# Homework # 7

## Section # 3.6

- **1.** Let $T_A : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the matrix transformation corresponding to

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}.$$

  Find $T_A(\mathbf{u})$ and $T_A(\mathbf{v})$, where

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

- **3.** Prove that the given transformation is a linear transformation:

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + y \\ x - y \end{bmatrix}$$

- **7.** Show that the given transformation is not a linear transformation:

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x^2 \end{bmatrix}$$

- **11.** Find the standard matrix of the linear transformation:

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + y \\ x - y \end{bmatrix}$$

- **13.** Find the standard matrix of the linear transformation:

$$T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - y + z \\ 2x + y - 3z \end{bmatrix}$$

- Show that the given transformation $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ is linear by showing that it is a matrix transformation:

  **15.** $F$ reflects a vector in the $y$-axis.

  **16.** $R$ rotates a vector 45° counterclockwise about the origin.

- **22.** Find the standard matrix of the linear transformation $\mathbb{R}^2 \rightarrow \mathbb{R}^2$: projection onto the line $y = 2x$

**Section # 4.1**

- **4.** Show that $\mathbf{v}$ is an eigenvector of $A$ and find the corresponding eigenvalue.

$$A = \begin{bmatrix} 4 & -2 \\ 5 & -7 \end{bmatrix}, \ \mathbf{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

- **11.** Show that $\lambda$ is an eigenvalue of $A$ and find the corresponding eigenvector.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}, \ \lambda = -1.$$

- **13.** Find the eigenvalues and eigenvectors of $A$ geometrically

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

- **23.** Find all eigenvalues and eigenvectors of the matrix $A$ using characteristic equation $det(A-\lambda I) = 0$. Give bases of the corresponding eigenspaces. Illustrate the eigenspaces and the effect of multiplying eigenvectors by $A$.

$$A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}.$$

- **28.** Find all eigenvalues and eigenvectors of the matrix $A$ over the complex numbers $\mathbb{C}$. Give bases of the corresponding eigenspaces.

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 0 \end{bmatrix}.$$

- **35a.** Show that the eigenvalues of the $2 \times 2$ matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

are the solutions of the quadratic equation $\lambda^2 - \lambda tr(A) + det(A) = 0$, where $tr(A) = a+d$ is the trace of $A$.

- **38.** Let $a$ and $b$ be real numbers. Find the eigenvalues and corresponding eigenspaces of

$$A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}.$$

over the complex numbers $\mathbb{C}$.