



**PROJECT: HOW INSECTS FLY**

**BASED IN THE PAPER:**

**Nonlinear time-periodic models of the longitudinal flight dynamics of desert locusts *Schistocerca gregaria*. by Graham K. Taylor, and Rafał Zbikowski.**

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Through Newton-Euler equations (with four variables: position of the insect, horizontal and vertical velocity and angular velocity) there is a simulation of a longitudinal flight considering forces from wing flapping. For three types of locust forces data were obtained from experiments and they were fitted into Fourier series until the eight harmonic order. These forces consider the effect at different velocities of the air and also at different angles of attack. After that, a first simulation includes the zero order harmonics which is going to be a model nonlinear time invariant, in this simulation is seen the effect of perturbing the initial condition of the horizontal velocity. In the following set of plots are considering the Fourier series until the eight order harmonics in order to show the effect of two different initial time conditions. Finally, for the nonlinear time invariant system there is a 3D plot showing how the position of the insect is changing with reference to the other three variables considered in the system.

## **Introduction**

Many investigations have been done to analyze stability of aircraft which has been explored from different approaches. These approaches can be split in three ways: the first one based in experiments commonly cited as flow visualization and the second one based in theoretical frame usually called computer fluid dynamics. Flow visualization requires of using complex and well-designed set up in order to recover appropriate information while computer fluids dynamics requires of representative values in the coefficients which are part of the equations to simulate the system. In both ways a strong background is required and also an adequate interpretation of results would be important. However, there is an alternative approach which is simplified in terms of application and also with wide versatility. This third approach is called aircraft stability which in general follows a framework similar to the one used to study airplanes or helicopters stability.

Adapting aircraft stability to the flight of insects should be made since insects can fly by means of forces produced in the wings flapping. This implies that insects follow a particular kinematics. But, in this work what matters from wings are the total force and/or moment produced [1].

Although, any insect follows similar characteristics or patterns, it is easily observed that size, shape, weight and others are sufficiently different to be impossible achieve a representative and general model. Under this situation have better been explored possibilities to simulate a flight of a locust [2]. But, in the first part develops a methodology to log a set of data from experiments which are going to be used in a set of equations to reproduce a flight under a linearized model

[1]. This set of data rather consider some properties as size, weight, moment of inertia, forces produced because of the flapping, among others.

### **Main characteristics of the locust**

As the model is determined by physical properties of the locust, then it is important to cite some of them. For example, to have an idea the wing beat frequencies of the largest butterflies is around (c. 40 rad/s in the  $1 \times 10^{-3}$  kg birdwing *Troides rhadamantus*<sup>3</sup>) which is comparable to the rotor frequencies of the smallest helicopters (44 rad/s in the  $2 \times 10^3$  kg Eurocopter Bo105<sup>4</sup>) [3].

The desert locust is a robust, migratory, four-winged insect of 0.1m span, optimized for endurance, rather than maneuverability, and with a range of several kilometers at its cruise speed of 4 m/s. Although locusts do not vary in overall body plan, different individuals vary markedly in size. The wings beat at c. 20 Hz, with the hindwing leading the forewing by  $\pi/6$  rad. The hindwing typically sweeps  $110^\circ$  through every half-stroke, while the forewing sweeps through  $70^\circ$ . The hindwing comprises a collapsible fan consisting of a flexible membrane with supporting ribs radiating from the root. Each wing is moved by 10 muscles, and with no separate control surfaces, flight control is made by changes in the wing kinematics. Slower flight control is made by moving the legs and abdomen, which act as a rudder and also shift the center of mass. Locusts are equipped with a range of sensors known to be used in correctional flight control. The antennae sense airspeed, analogous to a Pitot tube. Wind-sensitive hairs on the head sense aerodynamic incidence. Each wing has at its base strain receptors involved in flight control that

may measure total wing loading, but perhaps more likely monitor wing kinematics: they have no direct or functional analog in conventional aircraft [3].

From observations of biologists it is pointed out that compound eyes works as horizon detectors measuring bank and pitch angle, and as optic flow detectors measuring heading and perhaps pitch, roll and yaw rate. Additionally, locusts are not known to have any sense of gravity in flight. Although vision is certainly important in insect flight control, it cannot give absolute measurements of bank and pitch angle unless referenced to gravity, and is ambiguous in respect of certain aspects of self-motion [3].

### **Newton-Euler Equations**

To simulate the flight, this framework is going to be based in the set of equations called Newton-Euler equations which are showed below:

$$\begin{aligned}\dot{u} &= -wq + \frac{X}{m} - g \sin \theta \\ \dot{w} &= uq + \frac{Z}{m} + g \cos \theta \\ \dot{q} &= \frac{M}{I_{yy}} \\ \dot{\theta} &= q\end{aligned}\tag{1}$$

From this set of equations is important to say that simulation is going to consider a 2D simulation flight which is going to be longitudinal with respect to the insect. Formally,  $u$  and  $w$  are the velocity with respect to the horizontal and vertical axis correspondingly,  $q$  means the

angular velocity with respect to the center of mass of the insect and  $\theta$  is the angle with respect to the horizontal axis. This can be better explained in the Figure 1 showed below:

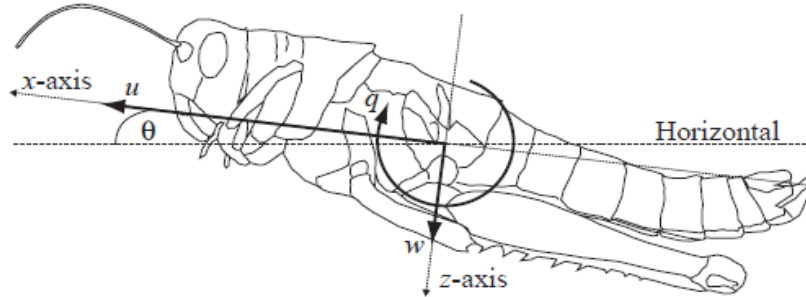


Fig.1 Variables considered in the model.

### Experiments

There were several concerns regarding to data obtained from experiments. Basically, attending needs of the Newton-Euler equations some values as the weight, moment of inertia and forces should be as accurate as possible. Considering three types of locusts some data were considered in the simulation and that information can be summarized in Table 1 [1].

Table 1 Reference values for three type of locust

Locust	Reference body mass (g)	Reference body length (mm)	Moment of inertia $I_{yy}$ (10 <sup>-9</sup> kg/m <sup>2</sup> )
“R”	1.8490	46.0	232.8
“G”	1.4357	40.5	140.3
“B”	1.8610	46.0	236.0

Although, in Table 1 considers three type of locust, in this midterm progress report is considered a simulation for the locust type “R”.

### **Logging forces**

Logging forces is important since they are going to be the reference in the simulation. As the main goal of this work is analyze stability and forces plays an important role in the model; a brief explanation of the experiments is mentioned. Essentially, a locust was attached to the top of a needle. That locust was inside of a wind tunnel (see Fig. 2) and the forces were recorded at different wind speeds and also at different position of the insect (angle with respect to the horizontal axis). Forces recorded were recorded in an interval from 2 to 5.5m/s and also at angles from  $0^\circ$  to  $14^\circ$ ; these ranges were given from observations when the locust is found in a free flight [1]. At this stage, it is expected to have a reaction of the locust as in a real free flight, however, a locust might recognize that is not a real flight but rather a reaction to move away of a “risky situation”; then forces might not be represented accurately.

### **Results**

Once forces were recorded, it was needed to preprocess data. This preprocess implies that was done a simplification of forces variability to one representative flapping. This means that there is an assumption that flapping is uniform during the flight and one period of flapping is going to be exactly equal to the next one; which it does not occur in a real flight but it reduces the set of data significantly. Then, data obtained from experiments can be visualized in Fig 3, (left plot) where variability of forces for one cycle of flapping (X correspond to the horizontal direction) at different angles of the locust (from  $0^\circ$  to  $14^\circ$ ).

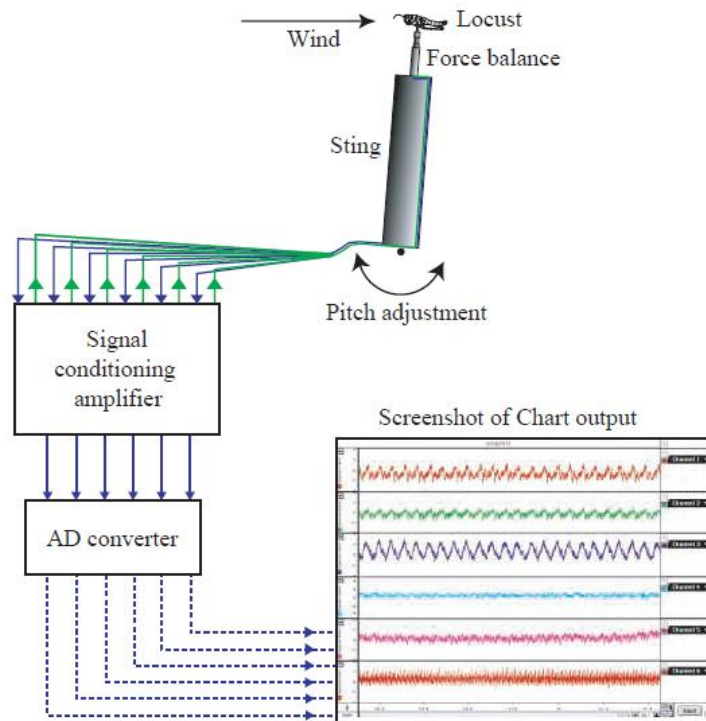


Fig. 2 Recording forces from a locust.

Fig 3 (left plot) includes thousands of points and the set of data may not be easy to handle. To overcome this situation was used Fourier series. Fourier series helps a lot since the set of data was reduced to a few coefficients which are able to reproduce original data with an acceptable approximation, as it is seen in the Fig 3 (right plot). Once again, data is compacted to a set of coefficients to be used in Fourier series. The maximum order of the harmonics is eight and it is good enough to have high accuracy in the representation of the original set of data.

