m^g \) is an asymptotically stable equilibrium solution of equation (20).

Equation (20) appears to fit our purpose well except the fact that it would not allow overshoot of production. For a solution \( q_g(t) \) of equation (20), we can either have \( q_g(t) < q_m^g \) for all \( t \) or \( q_g(t) > q_m^g \) for all \( t \). If we have a moment at which the quantity of production is under \( q_m^g \), then according to this model we would never over-produce. Similarly, if we have a moment at which the quantity of production of \( g \) is over \( q_m^g \), then according to this model we would never under-produce. In reality, however, production overshots all the time. In particular, economic cycles, a distinguished symptom of free market capitalist economic system, are mainly a phenomenon of simultaneous back and forth swing of productions of all consumption-goods in between over-production and under-production. Models based on (20), which we would classify as first derivative models, are intrinsically at disadvantage in modeling economic cycles.

For this reason we would move to second order equations. We assume that the production of \( g \) carries a momentum, which is \( \dot{q}_g(t) \). Entrepreneurs, of course, are able to monitor the marginal profit \( P'(q_g(t)) \), but they do not directly adjust the quantity of production in a way that is proportional to marginal profit. Instead they adjust the momentum of production in proportion to marginal profit to quantity of production. This is to say that we would model the behave of entrepreneurs by using a second order equation

\[
\ddot{q}_g(t) = kP'(q_g(t)).
\]

Observe that \( q = q_m^g \) is again an equilibrium solution of equation (21) but instead of being an asymptotically stable equilibrium, it is now a stable center with all solutions oscillating around. Forever oscillation, however, is an highly undesirable character and we better allow our entrepreneurs to approach equilibriums. To improve, we would add a dissipative term to (21) to obtain a new equation

\[
\ddot{q}_g(t) + \lambda \dot{q}_g(t) = kP'(q_g(t)).
\]

This is to say that entrepreneurs also act to slow down the momentum of production as a precaution against over-shooting. It would then follow that, assuming \( \lambda, k > 0 \) are constants, all solutions of equation (22) would take \( q = q_m^g \) as an asymptotically stable equilibrium. After weighing the two options in between equations (20) and (22), we are compelled to adopt the latter, again because first derivative models are intrinsically unfit in modeling economic cycles.

To adopt (22) is to adopt a modeling strategy of imitating the deed of classical mechanics. As a general principle we would treat various factors that influence the performances of an economic system as economic forces, the effect of which would be additive. We then use total force to determine acceleration, that is, the second derivative with respect to time, of the economic measures involved, introducing a set of second order differential equations to describe the change of production, cost and profit of all consumption-goods over the time. Again, our adoption of second derivative models are based on the fact that first derivative models are at an intrinsic disadvantage in modeling economic cycles. Besides, among mathematical equations
that have been used to model grand natural phenomenon, equations of second order
tend to be more successful. They usually admit a rich array of complicated phenom-
enon; and more than often they could also be subjected to effective mathematical
analysis.

C. Our Second Derivative Model We now move to a dynamical environment in
which cost and revenue schedules are as in (17) and (18) respectively. This is to say that
\[
S_g = S_g(q_g; q_{C(g)}), \quad R_g = R_g(q_g; q_{\tilde{C}(g)})
\]
where \(C(g) \subset L(t)\) is the collection of consumption-goods, the productions of which
directly compete either natural resources or production-goods against that of \(g\); and
\(\tilde{C}(g)\) is the list of consumption-goods, the consumption of which is either directly
complementary to or direct competing against \(g\). We assume (19) for \(S_g\) and \(R_g\).
This is to say that we have for all \(g \in L(t)\),
\[\partial_{q_g} S_g > 0, \quad \partial^2_{q_g q_g} S_g > 0; \quad \partial_{q_g} R_g > 0, \quad \partial^2_{q_g q_g} R_g < 0.\]
Entrepreneurs remain to monitor marginal profit to quantity of production. If we
adopt a first derivative model by using (20), that is, to adjust quantity of production
in proportion to marginal profit, we would obtain a first order equation for \(q_g\) as
\[
\dot{q}_g(t) = k_{\varphi} \partial_{q_g}[R_g(q_g(t); q_{\tilde{C}(g)}) - S_g(q_g(t); q_{C(g)})].
\]
If we adopt a second derivative model by using (22), that is, to adjust the momen-
tum of production in proportion to marginal profit, we would obtain a second order
equation for \(q_g\) as
\[
\ddot{q}_g(t) + \lambda \dot{q}_g(t) = k_{\varphi} \partial_{q_g}[R_g(q_g(t); q_{\tilde{C}(g)}) - S_g(q_g(t); q_{C(g)})]
\]
where \(\lambda, k_{\varphi} > 0\) are constants.

