

## Math 587 - Take-Home Midterm

Choose one problem to do from each of the categories below (a *category* is labelled by a Roman numeral) for a total of four problems. The solutions are due, in class, on Thursday, November 12. (Most of the problems referred to below are exercises from the course text [Miller]).

I.

- A) Exercise 3.1 OR Exercise 3.2
- B) Exercise 3.3
- C) Exercise 3.5

II.

- A) Exercise 4.8
- B) Exercise 4.9
- C) Exercise 4.13

III.

- A) Exercise 5.1
- B) Exercise 5.9
- C) Exercise 5.10

IV.

- A) Exercise 8.5
- B) Exercise 8.7
- C) We want to solve the differential equation

$$\epsilon u'' - uu' + u^3 = 0, \quad 0 \leq x \leq 1, \quad (1)$$

subject to  $u(0) = 2/3$  and  $u(1) = -1/2$ .

- a) Find the leading order behavior of the outer solution to

$$\epsilon u'' - uu' + u^3 = 0$$

regardless of boundary conditions.

- b) Find the leading order behavior of the matched expansions  $u_{\text{match}}^{0,0}$  for solutions of (1) that have boundary layers at  $x = 0$  and  $x = 1$  respectively.
- c) Seek a solution for (1) which has an inner layer: find the proper form of the inner solution at leading order. You will need to justify the sign of the integration constant as well as the form you are using for the solution. By matching this solution to the outer ones, find the location,  $x = x_0$ , of the interior layer and write the leading order solution  $u_{\text{match}}^{0,0}$ .