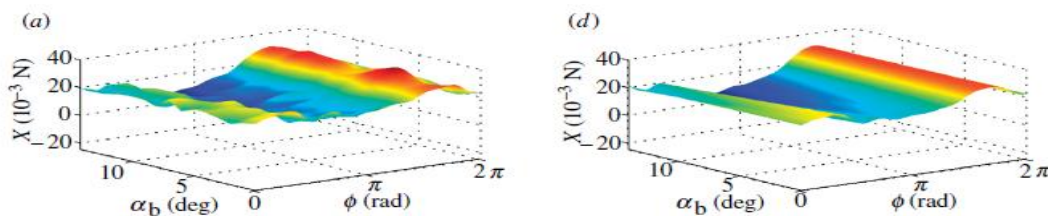


Fig 3 Forces obtained from experiments.

Fourier series can be represented by the equation (2) and it was used for the force X in the horizontal direction, the force Z in the vertical direction and the moment M at the center of mass of the insect.

$$P(t) = \sum_{n=0}^h (a_n \cos n\omega t + b_n \sin n\omega t) \quad (2)$$

As the first task was reproduced force X at different positions of the locust for once cycle of flapping. Results are showed in the Fig 4 (left plot), while results obtained in the paper are showed in Fig 4 (right plot). In the same way was done for the force X at different wind speeds also for one cycle of flapping as it is showed in Fig 5.

Forces in the vertical direction were reproduced at different angles and different wind speeds. In the same way for the moment as it is seen Fig. 6; where right plots corresponds to original plots and left plots corresponds to the ones reproduced.

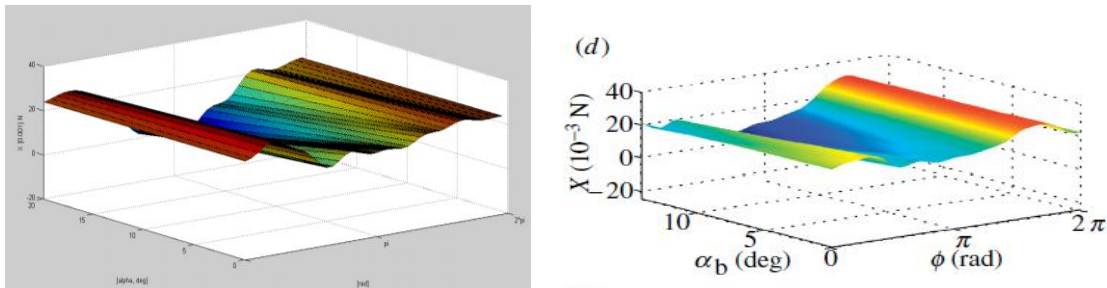


Fig 4. Reproducing X force at different angles of the locust for one cycle of flapping.

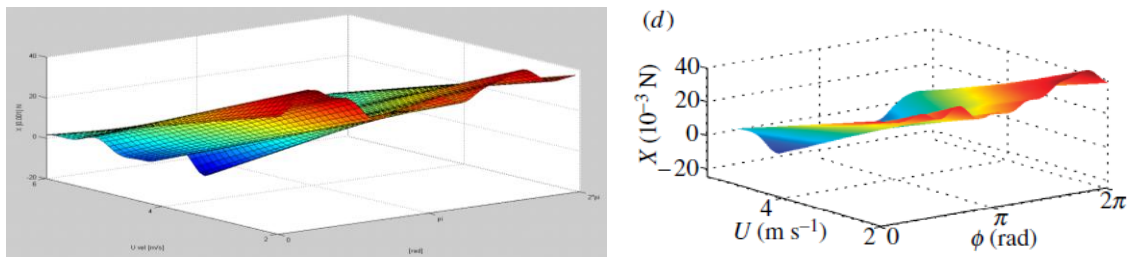


Fig 5. X force at different wind speeds for one cycle of flapping.

