Math 464 Solution of HW # 1

1. (a) (i) $(\Omega\setminus A)\cap(\Omega\setminus B)\cap(\Omega\setminus C)$ or $\Omega\setminus(A\cup B\cup C)$

(ii) $A\cup B\cup C$

(iii) $\big(A\cap(\Omega\setminus B)\cap(\Omega\setminus C)\big)\cup\big((\Omega\setminus A)\cap B\cap(\Omega\setminus C)\big)\cup\big((\Omega\setminus A)\cap(\Omega\setminus B)\cap C\big)$

(iv) $\big(A\cap B\cap(\Omega\setminus C)\big)\cup\big(A\cap(\Omega\setminus B)\cap C\big)\cup\big((\Omega\setminus A)\cap B\cap C\big)$

(v) $A\cap B\cap C$

(b) (i) $P(\Omega\setminus(A\cup B\cup C)) = 1-P(A\cup B\cup C)$

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$$

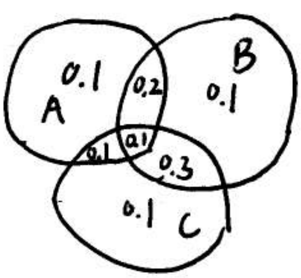
$$= \frac{5}{10}+\frac{7}{10}+\frac{6}{10}-\frac{3}{10}-\frac{4}{10}-\frac{2}{10}+\frac{1}{10}$$

$$= 1$$

$\therefore\ P(\Omega\setminus(A\cup B\cup C)) = 1-1=0$

(ii) $P(A\cup B\cup C)=1$

(iii) We can draw a Venn diagram for them.



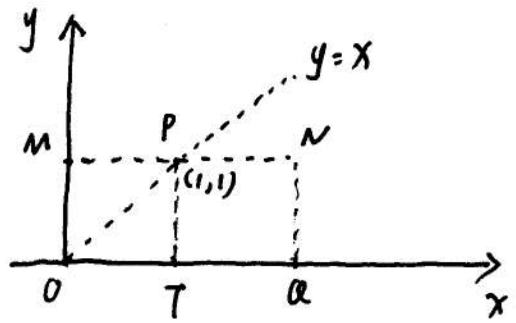
Then $P(\text{Exactly one of } A.B.C \text{ occurs}) = 0.1+0.1+0.1 = 0.3$

(iv) P(Exactly two of A.B.C occurs)

$= 0.2+0.1+0.3 = 0.6$

(v) P(Exactly three of A.B.C occurs) $=0.1$

2.



(a) $P(X>Y) = \dfrac{S(OPNQ)}{S(MNQO)} = \dfrac{\frac{3}{2}}{2} = \dfrac{3}{4}$

(b) $P(X>Y \mid X\le 1)$

$$= \frac{P(X>Y \cap X\le 1)}{P(X\le 1)} = \frac{\frac{S(OPT)}{S(MNQO)}}{\frac{S(MPTO)}{S(MNQO)}} = \frac{S(OPT)}{S(MPTO)} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

3. P(We roll the 3) $= \frac{1}{6}$

(a) P(it takes exactly 4 rolls to get a 3)

$= \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} = \frac{125}{1296}$

(b) P(it takes exactly 1 rolls to get a 3) $= \frac{1}{6}$

P(it takes ..... 2 ....) $= \frac{5}{6}\cdot\frac{1}{6} = \frac{5}{36}$

P(it takes ..... 3 ....) $= \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} = \frac{25}{216}$

~~P(it takes ..... 4 ....) $= \frac{125}{1296}$~~

Therefore, P(it takes at least 4 rolls)

$$= 1 - \frac{1}{6} - \frac{5}{36} - \frac{25}{216} = \frac{125}{216}$$

(c) P(it takes exactly 10 rolls | it takes at least 4 rolls)

$$= \frac{P(\{\text{it takes exactly 10 rolls}\} \cap \{\text{it takes at least 4 rolls}\})}{P(\text{it takes at least 4 rolls})}$$

$$= \frac{(1-\frac{1}{6})^9 \cdot \frac{1}{6}}{\frac{125}{216}} = \frac{5^6}{6^7}$$