Let us assume for the moment that \(C(g), \tilde{C}(g)\) are proper subset of \(L(t)\), and
\(C(g) = \tilde{C}(g)\) are such that \(C(\hat{g}) = \tilde{C}(\hat{g}) = C(g)\) for all \(\hat{g} \in C(g)\). Then for both
models, the collection of equation (24) for all \(g \in C(g)\) would be a self-contained set,
modeling the dynamics of production of all items in \(C(g)\). We caution that without
the above assumption on \(C(g)\) and \(\tilde{C}(g)\), we might very well need equation (24) for all
\(g \in L(t)\) to obtain a self-contained set of equations. Let \(C(g) = \tilde{C}(g)\) be as assumed.
then equilibrium solutions for both equations (23) and equations (24) are defined by
\[
\partial_{q_g}[R_g(q_g(t); q_{C(g)}) - S_g(q_g(t); q_{C(g)})] = 0
\]
for all \(g \in C(g)\).

Unlike the static case, however, (19) for all \(g \in C(g)\) would be insufficient to
assure the existence of an equilibrium solution. One might attribute our inability in
acquiring an equilibrium to our lacking of full knowledge of the properties of the cost
and supply schedules. But we incline to believe that this is more likely a reflection of
a reality that free competition does not always end up approaching an equilibrium.
Even if equation (25) admits an equilibrium, the importance of such equilibrium
solution would remain questionable because (1) there is unlikely a compelling reason
for a specific solution of (23) or (24) to approach to a particular equilibrium; (2) a
solution of (23) or (24) might very well leave the first quadrant of the phase space
where all \(q_g\) are positive before getting close to an equilibrium, implying that the
production of at least one product is pushed out by competition; and (3) whenever a new consumption product is added to $C(g)$, cost and revenue schedules ought to be re-assigned non-trivially so to render the old equilibrium meaningless. Therefore concerning the behavior of an economic system in a dynamical environment, that is, with creation of new products and dropout of the existing ones, equilibriums are not very much meaningful both in short term and in the long run. In short term, real solutions are usually a distance away, more likely to leave the positive quadrant of $q_g$ before approaching any equilibrium. In the long run, the process of creative-destruction would constantly change the contents of the system, rendering any equilibrium solution short-lived.

In the rest of this essay we would adopt a second derivative model represented by equation (24). Let

$$P_g(q_g; q_{C(g)}; q_{\tilde{C}(g)}) := R_g(q_g; q_{C(g)}) - S_g(q_g; q_{C(g)}).$$

We add two more considerations into the modeling equations. The first concerns the difference of Fig. 4(a) and Fig. 4(b). We observe that profit maximization, implemented by adjusting momentum of production in proportion to $k_g \partial q_g P_g(q_g; q_{C(g)}; q_{\tilde{C}(g)})$, does not distinguish Fig. 4(b) from Fig. 4(a). When an investor is in the process of deciding if he is going to make a copy-cat investment to add to the production of $g$, he is likely to consider not only the micro-trend represented by marginal profit to production but also the overall profit history of $g$, which we represent by using the quantity

$$\int_{t_0(g)}^t P_g(q_g(s); q_{C(g)}(s); q_{\tilde{C}(g)}(s)) \, ds$$

where $t_0$ is the time $g$ was added into the process of production. Let us also assume that he would give recent data more weight in his consideration therefore the quantity that actually effects his decision would be in the form of

$$\int_{t_0(g)}^t e^{\theta_g(s-t)} P_g(q_g; q_{C(g)}; q_{\tilde{C}(g)}) \, ds$$

where $\theta_g > 0$ is a constant. Add this term as a dragging force to the right hand of equation (24) we obtain

$$\ddot{q}_g(t) + \lambda \dot{q}_g(t) = k_g \partial q_g P_g(q_g; q_{C(g)}; q_{\tilde{C}(g)}) + \gamma_g \int_{t_0(g)}^t e^{\theta_g(s-t)} P_g(q_g; q_{C(g)}; q_{\tilde{C}(g)}) \, ds$$

where $\gamma_g > 0$ is a constant. This is to say that a recent positive profit history would drag to increase the production of $g$ but a recent negative profit history would drag to reduce the production of $g$.