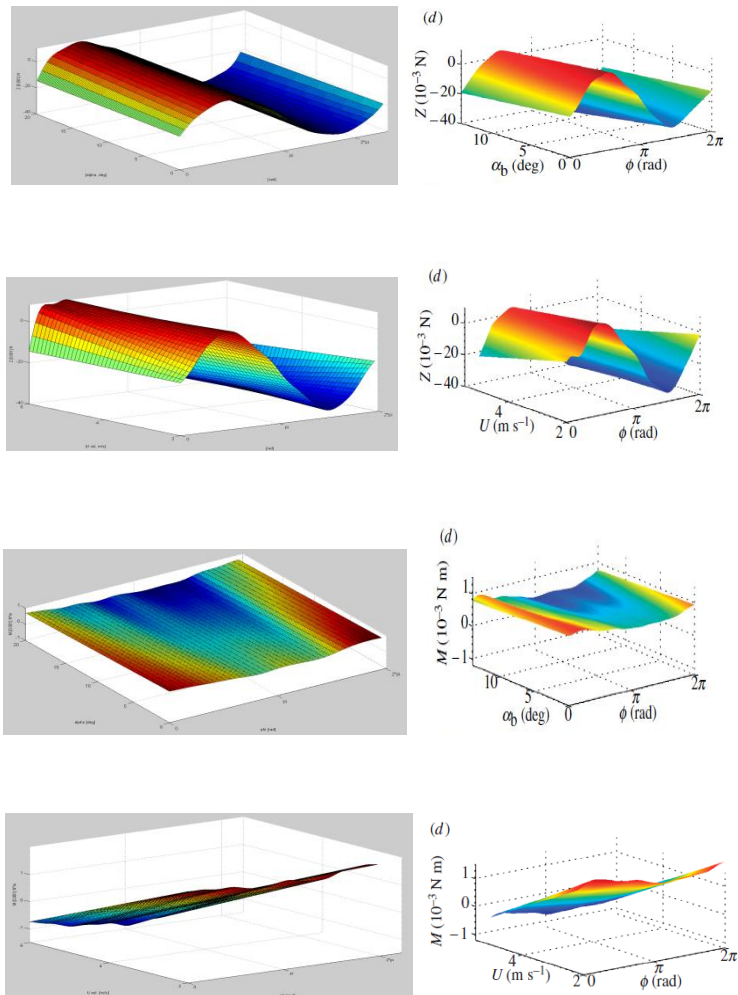


Fig 6. Vertical force  $Z$  and moment  $M$  at different angles of the locust and different wind speeds.

Forces and moment showed previously follows equations (3):

$$\begin{aligned}
 X(\alpha, U, t) &= \sum_{n=0}^8 (a_{1,n} \cos n\omega t + b_{1,n} \sin n\omega t) + (\alpha - \alpha_{ref}) \sum_{n=0}^8 (a_{2,n} \cos n\omega t + \\
 &\quad b_{2,n} \sin n\omega t) + (U - U_{ref}) \sum_{n=0}^8 (a_{3,n} \cos n\omega t + b_{3,n} \sin n\omega t) \\
 Z(\alpha, U, t) &= \sum_{n=0}^8 (a_{1,n} \cos n\omega t + b_{1,n} \sin n\omega t) + (\alpha - \alpha_{ref}) \sum_{n=0}^8 (a_{2,n} \cos n\omega t + \\
 &\quad b_{2,n} \sin n\omega t) + (U - U_{ref}) \sum_{n=0}^8 (a_{3,n} \cos n\omega t + b_{3,n} \sin n\omega t) \quad (3) \\
 M(\alpha, U, t) &= \sum_{n=0}^8 (a_{1,n} \cos n\omega t + b_{1,n} \sin n\omega t) + (\alpha - \alpha_{ref}) \sum_{n=0}^8 (a_{2,n} \cos n\omega t + \\
 &\quad b_{2,n} \sin n\omega t) + (U - U_{ref}) \sum_{n=0}^8 (a_{3,n} \cos n\omega t + b_{3,n} \sin n\omega t)
 \end{aligned}$$

which in general are similar and follows the equation (4).

$$P(\alpha, U, t) = P_{ref}(t) + P_{\alpha}(t)(\alpha - \alpha_{ref}) + P_U(t)(U - U_{ref}) \quad (4)$$

The equation mentioned before represents how the forces and moment can be handled once they fitted in Fourier series; and basically considers that a force or moment have a reference which is going to be affected by the position of the locust and also by the velocity of the locust. For example for the horizontal force X can be represented as equation (5).

$$X(\alpha, U, t) = X_{ref}(t) + X_{\alpha}(t)(\alpha - \alpha_{ref}) + X_U(t)(U - U_{ref}) \quad (5)$$

But P involves terms cited in equation (6)

$$P = [X, \quad Z, \quad M] \quad (6)$$

Now connecting forces and moment in Fourier series way with the set of Newton-Euler equations will be equation (7).

$$\begin{aligned}
 \dot{u} &= -wq + \frac{X_{\text{ref}}(t)}{m} + \frac{X_{\alpha}(t)}{m} \left( \tan^{-1} \frac{w}{u} - \alpha_{\text{ref}} \right) \\
 &\quad + \frac{X_U(t)}{m} \left( \sqrt{u^2 + w^2} - U_{\text{ref}} \right) - g \sin \theta \\
 \dot{w} &= uq + \frac{Z_{\text{ref}}(t)}{m} + \frac{Z_{\alpha}(t)}{m} \left( \tan^{-1} \frac{w}{u} - \alpha_{\text{ref}} \right) + \frac{Z_U(t)}{m} \left( \sqrt{u^2 + w^2} - U_{\text{ref}} \right) \\
 &\quad + g \cos \theta \\
 \dot{q} &= \frac{M_{\text{ref}}(t)}{I_{yy}} + \frac{M_{\alpha}(t)}{I_{yy}} \left( \tan^{-1} \frac{w}{u} - \alpha_{\text{ref}} \right) + \frac{M_U(t)}{I_{yy}} \left( \sqrt{u^2 + w^2} - U_{\text{ref}} \right) \\
 \dot{\theta} &= q
 \end{aligned} \tag{7}$$

With this new set of equations was looked for the quasi-static equilibrium in order to start simulations from those values. It is said that is quasi-static because the equations are set equal to zero in the left hand side, then specific values are found for every one of the four variables, but velocity in the vertical and horizontal are different from zero. This non zero value for the velocity variables means that the insect has an equilibrium during the flight considering this set of equations. Values found and used for the simulation are showed in Table 2.