4. $P(A \cup B \cup C) = P((A \cup B) \cup C) = P(A \cup B) + P(C) - P((A \cup B) \cap C)$

$= P(A) + P(B) - P(A \cap B) + P(C) - P[(A \cap C) \cup (B \cap C)]$

$= P(A) + P(B) - P(A \cap B) + P(C) - [P(A \cap C) + P(B \cap C) - P((A \cap C) \cap (B \cap C))]$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

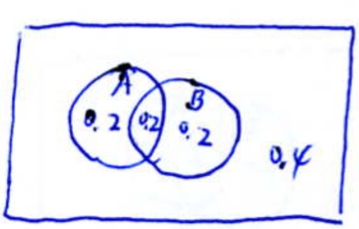
(Using the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$)

5. (a) Since $P((A^c \cap B) \cup (A \cap B)) = P(B)$ & $P((A^c \cap B) \cap (A \cap B)) = 0$

Events $A^c \cap B$ and $A \cap B$ are exclusive.

$$P(A^c | B) = \frac{P(A^c \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = 1 - \frac{P(A \cap B)}{P(B)} = 1 - P(A|B)$$

(b)



$$P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{0.2}{0.6} = \frac{1}{3}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = \frac{1}{2}$$

Then $P(A|B^c) \neq 1-P(A|B)$

Hence $P(A|B^c)$ is not always equal to $1-P(A|B)$

6. T^+: Positive Test $\quad D^+$: have disease $\quad D^-$: not have disease

$P(D^+) = 1\%$ $\quad P(T^+|D^+) = 90\%$ $\quad P(T^+|D^-) = 5\%$

$$P(D^+|T^+) = \frac{P(D^+ \cap T^+)}{P(T^+)} = \frac{P(T^+|D^+) \cdot P(D^+)}{P(T^+)}$$

$$= \frac{P(T^+|D^+) \cdot P(D^+)}{P(D^+) \cdot P(T^+|D^+) + P(D^-) \cdot P(T^+|D^-)} = \frac{0.9 \times 0.01}{0.9 \times 0.01 + 0.05 \times 0.99} = 0.154$$

7. (a) P (the coin in the other hand is also gold | the coin in the first hand is gold)

$$= \frac{P(\text{both hands have gold})}{P(\text{the first hand has gold})} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2}} = \frac{1}{2}$$

(b) P(the coin in the other hand is silver | the coin in the first hand is gold)

$$= \frac{P(\text{one hand has gold and the other has silver})}{P(\text{the first hand has gold})}$$

$$= \frac{\frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2}}{\frac{1}{4} + \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2}} = \frac{1}{2}$$

8. (a) $$P(E) = \frac{C_{10}^2}{C_{15}^2} = \frac{45}{105} = \cancel{\frac{9}{35}}\ \frac{3}{7}$$

Note: $C_n^k = \binom{n}{k}$

$$P(F) = \frac{C_5^2}{C_{15}^2} = \frac{10}{105} = \frac{2}{21}$$

$$P(G) = \frac{C_{10}^1 \cdot C_5^1}{C_{15}^2} = \frac{10}{21}$$

(b) $$P(E) = \frac{C_{10}^1 \cdot C_{10}^1}{C_{15}^1 \cdot C_{15}^1} = \frac{100}{225} = \frac{4}{9}$$

$$P(F) = \frac{C_5^1 \cdot C_5^1}{C_{15}^1 \cdot C_{15}^1} = \frac{25}{225} = \frac{1}{9}$$

$$P(G) = \frac{C_{10}^1 \cdot C_5^1 \cdot 2!}{C_{15}^1 \cdot C_{15}^1} = \frac{100}{225} = \frac{4}{9}$$

(c) $$P(E) = \frac{C_{10}^1 \cdot C_9^1}{C_{15}^1 \cdot C_{14}^1} = \frac{90}{15 \times 14} = \frac{3}{7}$$

$$P(F) = \frac{C_5^1 \cdot C_4^1}{C_{15}^1 \cdot C_{14}^1} = \frac{20}{15 \times 14} = \frac{2}{21}$$

$$P(G) = \frac{C_{10}^1 \cdot C_5^1 \cdot 2!}{C_{15}^1 \cdot C_{14}^1} = \frac{100}{15 \times 14} = \frac{10}{21}$$