Second we would add to the productions of all $g \in L(t)$ the consideration of constraints (15) of Sect. 2B. This constraint is critically important because it is the cause of the regular economic cycles. To model the impact of (15), we add

$$-\Theta_g \left( r_0 + \frac{\sum_{g \in L(t)} P_g(q_g; q_{C(g)}; q_{\tilde{C}(g)})}{\sum_{\tilde{g} \in L(t)} S_{\tilde{g}}(q_{\tilde{g}}; q_{C(\tilde{g})})} \right)$$
to the right side of equation (26) to obtain

$$\ddot{q}_g + \lambda \frac{d}{dt} q_g = \partial_{q_g} P_g(q_g; q_{C(g)}; q_{\tilde{C}(g)}) + \gamma_g \int_{t_0(g)}^t e^{\theta_g(t-s)} P_g(q_g; q_{C(g)}; q_{\tilde{C}(g)}) ds$$

(28)

$$- \Theta_g \left( r_0 + \frac{\sum_{g \in L(t)} P_g(q_g; q_{C(g)}; q_{\tilde{C}(g)})}{\sum_{g \in L(t)} S_g(q_g; q_{C(g)})} \right),$$

where $\Theta_g > 0$ is a constant. Let

$$r = \frac{\sum_{g \in L(t)} P_g(q_g; q_{C(g)}; q_{\tilde{C}(g)})}{\sum_{g \in L(t)} S_g(q_g; q_{C(g)})}.$$

Because of (15) we would expect $r < 0$. To add (27) to equation (26) is to assume that there is an optimal ratio in between aggregate value of profit function and aggregate value of business cost function $-r_0 < 0$, at which the influence of (15) is neutral on the production and consumption of all $g \in L(t)$. Overall business environment is more favorable for established businesses to turn in a profit if $r < -r_0$, for more money are passed to consumers through entrepreneur investments. Similarly, overall business environment is less favorable for established businesses to turn in a profit if $r > -r_0$, for the amount of money passed to consumers through entrepreneur investment is in shortage. In (27), $\Theta_g > 0$ is a proportional constant that measures the sensitivity of production and consumption of $g$ to overall economic environment. For $g \in L(t)$, $\Theta_g$ is determined mainly by the position of $g$ in the priority list of the consumers. If consumers are less willing to cut down the consumption of $g$ in a less favorable economic environment, then $\Theta_g$ would be smaller. Otherwise $\Theta_g$ would be larger. Consumption-goods of subsistence are more likely to have a small $\Theta_g$ and luxury consumption items are more likely to have a large $\Theta_g$. In general, $\Theta_g$ varies wildly on different $g \in L(t)$.

The collection of equation (28) for all $g \in L(t)$ is our dynamics model on production, cost and profit of all businesses. We caution that this is not a self-contained set of differential equations because $L(t)$ is ever changing. New products are constantly added into, and old products are constantly driven out, of $L(t)$ through the process of creative destruction. Whenever a product is added into or dropped from $L(t)$, functions on the right side of equation (28) experience discontinuous jump. This incompleteness, on the other hand, is a distinctive character that works actually towards our favor. Economic system is not only complicated but also dynamic, and the future of the system depends not only on what have happened so far but also on what will happen in the future. Our objective is to try to comprehend the main factors in play through the construction of an intellectual framework, and to understand economic trend associated to these factors. Efforts such as to first make a seemingly plausible quantitatively fitting curve using past economic data, then to project the future trajectory using the curves acquired are intrinsically futile.

