

### Math 464 - Homework 6

The exercises referred to in 1-5 below are exercises from the course text (Grimmett and Welsh).

1. Exercise 2 in Chapter 6.
2. Exercise 5 in Chapter 6.
3. Exercise 8 in Chapter 6.
4. Exercise 11 in Chapter 6.
5. Exercise 13 in Chapter 6.
6. Suppose the joint density of  $X, Y$  is uniform over the disc centered at the origin of radius 1.
  - a) Find the marginal pdf's of  $X$  and  $Y$ .
  - b) Are  $X$  and  $Y$  independent? Give a proof or a counter-example.
7. Let  $X$  and  $Y$  be random variables and suppose

$$E [X^n Y^k] = \frac{2^n 3^k}{(n+1)(k+1)}$$

for positive integers  $n$  and  $k$ . Find the mean and variance of the four random variables  $X^2, Y^2, X + Y$  and  $(X + Y)^2$ .

8. Let  $X$  be a random variable with the exponential distribution for some positive parameter  $\lambda$ . Show that

$$P(X > u + v | X > u) = P(X > v)$$

for all  $u, v \geq 0$ . (This is called the *lack-of-memory* property).