Table 2. Conditions for quasi-static equilibrium of each locust.

Locust	$\theta$ (deg)	$\alpha$ (deg)	U(m/s)
“R”	26	6	4.48
“G”	5	12	2.93
“B”	23	9	4.79

Quasi-static equilibrium estimation is coming from equations (1) which will be used as the initial conditions. Reproducing for locust type “R”, velocity u is equal to  $4.48 \cdot \cos(6)$ ; velocity w is equal to  $4.48 \cdot \sin(6)$ , q is equal to zero and  $\theta = 26/57.29$  (converted to radians). In this stage is

important to indicate that  $\alpha$  is the angle formed of the speed vector ( $U$ ) with respect to the horizontal axis. Furthermore, this quasi-static equilibrium will be there initial conditions, since it is assumed that is a point with stable flight. A first simulation shows a nonlinear model time invariant; in other words, implying that forces or moment during one cycle of flapping are constant but relation among variables are nonlinear, see multiplication of variables in equation (1). Results are showed in Fig. 7, 8 and 9. Plots in the right side correspond to the ones from the

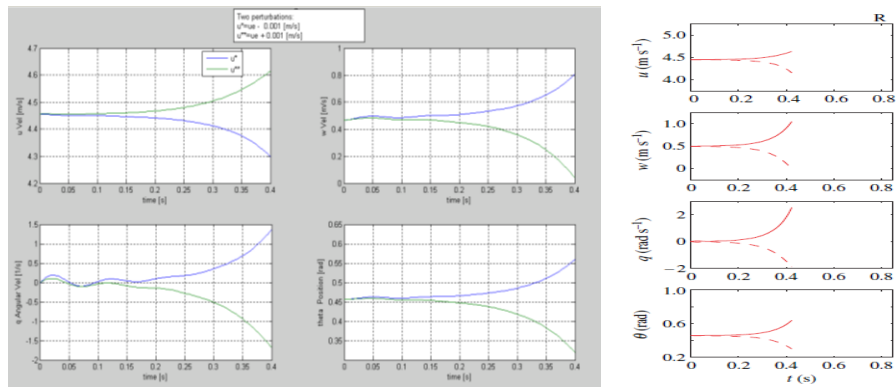


Fig 7. Simulation for a Non linear time invariant model with perturbation in the u-velocity for locust type “R”.

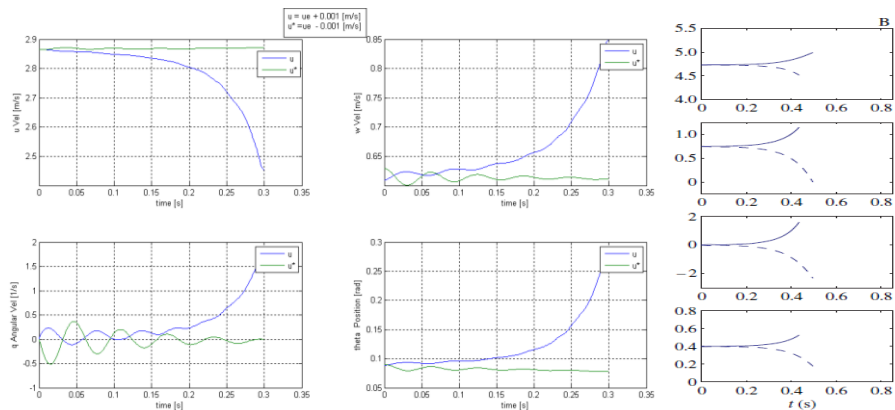


Fig 8. Simulation for a Non linear time invariant model with perturbation in the u-velocity for locust type “G”.

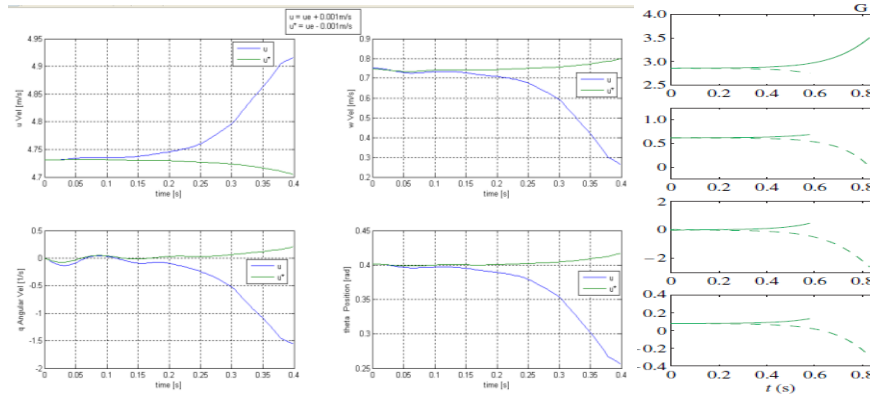


Fig 9. Simulation for a Non linear time invariant model with perturbation in the u-velocity for locust type “B”.

paper while plots in the left side correspond to the ones reproduced using the function “ode45” of Matlab. This work consider three different types of locust called “R”, “G” and “B” where main characteristics are cited in Table 1 and 2. But the set of plots regarding to forces are not included for locust type “G” and “B”, but coefficients are added at the end of this document.

