

PROBLEM SET 16

PROBLEM 1

Recall that a function $f(x)$ satisfies Dini's condition at a point x_0 if $f(x_0 \pm 0)$ exist and

$$\int_0^\eta \frac{|f(x_0 + t) + f(x_0 - t) - f(x_0 - 0) - f(x_0 + 0)|}{t} dt < \infty$$

for some $\eta > 0$. Give an example of a function, which is of bounded variation in a neighborhood of x_0 , and that does not satisfy Dini's condition at x_0 ; give an example of a function that satisfies Dini's condition at x_0 , but it is not of bounded variation in any neighborhood of x_0 . For the second example, one can arrange the function to be differentiable at every point.

From Folland's book: problems 27, 32, 35, 36, 37, 40, pages 164–165; 35, 37, pages 269–270