

Math 464 - Homework 2

1.

a. Find the coefficient of a^3b^2c in $(a + b + c)^6$.

b. The following identity is true

$$\binom{n+m}{k} = \binom{n}{0} \binom{m}{k} + \binom{n}{1} \binom{m}{k-1} + \binom{n}{2} \binom{m}{k-2} \\ + \cdots + \binom{n}{k} \binom{m}{0}.$$

First, check this for $n = 3; m = 3; k = 3$. Give an interpretation of this formula in terms of a counting problem. There might be more than one answer.

2. By *digits* I mean $0, 1, 2, \dots, 9$.

a. How many license plates can be made out of 3 letters and 2 digits if the digits must come first and last?

b. How many license plates can be made out of 3 letters and 3 digits if the first and last symbol must be a digit?

3. A lunch counter has 10 seats, of which 3 are unoccupied. What is the probability that these 3 are consecutive?

4. Ann and Bob are two of n people who are standing in a circle. The probability that there are k people between A and B, when *between* means: counting from A to B in the clockwise direction, is the same for all k . (Note that B could be standing just at A's right, but there would be $n-2$ people between them, by this rule).

a. Find it (this probability), given that it is independent of k .

b. Find it by counting, to verify that it is indeed independent of k .

5. What proportion of families with six children should be expected to have exactly two boys (assuming boys and girls are equally likely)?

6. Past experience shows that when I play darts, I hit the target $1/3$ of the time. What is the probability that in my next ten attempts, I will hit it at least twice? (I am not getting better with time.)

7. 20 balls are distributed at random into 8 boxes. Find the probability that the first 3 boxes, together, contain 6 balls.