**Problem Set 17**

**Problem 1**

Let \( K(x, y) \) be a continuous function on \([0, 1]^2\). In the space \( L^2([0, 1], (1 + x)dx) \), we define an operator \( K \) by the formula

\[
Ku(x) = \int_0^1 K(x, y)u(y)dy.
\]

Show that the operator \( K^* \) can be written down as

\[
K^*v(x) = \int_0^1 K^*(x, y)v(y)dy
\]

and find the expression of \( K^*(x, y) \) in terms of \( K(x, y) \).

**Problem 2**

Let

\[
Au(x) = \int_0^1 e^{x-y} \sin(x + y)u(y)dy.
\]

Find a Hilbert space \( H \) of functions on \([0, 1]\) such that formula (1) defines a bounded, self-adjoint operator in \( H \).