Problem 1.3.22(d)

The first operation is subtracting 1/2 of the first row from the second row. One gets

$$\begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & -1/2 \\ 0 & 1 & 1 \end{pmatrix}.$$

The corresponding elementary matrix

$$L_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The second operation is subtracting 1/3 of the second row from the third row. Then

$$U = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & -1/2 \\ 0 & 0 & 7/6 \end{pmatrix}.$$

The second elementary matrix

$$L_2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1/3 & 0 \end{pmatrix},$$

and

$$L = L_2 L_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 1/3 & 1 \end{pmatrix}.$$

Problem 1.4.10(d)

$$P = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Problem 1.4.20(a)

The matrix of the system is

$$\begin{pmatrix} 4 & -4 & 2 \\ -3 & 3 & 1 \\ -3 & 1 & -2 \end{pmatrix}.$$

Add 3/4 of the first row to both the second and the third rows to get

$$\begin{pmatrix} 4 & -4 & 2 \\ 0 & 0 & 2.5 \\ 0 & -2 & -0.5 \end{pmatrix}.$$

The lower triangular matrix that corresponds to this operarion is

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -0.75 & 1 & 0 \\ -0.75 & 0 & 1 \end{pmatrix}.$$

Now, interchange the second and the third rows to get

$$U = \begin{pmatrix} 4 & -4 & 2 \\ 0 & -2 & -0.5 \\ 0 & 0 & 2.5 \end{pmatrix}.$$

The matrix L does not change, and the permutation matrix

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

To solve the system, we write

$$LUx = PAx = \begin{pmatrix} 1 \\ -5 \\ 3 \end{pmatrix}.$$

Let $y = Ux$. Then

$$Ly = \begin{pmatrix} 1 & 0 & 0 \\ -0.75 & 1 & 0 \\ -0.75 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ 3 \end{pmatrix}.$$

By forward substitution,

$$y_1 = 1, \quad y_2 = 0.75y_1 - 5 = -4.25, \quad y_3 = 0.75y_1 + 3 = 3.75.$$

Then

$$Ux = \begin{pmatrix} 4 & -4 & 2 \\ 0 & -2 & -0.5 \\ 0 & 0 & 2.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -4.25 \\ 3.75 \end{pmatrix}.$$

We find x by back substitution: $x_3 = 3.75/2.5 = 1.5$; $2x_2 = 4.25 - 0.5x_3 = 3.5$, so $x_2 = 1.75$; $4x_1 = 1 + 4x_2 - 2x_3 = 5$, so $x_1 = 1.25$.
Answer: $x_1 = 1.25$, $x_2 = 1.75$, $x_3 = 1.5$.

Problem 1.5.13

One has

$$3A - A^2 = I,$$

so

$$A(3I - A) = I.$$

Therefore,

$$A^{-1} = 3I - A.$$

Problem 1.6.25(b)

To find the LU decomposition of the matrix, we add 1.5 times the first row to the second row:

$$U = \begin{pmatrix} -2 & 3 \\ 0 & 3.5 \end{pmatrix}.$$

The corresponding matrix L is given by

$$L = \begin{pmatrix} 1 & 0 \\ -1.5 & 0 \end{pmatrix}.$$

The matrix U factorizes further:

$$U = \begin{pmatrix} -2 & 0 \\ 0 & 3.5 \end{pmatrix} \begin{pmatrix} 1 & -1.5 \\ 0 & 1 \end{pmatrix} = DL^T.$$