Statement of Accomplishments and Objectives on Research, Teaching, and Service/Outreach

1 A brief statement on Research

In mathematics we often adopt the point of view that systems in nature are *deterministic* in the sense that the future is determined completely by the present. The future, however, is not always *predictable* in the long run, for a minor error in the measurement of the present might seriously impact our prediction about the future. This possibility has been observed in some science fiction novels: people went back in time, making minor changes, only come back to see a completely different world.

For systems with dissipation, the general perception was rather different for a long time. Pendula would eventually stop swinging because of the air resistance. Friction would cause all moving objects to come to rest. Dissipative systems, in general, were expected to approach a stable equilibrium or, if subject to forcing, simple periodic oscillations independent of initial conditions. Therefore when unpredictable behavior was observed in experiments, it was thought of as *strange*, and the complicated destination of the system was thus called a *strange attractor*. These systems were also regarded as *chaotic*, soon a popular name for natural phenomena characterized by sensitive dependence of future behavior on current conditions.

It has eventually turned out that strange attractors are rather common and they have been encountered in many scientific disciplines. One example is the forced van der Pol system, an electric circuit in which strange attractors were first discovered. Another famous example of the occurrence of strange attractors is in the Lorenz system from meteorology, the existence of which has darkened the hope of precise, long term weather forecasting. In the past fifty years, mathematical studies of chaos and strange attractors have influenced the modern development of many scientific disciplines, and the importance of these studies is now widely acknowledged in the world of non-linear sciences.

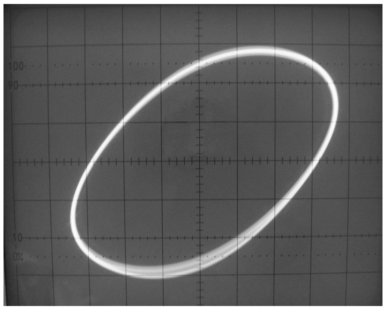

Fig. 1 A stable limit cycle

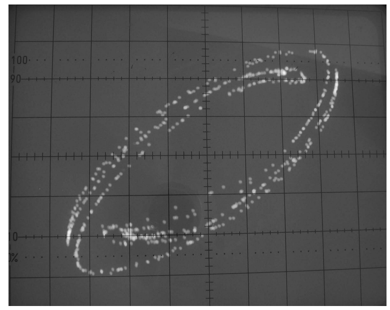

Fig. 2 A strange attractor

Figures 1 and 2 illustrate the difference between a simple limit cycle and a strange attractor. Both figures show a certain voltage measured at regular time intervals. In Fig. 1, one sees a periodic oscillation. In Fig. 2, the limit of the system is a fractal set of complicated geometry. These pictures are taken from my own study of a well-known circuit introduce by the electric engineer O. Chua in 1970's.

As a mathematic subject, the theory of chaos and strange attractors is organized around the following two themes:

(1) to develop mathematical theories that help us to understand and to describe the complicated structure of chaos and strange attractors; and

(2) to apply these theories to analyze differential equations arising from real world applications.

Concerning theme (1), there have been spectacular developments and the literature in the last sixty years is vast. It has turned out that some types of chaos are easier to understand than others (though all complicated). In particular, comprehensive mathematical theories have been developed for chaotic systems with fixed directions of expansion and contraction. These are called the *uniformly hyperbolic* systems. When the directions of expansion and contraction are confused in phase space, mathematical analysis become much harder. Nonetheless, profound mathematical theories has been developed during the last thirty years for such "non-uniformly hyperbolic" systems. These theories indicate that there tends to be *statistical laws* that govern the evolution of the chaotic systems. This is to say that, although we are unable to tell exactly what happens at a given time in the future, we can predict, on a large time scale, the percentage of time that a certain event will occur and can assert that this percentage is independent of the current state of the system.[1]

On the other hand, these theories have been mostly developed under mathematical assumptions that are intrinsically difficult to verify for models from the real world applications. Mathematical progresses on theme (2) has thus been limited.

My research in the last ten years have been on both themes, but my emphasis have been mainly on theme (2). This research work is targeted at bridging the gap between the existing mathematical theories on non-uniformly hyperbolic systems and their rigorous applications to real world problems.

Previously, there existed only one successful case in the analysis of (genuinely) non-uniformly hyperbolic systems.[2] This is the theory of Benedicks and Carleson on dissipative Hénon maps. The strategy of my study is to first generalize the analysis of Benedicks and Carleson to a wider class of maps in order to build a theory that is flexible enough to be applied to differential equations, and then to apply the theory developed to systems of practical and historic importance. My long term collaborator

[1] I am talking here about the existence of SRB measures.

[2] Here I mean systems without the invariant cone property.

on this research program is Lai-Sang Young, currently at Courant Institute, NYU. I have also collaborated with others, including Ali Oksasoglu at Honeywell corporation, Kening Lu at BYU, and William Ott at University of Houston, on various research projects related to the analysis of non-uniformly hyperbolic systems.