In Fig. 7, 8 and 9 there is a corresponding initial conditions following the values given in Table 2. One way to analyze performance of the model is considering a small perturbation in one value of the initial conditions. In this case, horizontal velocity ( $u$ ) is perturbed by a small value of 0.002m/s to check response of the model. In Figure 7, there is the response for 0.4 s (which correspond to 8 cycles of flapping) and there is a significant change in the final path. It is important to remark that combination of the four variables is going to give a specific path; which is completely different since Figure 7 show up different curve at every variable. In the same way, in Figure 8, there is a similar response when perturbation is assigned also to the horizontal velocity with  $u = u_e \pm 0.001 \text{ m/s}$  getting similar plots in Figure 7. And following the same response is found in Figure 9; then three type of locust models similar response. From here it is

possible to say that the three types of locust follows the same pattern; which can be led to the idea of instability.

However, these simulations consider average of the forces during one cycle of flapping, but it is still possible to explore other type of model called nonlinear time periodic (NLTP). In this model forces are going to change during one cycle of flapping, then more information from forces might help to better results.

NLTP model considers forces in Fourier series fitted until the eight order of harmonics, which covering variability of forces or moment during one cycle of flapping. It will be showed now in Fig. 10, 11 and 12 results for a model called nonlinear time periodic model. In this case, initial conditions are set equal to the quasi-static equilibrium, following the same way as in the NLTI models and based in Table 2. In this case, instead of perturbing horizontal velocity, initial time is going to be altered for 0.025s; a value equivalent to a half cycle of flapping. This perturbation accomplishes the idea of how much would change a possible trajectory of the flight because starting of flapping with open wings instead of closed wings (when initial time is equal to zero). Figure 10 indicates that the simulated final path is completely different for locust type “R” because of different initial times. In the same way in Figure 12 there is a similar result. However, in Figure 11 which corresponds to locust type “G”, there is a parallel path for the four variables. This situation may tell that this type of locust might be stable.

At this point, using all the information from experiments; some conclusions come up. For instance, the model is still weak to show simulation of an insect flight, several reasons are valid as data are not representative of a free flight, methodology to obtain data can be improved and the framework used does not represent the phenomena under analysis. Additionally, model for

locust type “G” may work slightly different; but this situation does not help to have convincing conclusions.

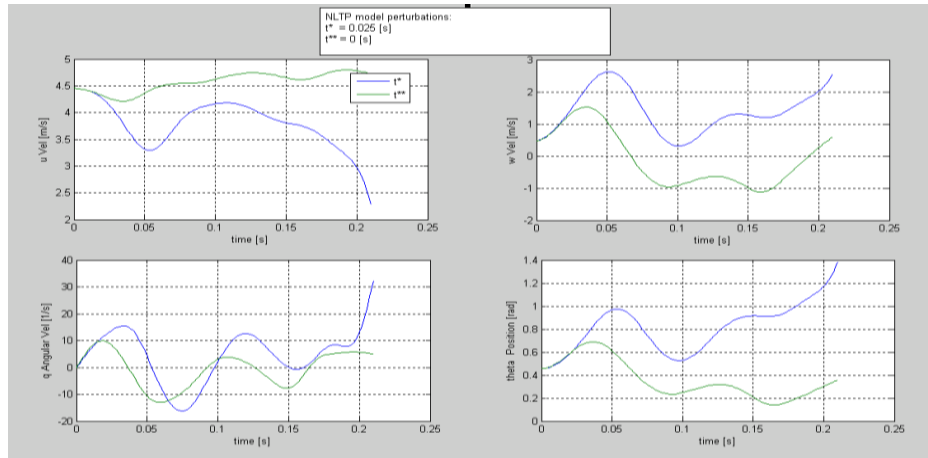


Fig 10. Simulation of a nonlinear time periodic model with perturbation in the initial time for locust type “R”.

