Homework # 5

Section # 3.2

- **2.** Solve the equation $2X = A - B$, given that

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}.$$

- **14.** Determine whether the given matrices are linearly independent.

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

- **44.** If A and B are $n \times n$ matrices, prove the the following properties of the trace:

  a. $tr(A + B) = tr(A) + tr(B)$

  b. $tr(kA) = k\ tr(A)$, where k is a scalar.

Section # 3.3

- **2.** Find the inverse of given matrix (if it is exists).

$$\begin{bmatrix} 4 & -2 \\ 2 & 0 \end{bmatrix}.$$

- **13.** Let A, $\mathbf{b}_1$, and $\mathbf{b}_2$ and $\mathbf{b}_3$ are defined as follows

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

  a. Find A^{-1} and use it to solve the three systems $A\mathbf{x} = \mathbf{b}_1$, $A\mathbf{x} = \mathbf{b}_2$, $A\mathbf{x} = \mathbf{b}_3$.

  b. Solve all these systems at the same time by raw reducing the augmented matrix $[A|\mathbf{b}_1\mathbf{b}_2\mathbf{b}_3]$ using Gauss-Jordan elimination.

  c. Count the total number of individual multiplications that you performed in (a) and in (b). You should discover that, even for this 2×2 example, one method uses fewer operations.

- **20.** Solve the matrix equation $XA^2 = A^{-1}$. Simplify answer as much as possible. Assume that all matrices are invertible.

- **31.** Find the inverse of the given elementary matrix.

$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

- **48.** Use the Gauss-Jordan method to find the inverse of the matrix

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

(if it is exists).

Section # 3.4

- Solve the system $A\mathbf{x} = \mathbf{b}$ using the given LU factorization of A.

1.

$$A = \begin{bmatrix} -2 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 0 & 6 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

4.

$$A = \begin{bmatrix} 2 & -4 & 0 \\ 3 & -1 & 4 \\ -1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix}$$

- **19.** Write the given permutation matrix as a product of elementary (raw interchange) matrices.

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- **27.** Solve the system $Ax = b$ using the given factorization $A = P^T LU$.

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & -\frac{5}{2} \end{bmatrix} = P^T LU, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$