My work in the past six years have been along three separate but related lines. They are (1) to complete the development of my theory with Lai-Sang Young on the dynamics of rank one maps; (2) to apply this theory to the study of differential equations of practical and theoretic importance; and (3) to develop a comprehensive theory on the dynamics of non-autonomously perturbed second order equations.

On the theoretical front, Young and I have developed a systematic theory on the dynamics of rank one maps, publishing three papers, including one paper of 138 pages long appeared on the Annals of Mathematics last year. On the application front, Young and I have applied our theory to certain periodically kicked systems. Oksasoglu and I have extended such applications to switch controlled Chua circuits, a system well-known in electric engineering. Our work on this project include rigorous mathematical analysis, numerical simulations and actual lab implementation of the proposed circuits. Also in the last two years, I have gradually shifted the focus of my research to the study of systems with time dependent forcing. I have introduced fresh techniques in my study and developed a new dynamics theory on chaos in non-autonomously forced second order equations.

2 Six selected publications and future projects

The Integral Manifold of the Three-body Problem (Item [7] of my publication list)

Celestial mechanics was a major area of my research. This paper is selected to reflect this part of my academic career in the past. My studies on the N-body problem include (a) refuting a long standing conjecture made by G.D. Birkhoff on the bifurcation of the integral manifolds of the three-body problem, and (b) resolving a problem proposed more than one hundred years ago by Weierstrass, who asked for the construction of a convergent power series solution for all initial conditions leading to no singularities of collision. In addition, I also published (mostly joint with Ken Meyer) a sequence of research papers on the dynamical properties of gravitational systems of lower degrees of freedom.

Toward a Theory of Rank one Attractors (item [18] of my publication list)

Nonuniformly Expanding 1D Maps (item [15] of my publication list)

Lai-Sang Young and I have systematically developed a chaos theory for what we call *the rank one maps*. This name emphasizes the fact that there is only one direction of instability in the setting we introduced. The first paper we published was on 2-dimensional maps ([9], 97 pages, Commun. Math. Physics, 2001). We then published two more papers, building a similar theory for 1-dimensional maps ([15], 30 pages, Commun. Math. Physics, 2006) and for maps of dimension higher than two ([18], 138 pages, Annals of Mathematics, 2008).

The main purpose of this long project is to develop a systematic theory on the dynamics of non-uniformly hyperbolic maps that can be applied to differential equations of practical and historic importance. The results of this project have substantially improved the applicability of many profound mathematical theories developed in the past forty years, including the Newhouse theory, the theory of SRB measures, and the theory of Benedick and Carleson on Hénon maps, to application problems from the real world modelled by differential equations. Based on what was available to us when we began our investigations, we decided that our goal was achievable in *two steps*. First, we would introduce a flexible setting for non-uniformly hyperbolic maps and construct a comprehensive chaos theory. Second, we would try to find concrete equations of either practical or historic importance, to which we could apply our new theory. With these publications ([9], [15], 18]), we completed the *first step*.

Rank One Chaos: Theory and Applications (item [17] of my publication list)

Experimental Verification of Rank one Chaos in Switch-controlled Smooth Chua's Circuit (item [19] of my publication list)

To apply our theory developed in [9], [15] and [18] to ordinary differential equations, Young and I started with a periodically kicked second order equation. We computed the time-T map in the vicinity of a stable limit cycle under the assumption that the perturbations are in the form of a *periodic kick* in [10]. The kicking form of the forcing was imposed so we could gain analytical control on the time-T maps. We proved the existence of strange attractors with SRB measures by using the theory developed in [9]. We then extended our study to the dynamics of periodically kicked limit cycles in equations with supercritical Hopf bifurcations [12]. These two are then followed by a sequence of papers involving Young and myself, sometimes joint, sometimes separated with other collaborators, led to a divergent set of application problems under the general framework of periodically kicked equations.

These two papers ([17], [19] of my publication list) represent my own effort in finding applications for my theory with Young in circuit systems. My main collaborator on this project is Ali Oksasoglu from Honeywell corporation. We have created rank one chaos through periodically controlled switches, and applied this scheme to well-known circuit systems such as the Chua's circuit. Our work on this project include rigorous analysis, numerical simulations and actual lab implementation of the proposed circuits. This research project has resulted a sequence of publications on various applied journals ([13], [14], [16], [17], [19]). It is *the first systematic application of my theory with Young to differential equations of real world system of practical importance.*

Dynamics of Homoclinic Tangles in Periodically Perturbed Second Order Equations (item [3] in my submitted list)

In the last two years, I wrote a sequence of papers with a number of co-authors, developing systematically a new theory on the chaos dynamics of non-autonomously perturbed homoclinic solutions. In a joint paper with W. Ott (item [1] in my submitted list), we proved that, when the perturbed stable and unstable manifold are pulled apart, the separatrix maps are, in general, rank one maps. *With this proof we finally succeeded in applying my theory with Young on rank one attractors to a class of equations of substantial historic importance.* Concerning the case of homoclinic tangles, Oksasoglu and I (item [3] of my submitted list) proved that, as the magnitude μ of the forcing approaches to zero, homoclinic tangles move in and out of the Newhouse domain infinitely many times. We also obtained, *for the first time*, a comprehensive description on the global geometric and dynamic structure for these homoclinic tangle. In contrast, *all* results obtained in previous literature were on the existence of complicated *subsets* without such comprehensive overview. Lu and I (item [4] of my submitted list) extended these studies to non-autonomously perturbed equations without any time-periodicity, developing for these equations a chaos theory that is parallel to the classical Smale-Melnikov method in periodic case.

