Homework # 3

Section # 2.1

- Determine which equations are linear equations in the variables x, y and z. If any equation is not linear, explain why not.

  2. $x^2 + y^2 + z^2 = 1$

  4. $2x - xy - 5z = 0$

- Find the solution set of each equation.

  11. $3x - 6y = 0$

  13. $x + 2y + 3z = 4.$

- **20.** Solve the given system by back substitution

$$\begin{aligned} 2u - 3v &= 5 \\ 2v &= 6 \end{aligned}$$

- **26.** Solve the given system by forward substitution.

$$\begin{aligned} x_1 &= -1 \\ -\frac{1}{2}x_1 + x_2 &= 5 \\ \frac{3}{2}x_1 + 2x_2 + x_3 &= 7 \end{aligned}$$

- **28.** Find the augmented matrix of the following linear system

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 1 \\ x_1 + x_3 &= 0 \\ -x_1 + 2x_2 - 2x_3 &= 0 \end{aligned}$$

- **31.** Find a system of linear equations that has the given matrix as its augmented matrix

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 2 & -1 & 1 & 1 \end{array}\right]$$

- **37.** Solve the system of linear equations given in the exercise **31**.

- **39.**

  (a) Find a system of two linear equations in the variables x and y whose solution set is given by the parametric equations $x = t$ and $y = 3 - 2t$

  (b) Find another parametric solution to the system in part (a) in which the parameter is s and $y = s$.

- **43.** Find substitution (change of variables) that convert system into linear system and use this linear system to help solve the given system.

$$\begin{aligned} \tan x - 2\sin y &= 2 \\ \tan x - \sin y + \cos z &= 2 \\ \sin y - \cos z &= -1 \end{aligned} \tag{0.1}$$

Section # 2.2

- **6.**Determine whether the given matrix is in row echelon form. If it is, state whether it is also in reduced row echelon form.

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

- **14.** Use elementary row operations to reduce the given matrix to (a) row echelon form and (b) reduced row echelon form.

$$\begin{bmatrix} -2 & -4 & 7 \\ -3 & -6 & 10 \\ 1 & 2 & -3 \end{bmatrix}$$

- **17.** Show that the given matrices are row equivalent and find a sequence of elementary row operations that will convert A into B.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$$

- **19.** What is wrong with the following "proof" that every matrix with at least two rows is row equivalent to a matrix with a zero row?

  Perform $R_2 + R_1$ and $R_1 + R_2$. Now rows 1 and 2 are identical. Now perform $R_2 - R_1$ to obtain a row of zeros in the second row.

- **25.** Solve the given system of equations using either Gaussian or Gauss-Jordan elimination.

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 9 \\ 2x_1 - x_2 + x_3 &= 0 \\ 4x_1 - x_2 + x_3 &= 4 \end{aligned}$$

- **41.** For what value(s) of k, if any, will the system have (a) no solutions, (b) a unique solution, and (c) infinitely many solutions?

$$\begin{aligned} x + ky &= 1 \\ kx + y &= 1 \end{aligned}$$

- **49.** Determine whether the lines $\mathbf{x} = \mathbf{p} + s\mathbf{u}$ and $\mathbf{x} = \mathbf{q} + t\mathbf{v}$ intersect and, if they do, find their point of intersection.

$$\mathbf{p} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$