1. According to the Theorem 2 on multinomial coefficients from "Notes on counting", we know the coefficient of a^3b^2c in $(a+b+c)^6$ should be $M_6(3,2,1) = \frac{6!}{3!\,2!\,1!} = 60$

b. When $n=3$, $m=3$, $k=3$

$$\binom{n+m}{k} = \binom{3+3}{3} = \binom{6}{3} = 20 = \text{Left hand side}$$

$$\binom{n}{0}\binom{m}{k} + \binom{n}{1}\binom{m}{k-1} + \cdots + \binom{n}{k}\binom{m}{0}$$

$$= \binom{3}{0}\binom{3}{3} + \binom{3}{1}\binom{3}{2} + \binom{3}{2}\binom{3}{1} + \binom{3}{3}\binom{3}{0}$$

$$= 1 + 3\times3 + 3\times3 + 1 = 20 = \text{Right hand side}, \quad \text{Left} = \text{Right}$$

Hence, this ~~formula~~ identity is true for $n=3$; $m=3$; $k=3$

Interpretation of this formula: the number of ways to pick a set of k objects from $n+m$ different objects is equivalent to the number of ways to pick a set of i objects from n different objects and pick a set of j objects from m different objects with $i+j=k$, $0\le i\le k$, $0\le j\le k$.

2.

a. The number of such license plates

$= 10 \times 26 \times 26 \times 26 \times 10 = 1757600$

b. The number of such license plates

$= 10 \times \binom{4}{1} \times 10 \times 26 \times 26 \times 26 \times 10$

$= 4 \times 10^3 \times 26^3 = 70304000$

Note: $\binom{4}{1}$ comes from the fact that the third digit may be placed anywhere.

3. $p($these 3 are consecutive$) = \frac{8}{\binom{10}{3}} = \frac{8}{120} = \frac{1}{15}$

4. a. There are $n-1$ situations that are supposed to have equal probability: Ann and Bob have k people between them as k ranges from 0 to $n-2$. So totally there are $n-1$ different situations. Since the total probability must equal to 1, the probability of each of these situations must be $\frac{1}{n-1}$

b. There are $\binom{n-2}{k}$ ways to choose these k individuals who are between Ann and Bob and then there are $k!$ ways to order them. Besides, there are also $(n-k-2)!$ ways to order the remaining $n-k-2$ individuals who are not between Ann and Bob. Finally there is a total of $(n-1)!$ ways to order n people in a circle.

Therefore, $$p = \frac{\binom{n-2}{k} \times k! \times (n-k-2)!}{(n-1)!} = \frac{1}{n-1}$$

5. $p(\text{one boy}) = p(\text{one girl}) = \frac{1}{2}$

$$p = \binom{6}{2} (\tfrac{1}{2})^2 (1-\tfrac{1}{2})^4 = 15 \times \frac{1}{64} = \frac{15}{64}$$

6. $p(\text{hit it one time}) = \binom{10}{1} (\frac{1}{3})^1 (1-\frac{1}{3})^9 = 0.0867$

$p(\text{not hit it at all}) = \binom{10}{0} (\frac{1}{3})^0 (1-\frac{1}{3})^{10} = 0.0173.$

$\therefore$ $p(\text{hit it at least twice}) = 1 - 0.0867 - 0.0173 = 0.896$

7. $$p = \frac{\binom{20}{6} 3^6 \cdot 5^{14}}{8^{20}} = 0.1496$$