Future Projects

The new method introduced in our recently study of non-autonomously perturbed homoclinic solutions ([1], [3], [4] of my submitted list) has opened many opportunities for future research. It is an effective way of relating the study of non-autonomous equations to non-uniformly hyperbolic maps. So far, in both theory and application, we have focused on obtaining new results for the second order equations. An immediate next step is to generalize this theory to cover differential equations of higher order, and to find possible application to partial differential equations. Each of these extensions would be a substantial development. This method can also be extended to study heteroclinic tangles, and tangles with two instead of one homoclinic loops.

Since we no longer assume time-periodicity for forcing functions, our theory ([4] of my submitted list) has extended the scope of the chaos theory to the study of a much larger class of ordinary differential equations. One particularly promising direction is for us to develop new chaos theory first for randomly perturbed, then stochastically perturbed equations. We could also explore the possibility of generalizing the horseshoe part of our theory to study the non-periodically perturbed Hamiltonian equations.

We have obtained (in [3] of my submitted list), *for the first time*, a comprehensive overview on the dynamical structure of homoclinic tangles from a dissipative saddle point. We can use this overview as a guidance to perform systematic numerical investigations for many equations of practical and historic importance. This is also a direction yet to be fully explored.

3 Statement on Teaching

During the ten years I have been at the University of Arizona, I have taught various classes at both undergraduate and graduate level. I also taught at UCLA for four years and at Vanderbilt university for two years before joining the faculty at the University. I have always regarded teaching as a solemn responsibility and an essential component of my academic career. I have worked closely with both graduate and undergraduate students and strived to make a difference in their education.

For my "outstanding contributions to the Honors community" and my "commitment to teaching and mentoring talented undergraduates", I received an outstanding faculty award from the Honor College, University of Arizona this year (2009).

Undergraduate teaching:

I have taught undergraduate classes in University of Arizona at all levels, ranging from calculus for freshmen to modelling classes for math seniors.

The fundamentals of my teaching philosophy is to always place myself in a friendly position to encourage and help my students. In my lectures I try to present challenging mathematical materials in a way that is accessible to the least prepared students in my class. I invite questions and give in-class exercises to keep communication going in both directions. I give extensive office hours, and open my office whenever I am in to welcome students who have questions. I regard tests, especially multi-time mid-term tests, as a tool to help my students review the materials covered, and to catch up if behind; so I usually spend time one-on-one with students who fail the test and then allow them re-do the test to make up their grades.

In a scale of 1-5, the averages of teaching evaluations I have received from my students are constantly around 4.0. I have received many positive and encouraging remarks from students.

Graduate teaching:

I have graduated one Ph.D student (Predrag Punosevac). I also served as mentor for a visiting postdoc (Kuo-Chang Chen, who wrote a paper while he was here later published on Annals of Mathematics). I have been actively involved in the activities of the graduate program in my department, serving many years on the graduate committee for the Interdisciplinary Program in Applied Mathematics. I was also responsible for recruiting students from the People's Republic of China. I also served a two year term on graduate committee for the pure math program. In addition, I have delivered a sequence of Research Tutorial Group (RTG) lectures for the first year graduate students, and becoming advisor for a few of them in this RTG program.

I have taught math 557 (graduate class on dynamical systems and chaos) twice. After a careful search for a graduate text book which I felt suitable, I decided to develop my own lecture notes for the use of this class. I have spend a substantial amount of time and effort in writing these lecture notes, and have used them twice. They appear to be well-received by my Math 557 students.

Technology in teaching:

I design and set up my own web-page for classes I teach, using it as a main way of communicating with my student in terms of assigning homework, exchanging information, and making announcements. I have been involved in teaching calculus using advanced calculators, and have tried to use calculator and other computing instrument in creative ways to make a difference in helping my students to grasp complicated mathematical method and concepts.

4 Service/Outreach

I regard service at various levels as an integral part of my academic career. As a member of the department, the university, and the mathematical society, we are constantly supported by various administrative services provided generously by others in the organizations. Taking administrative responsibilities and providing excellent services at various levels both internally and externally are not only an obligation, but also a way of having influence.

I am currently the scriber of the dynamical systems group in the department. I have been co-organizing our seminar on dynamical systems since the time I came to the University. I have joint the IDP in Applied Mathematics and served as a member of its graduate committee for many years. As part of my duty on the committee I was responsible for recruiting graduate students from PRC. I have also served on the undergraduate committee, graduate committee, and library committee of the Department of Mathematics.

I have lectured in and out of the country at various institutions and professional conferences. I served on NSF grant panel on Ergodic Theory and Dynamics Systems. I have also served as referee for various professional journals (Including Inventiones, TAMS, ETDS, Non-linearity, JDE).