PROBLEM SET 2

PROBLEM 1

Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be a mapping given by the formulas

$$y^1 = (x^1)^3 + x^2,$$
$$y^2 = (x^1)^3 - (x^2)^3.$$

a) Is F a homeomorphism?
b) Is F a diffeomorphism?

PROBLEM 2.

Let S^3 be thought of as

$$\{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^2 = 1\},$$

and let S^2 be thought of as the complex plane $\mathbb{C}$, with ∞ added. Define a mapping
$H : S^3 \to S^2$ by the formula $H(z_1, z_2) = z_1/z_2$.
a) Show that H is smooth.
b) Find the rank of H at each point.
c) What is $H^{-1}(p)$ where $p \in S^2$?
From Spivak's book: problems 8, 9, 15(a), 33, 34, p.p. 53–62