Homework # 4

Section # 2.3

- Determine if the vector $\mathbf{v}$ is a linear combination of the remaining vectors.

  1.
  $$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

  2.
  $$\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

  5.
  $$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

- Determine if the vector $\mathbf{b}$ is in the span of the columns of the matrix A
  $$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

- **10.** Show that
  $$\mathbf{R}^2 = span\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right). \tag{0.1}$$

- **15.** Describe the span of given vectors (a) geometrically and (b) algebraically.
  $$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

- Determine if set of vectors are linearly independent. For any sets that are linearly dependent, find a dependance relationship.

  24.
  $$\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix}$$

  31.
  $$\begin{bmatrix} 3 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \\ 3 \end{bmatrix}$$

- **46.** Prove that every subset of a linearly independent set is linearly independent.

Section # 3.1

- Compute the indicated matrices (if possible).

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 4 & 2 \end{bmatrix}, \quad F = \begin{bmatrix} -1 \\ 2 \end{bmatrix},$$

  1. $A + 2D$

  6. BD

  12. EF

  13. $B^T C^T - (CB)^T$

- **21.** Show that the given matrices are row equivalent and find a sequence of elementary row operations that will convert A into B.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$$

- Let

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

  38(a) Show that

$$A^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}.$$

  38(b) Prove by mathematical induction that

$$A^n = \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}, \quad \texttt{for } n \geq 1.$$