D. Regular Economic Cycles Let

$$\mathcal{P} = \sum_{g \in L(t)} P_g(q_g(t); q_{C(g)}; q_{\tilde{C}(g)}), \quad \mathcal{S} = \sum_{g \in L(t)} S_g(q_g(t); q_{C(g)}).$$
We have
\[
\dot{\mathbf{P}} = \sum_{g \in L(t)} \sum_{\tilde{g} \in C(g) \cup \tilde{C}(g)} \partial_{\tilde{q}_g} P_g \cdot \tilde{q}_g,
\]
\[
\ddot{\mathbf{P}} = \sum_{g \in L(t)} \sum_{\tilde{g} \in C(g) \cup \tilde{C}(g)} \left( \partial_{\tilde{q}_g} P_g \cdot \tilde{q}_g + \sum_{\hat{g} \in C(g)} \partial_{\tilde{q}_g q_{\hat{g}}}^2 P_g \cdot \tilde{q}_g \hat{q}_{\hat{g}} \right)
\]
Similarly, we have
\[
\dot{\mathbf{S}} = \sum_{g \in L(t)} \sum_{\tilde{g} \in C(g)} \partial_{\tilde{q}_g} S_g \cdot \tilde{q}_g,
\]
\[
\ddot{\mathbf{S}} = \sum_{g \in L(t)} \sum_{\tilde{g} \in C(g)} \left( \partial_{\tilde{q}_g} S_g \cdot \tilde{q}_g + \sum_{\hat{g} \in C(g)} \partial_{\tilde{q}_g q_{\hat{g}}}^2 S_g \cdot \tilde{q}_g \hat{q}_{\hat{g}} \right)
\]
By definition,
\[
\dot{r} = \frac{1}{S^2} \left( \dot{\mathbf{P}} \mathbf{S} - \dot{\mathbf{S}} \mathbf{P} \right)
\]
\[
\ddot{r} = \frac{1}{S^2} \left( \ddot{\mathbf{P}} \mathbf{S} - \ddot{\mathbf{S}} \mathbf{P} \right) - \frac{2}{S^3} \left( \dot{\mathbf{P}} \dot{\mathbf{S}} - \dot{\mathbf{S}} \dot{\mathbf{P}} \right).
\]
It then follows that
\[
\ddot{r} = \frac{1}{S^2} \sum_{g \in L(t)} \sum_{\tilde{g} \in C(g) \cup \tilde{C}(g)} \left( \partial_{\tilde{q}_g} P_g \cdot \mathbf{S} - \partial_{\tilde{q}_g} S_g \cdot \mathbf{P} \right) \tilde{q}_g
\]
\[
+ \frac{1}{S^2} \sum_{g \in L(t)} \sum_{\tilde{g}, \hat{g} \in C(g) \cup \tilde{C}(g)} \left( \partial_{\tilde{q}_g q_{\hat{g}}}^2 P_g \cdot \mathbf{S} - \partial_{\tilde{q}_g q_{\hat{g}}}^2 S_g \cdot \mathbf{P} \right) \tilde{q}_g \hat{q}_{\hat{g}}
\]
\[
- \frac{2}{S^3} \sum_{g \in L(t)} \sum_{\tilde{g}, \hat{g} \in C(g) \cup \tilde{C}(g)} \partial_{\tilde{q}_g} S_g \cdot \left( \partial_{\tilde{q}_g} P_g \cdot \mathbf{S} - \partial_{\tilde{q}_g} S_g \cdot \mathbf{P} \right) \cdot \tilde{q}_g \hat{q}_{\hat{g}}
\]
By using (28) we obtain
\[
\ddot{r} = A(t)(r_0 + r) + B(t)
\]
where
\[
A(t) = \frac{1}{S} \sum_{g \in L(t)} \sum_{\tilde{g} \in C(g) \cup \tilde{C}(g)} \Theta_g \cdot \partial_{\tilde{q}_g} P_g + \frac{1}{S} \left( \sum_{g \in L(t)} \sum_{\tilde{g} \neq g} \Theta_g \cdot \partial_{\tilde{q}_g} S_g \right) \cdot r
\]
\[
+ \frac{1}{S} \left( \sum_{g \in L(t)} \Theta_g \cdot \partial_{\tilde{q}_g} S_g \right) \cdot r
\]
and
\[
B(t) = (I) + (II) + (III) + (IV) + (V),
\]
in which
\[
(I) = \frac{-\lambda}{S^2} \sum_{g \in L(t)} \sum_{\tilde{g} \in C(g) \cup \tilde{C}(g)} \left( \partial_{\tilde{q}_g} P_g \cdot \mathbf{S} - \partial_{\tilde{q}_g} S_g \cdot \mathbf{P} \right) \cdot \tilde{q}_g
\]
\[
(II) = \frac{1}{S^2} \sum_{g \in L(t)} \sum_{\hat{g} \in C(g) \cup \hat{C}(g)} (\partial_{\eta_q} P_g \cdot \mathcal{S} - \partial_{\eta_q} S_g \cdot \mathcal{P}) \cdot k_g \partial_{\eta_q} P_g
\]

