PROBLEM SET 17

Problem 1

Let E be a Banach space. Suppose the the norm in E satisfies the parallelogram identity

$$||x+y||^2 + ||x-y||^2 = 2(||x||^2 + ||y||^2).$$

Prove that the polarization identity

$$(x,y) = \frac{1}{4}\big(||x+y||^2 - ||x-y||^2 + i||x+iy||^2 - i||x-iy||^2\big)$$

defines a scalar product, and E, endowed with this scalar product, is a Hilbert space.

From Folland's book: problems 55, 56, 57, 63, 64, 66, 67