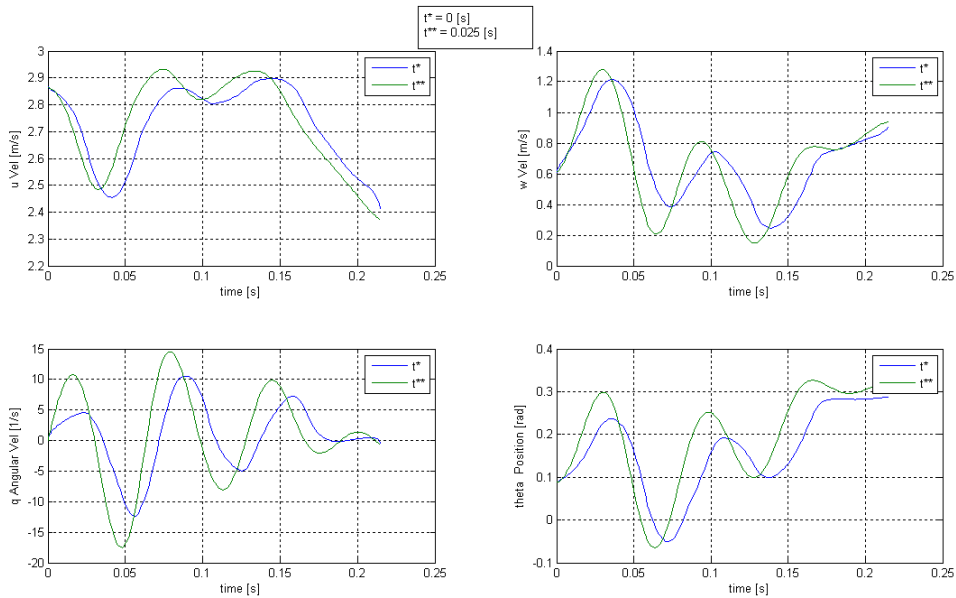


Fig 11. Simulation of a nonlinear time periodic model with perturbation in the initial time for locust type “G”.

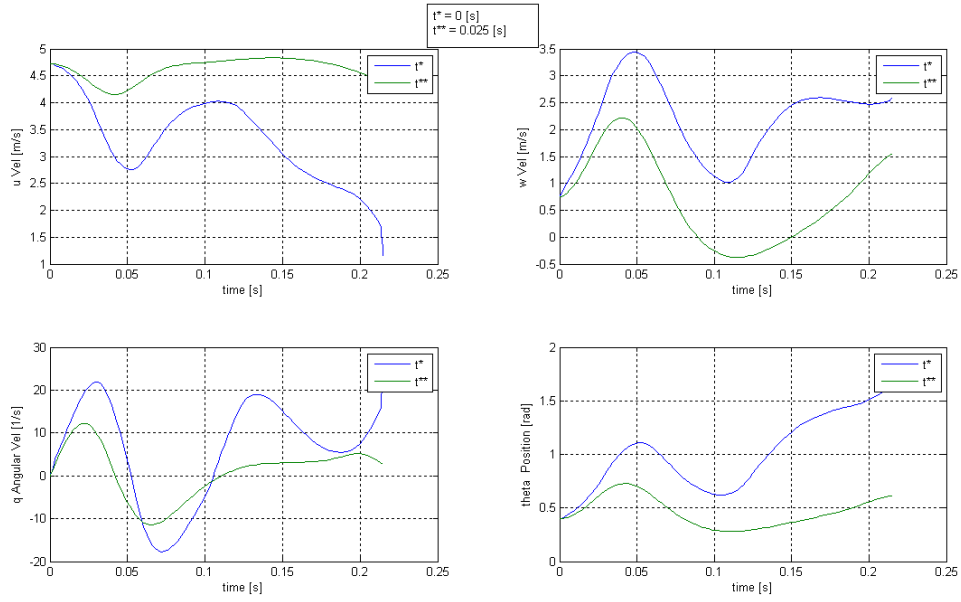


Fig 12. Simulation of a nonlinear time periodic model with perturbation in the initial time for locust type “B”.

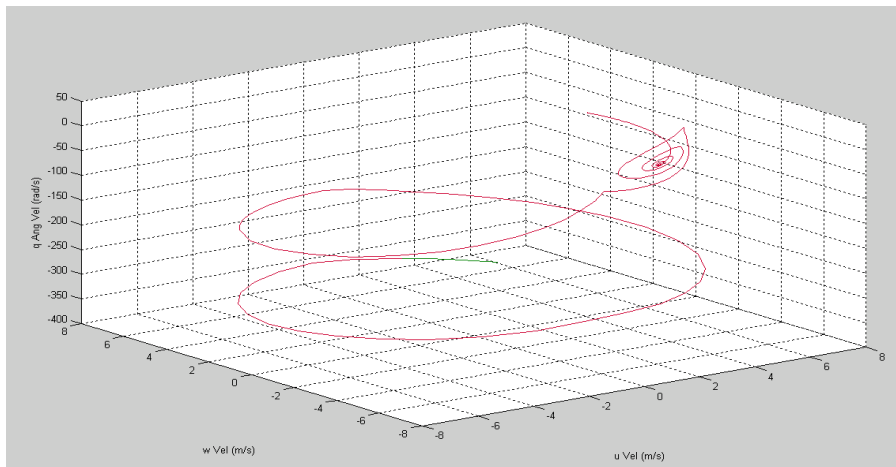


Fig 13. Phase portrait for  $\theta$  for the non linear time invariant model for locust type “R”.