\[
(III) = \frac{1}{S^2} \sum_{g \in L(t)} \sum_{\hat{g} \in C(g) \cup \hat{C}(g)} (\partial_{\eta_q} P_g \cdot \mathcal{S} - \partial_{\eta_q} S_g \cdot \mathcal{P}) \cdot \gamma_g \int_{t_0(g)}^t e^{\delta_g(s-t)} P_g ds
\]

\[
(IV) = \frac{1}{S^2} \sum_{g \in L(t)} \sum_{\hat{g} \in C(g) \cup \hat{C}(g)} \left( \partial^2_{\eta_q\eta_q} P_g \cdot \mathcal{S} - \partial^2_{\eta_q\eta_q} S_g \cdot \mathcal{P} \right) \hat{q}_g \hat{q}_{\hat{g}}
\]

\[
(V) = -\frac{2}{S^2} \sum_{g \in L(t)} \sum_{\hat{g} \in C(g) \cup \hat{C}(g)} \partial_{\eta_q} S_g \cdot \left( \partial_{\eta_q} P_g \cdot \mathcal{S} - \partial_{\eta_q} S_g \cdot \mathcal{P} \right) \cdot \hat{q}_g \hat{q}_{\hat{g}}
\]

At this point, there is no way for us to make even a semi-rigorous estimation on \( A(t) \) and \( B(t) \) so here we have to resolve to the use of a heuristic argument that is plausible. Observe that for the three sums of \( A(t) \) in (30), the natural of the first two are seemingly different from that of the third. For the first sum, the sign of \( \partial_{\eta_q} P_g \) varies as \( g \) and \( \hat{g} \) run over \( L(t) \) and \( C(g) \cup \hat{C}(g) \) respectively. The list \( L(t) \) is large, and at any given time, the effect of terms of positive sign are at least partially balanced by that of terms of negative sign. The second sum is similar but for the third sum, we have from (19) \( \partial_{\eta_q} S_g > 0 \) for all \( g \). This is to say that all terms in the third sum add to re-enforce each other. We can also go over sums in \( B(t) \) to conclude that the terms in each of these sums are with no particular preference in sign therefore they tend to cancel each other. Let us caution that, this is a very rough argument and it does not necessarily imply that the first two sums in \( A(t) \) or any of the sums in \( B(t) \) are in fact added to zero at any given time. These observations, however, indicate that the third sum of \( A(t) \) could somewhat dominate, and we could therefore propose to take

\[
(31) \quad \ddot{r} = \left( \frac{\sum \Theta_g \partial_{\eta_q} S_g}{S} \right) r (r_0 + r)
\]

as a modeling equation for \( r \). As a first approximation we could further simplify by replacing the sum on the right hand with a constant. This is to say we would model \( r \) by using

\[
(32) \quad \ddot{r} = A_0 r (r_0 + r)
\]

where \( A_0 > 0 \). Equation (32) takes \((r, \dot{r}) = (-r_0, 0)\) and \((r, \dot{r}) = (0, 0)\) as equilibrium points. \( (-r_0, 0) \) is a center and \((0, 0) \) is a saddle. Solutions oscillating around \((-r_0, 0)\) are such that \( r(t) < 0 \) for all \( t \). See Fig. 5 for the phase portrait of equation (32).

From equation (32), it follows that \( r \) varies cyclically over the time and cyclic motions of \( r \) would cause cyclic fluctuations of the aggregate production of the economic system. According to equation (28), aggregate production would be dragged down when \( r \) increases, and would be pushed up when \( r \) decreases. The periodicity of solutions of equation (32) therefore represents regular business cycles that are discussed previously in Sect. 2E. Let us recall that in Sect. 2E we assumed an economic environment in which current technological potential is fully realized by entrepreneurs.
investors and significant new advancement on science and technology is temporarily lacking. Such is an environment in which we do not expect substantial changes in $L(t)$. Therefore in computing $r(t)$, which is an aggregate measurement, we could practically ignore the changes in $L(t)$ and the associated worries about the discontinuities of equations (28).

Fig. 5 Phase portrait of equation (32) for $r$