

Math 464 - Homework 5

The exercises and problems referred to in 1-2 below are exercises and problems from the course text (Grimmett and Welsh).

1. Exercise 15 in Chapter 5. (See page 64 for the definition of the χ^2 distribution.)
2. Problem 1 in Chapter 5.
3. Let X be exponential. Suppose $P(X \geq .01) = 1/2$. Find t such that $P(X \geq t) = .9$.
4. Pick four numbers independently at random from $[0, 1]$ and let Y be the second largest. Find $P(Y > 1/2)$.
5. Let X be a random variable with uniform distribution on $(0,1)$ and $Y = -X$. Find f_Y .
6. Let X be a random variable with density f_X , and let $Y = |X|$. Find f_Y in terms of f_X .
7. Consider the experiment of counting cars as they drive past an observation point along a certain street (for example, one might be a policeman at a radar check along Campbell Avenue). This count is known to be a Poisson random variable. Suppose one knows that the average rate, λ , at which cars pass the observation point is 0.5 cars per minute. When a car passes by the observation point, one says that *it has arrived*, even though it keeps going. The periods between arrivals are called *waiting times*. We further assume that the waiting times between cars are independent. What is the probability that the first car will arrive at a time t satisfying $2 < t < 3$?
8. Let X_1, \dots, X_n be numbers drawn independently from the uniform distribution on $(0, 1)$. Let Y be the largest and Z the second largest number in this drawing. Find their expectations and variances.