PROBLEM SET 7

Problem 1

Let $X(x)$ be a smooth vector field in $\overline{B^n} = \{x \in \mathbb{R}^n : |x| \le 1\}$. Suppose that $X(x) \cdot x > 0$ for all $x \in S^{n-1} = \{x : |x| = 1\}$. Here $\cdot$ is the dot-product. Prove that there exists $x \in B^n$ such that $X(x) = 0$.

Problem 2

Let S^{2n} be an even-dimensional sphere.
a) For a continuous vector field $X(x)$ on S^{2n}, prove that $X(x) = 0$ for some point $x \in S^{2n}$.
b) For a continuous mapping $F : S^{2n} \to S^{2n}$, prove that there exists a point $x \in S^{2n}$ such that either $X(x) = x$ or $X(x) = -x$.
Hint. For a), you can show that if $X(x) \ne 0$ everywhere then the identity mapping of S^{2n} is homotopic to the mapping $x \mapsto -x$. Prove that they are actually not homotopic.

From Hatcher's book: problems 1, 4, 9, 10, 11. p.p. 18–20