Sample problems for review for the Final Exam

1. Write the number in the form $a + ib$.
   (a) $\frac{1}{3 + 4i}$

   (b) Log($i$)

   (c) $(1 - i)^{16}$

   (d) $\exp(2 - 3i)$

   (e) $\arctan\left(\frac{1 + i}{\sqrt{2}}\right)$, give all values (hint: $\arctan(z) = \frac{i}{2} \log \frac{1 - iz}{1 + iz}$)

2. Find all the values of $(-64)^{1/6}$. Sketch the “values of $(-64)^{1/6}$” = “solutions of $z^6 + 64 = 0$” on the complex plane.

3. Check where the Cauchy–Riemann equations for the function $f(z)$ hold ($z = x + iy$).
   (a) $f(z) = \frac{x}{x^2 + y^2} - i \frac{y}{x^2 + y^2}$

   (b) $f(z) = x^3 - 3xy^2 + iy^3 - 3ix^2y$

4. Find the radius of convergence of the power series
   (a) $\sum_{n=0}^{\infty} n^{2008} z^n$

   (b) $\sum_{n=0}^{\infty} (12 - 5i)^n z^{2n}$
(c) Taylor series for the function \((2+z)^{1/3}\) about \(z = 0\) (which goes as 2\(^{1/3}\) + \(\frac{z}{2^{2/3}3}\) - \(\frac{z^2}{2^{2/3}18}\) + ...)

5. Evaluate the integrals

(a) \[\int_{-\infty}^{\infty} \frac{dx}{(1 + x^2)^3}\]

(b) \[\oint_{|z|=2} \frac{\cos(\pi z)}{z(z - 1)} \, dz\]

(c) \[\oint_{|z|=11} \frac{\sin z}{z^2} \, dz\]

(d) \[\int_{\gamma} \frac{dz}{z(z^2 - 1)}\]

6. Evaluate integrals of trigonometric functions over \([0, \pi]\) and integrals involving fractional powers

(a) \[\int_{0}^{2\pi} \frac{d\theta}{2 + \cos^2 \theta}\]

(b) \[\int_{0}^{2\pi} \frac{d\theta}{(2 - \sin \theta)^2}\]
(c) \[ \int_{0}^{\infty} \frac{\sqrt{x}}{x^2 + 2x + 5} \, dx \]

7. Find the Laurent series for the given functions about the indicated point.

(c) \( \exp\left(\frac{1}{z-1}\right) : z_0 = 1 \)

(b) \( \frac{1}{z^2 + 1} : z_0 = 0 \), write an expansion that works for \( |z - z_0| > 1 \)

(c) \( \sqrt{1 + \sin z} \quad z_0 = 0 \) \( \) (three first non-zero terms of the (in this case) Taylor series)

8. Find the linear fractional transformation that maps the triple \( (-2, 0, i) \) to the triple \( (3, 1, \frac{3+i}{2}) \).

9. Find \( f(z) \) that maps the half plane \( U = \{ z: \text{Im} \, z > 0 \} \) onto the disk \( \Delta = \{ w: |w| < 1 \} \)

10. Find \( f(z) \) that maps the strip \( \pi \leq y \leq \pi \) onto the punctured plane, those \( w \) with \( w \neq 0 \)

11. See examples 3, 4, 5 page 226-227