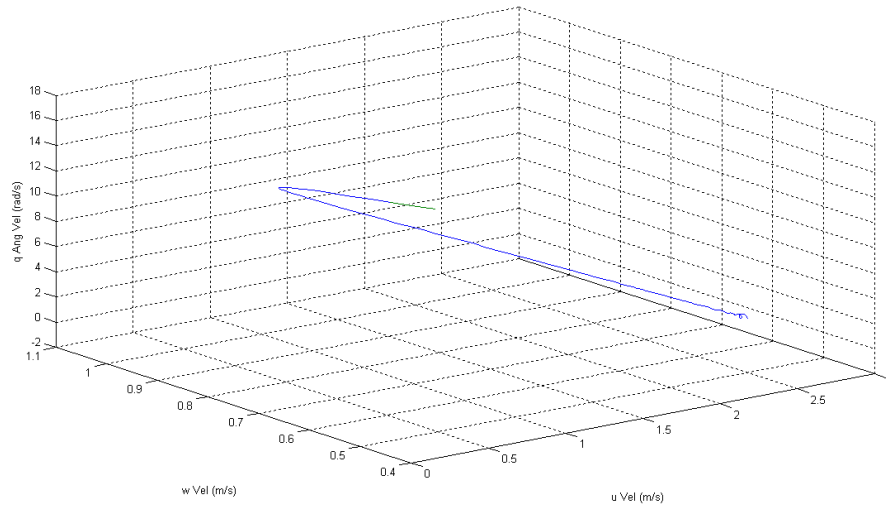


Fig 14. Phase portrait for  $\theta$  for the non linear time invariant model for locust type "G".

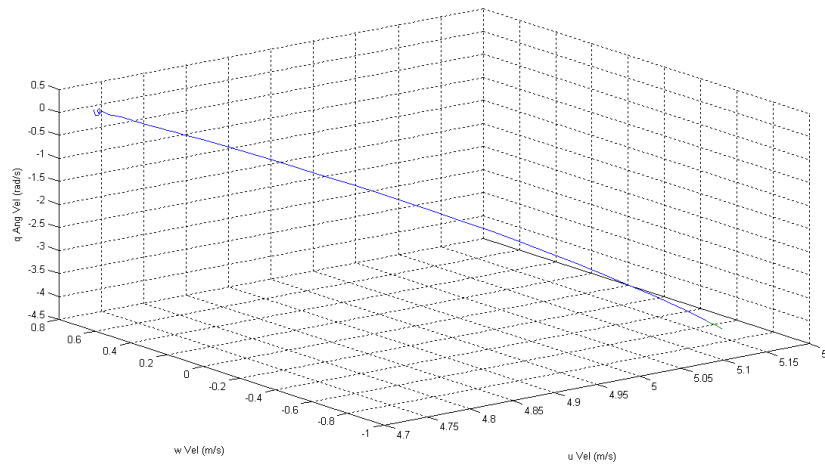


Fig 15. Phase portrait for  $\theta$  for the non linear time invariant model for locust type "B".

Finally, it is showed a last set of Figures: 13, 14 and 15 which helps to have a better idea of the variability of  $\theta$  with respect to the other three variables (u, w and q). Based in the NLTI model; for instance for locust type "R" results are in Fig. 13. It is easy to see that there are at least two points where  $\theta$  changes abruptly when simulation time correspond to 0.6s. These rapid changes in the path of  $\theta$  implies that once the insect falls in this point then the insect might have change

rapidly the path. This situation does not happen in reality; in other words, there is instability in the simulation which is not present in a real free flight. In Figure 14 there is the path of  $\theta$  corresponding to locust type “G”, in this case the path is smooth, but at the beginning there are some spirals probably difficult to reproduce in a real insect, however it is notorious an improved stability of the flight. In Figure 15 for locust type “B” also follows a smooth path but at the end starts a set of small spirals which also might be difficult to see in a free flight.

### **Conclusions**

Since the model is based in forces recorded, it is suspicious that forces do not represent a real free flight. This is possible since several factors were limiting it; for instance, forces were not accurate because of the instruments used, the insect can have different reactions under this “altered conditions” to simulate the flight. Additionally, some assumptions were made as the gravity, weight and moment of inertia are constant which is not true and it is easily noticed in the moment of inertia because of the flapping.

Also, when the experiments were performed, there was a recording of forces in open loop. Under this situation, it is assumed that the insect does not control the flight, which is not true. Then forces to manipulate the flight might already be included in the set of data given in the paper however so far it is not possible to obtain those forces apart.

Additionally, exploring possibilities to control the flight is difficult; then two chances are present: improved methodology to get accurate values from experiments and estimation of forces from the set of equations (1). Both ways are equal challenging. If in the set of equations based in (1) is possible to find correct values of forces; then this problem might be solved and

also will give a better picture of the whole system. But also, the identification of forces through experiments also is possible and worthy in order to have better idea of the system.

## **References**

[1] Graham K Taylor, Adrian L. R. Thomas. 2003 “Dynamic flight stability in the desert locust *Schistocerca gregaria*” *Journal of Experimental Biology* 206, 2803-2829

[2] Graham K Taylor, Rafal Zbikowski. 2005 “Nonlinear time-periodic models of the longitudinal flight dynamics of the desert *Schistocerca gregaria*” *Journal of the Royal Society Interface* 2, 197-221

[3] Graham K Taylor, Richard J. Bomphrey and Jochem Hoen. 2006 “Insect flight dynamics and control” *Aerospace Science Meeting and Exhibit*. Reno Nevada.