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Recognition of Spectral Patterns

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The recognition of spectral patterns through the utilization of an optical correlator is an integral category of the mathematical domain of pattern recognition - and more generally - pattern classification. As described in this research, the basics of this field are founded in considerable mathematical rigor.

> "Pattern recognition – the act of taking in raw data and making an action based on the category of the pattern - has been crucial for our survival, and we have evolved highly sophisticated neural and cognitive systems for such tasks."
>
> Richard O. Duda

Introduction

The recognition of spectral patterns through the utilization of an optical correlator is an integral category of the mathematical domain of pattern recognition - and more generally - pattern classification. The domain of pattern recognition is applicable to disparate fields of inquiry such as handwriting and gesture recognition; DNA sequence identification, and geological analysis. It is interesting to note the biological (neurological) root s in our human development of pattern recognition systems. [1] Correlation filters have been utilized with much success in automatic target recognition (ATR) applications. Specifically, the functional domain of this research specifically relates to the utilization of optical correlation filters to detect and locate ground-based tactical targets in observed scenes. [6] More specifically, this research evaluates the MACH (Maximum Average Correlation Height) filter for ATR and aim point analysis against the high-value, defense targets with background clutter using an infrared seeker. As a result, target signatures may be considered as a signal detection approach to ATR. In other words, correlation filters may be thought of as a general-purpose "matching" engine. [3] As described in this research, the basics of this field are founded in considerable mathematical rigor.

Walls, 1999

The rationale for the implementation of correlation filters is that the algorithm strikes a balance between robustness (with respect to noise) and simplicity in order to be realizable in the real world. Specifically, the MACH correlation filter algorithm has excellent robustness to noise in the input scene as well as a high tolerance to distortions. Using the MACH correlation filter algorithm, the image is cross-correlated with a set of carefully designed correlation templates with regard to shift invariance as well as distortion-tolerance. These spatial (or correlation) filters are used to detect and locate targets in observed scenes. The resulting output is searched for large peaks. Specifically, the implementation of the correlation filter algorithm results in sharp correlation peaks for targets of interest, as well as high discrimination against unwanted objects; specifically clutter, decoys (i.e., noise). [7]

Mahalanobis, 2000

There exists a distinct real-time performance consideration in the utilization of spatial filters for ATR in this context. Specifically, since a missile intercept has a closing rate of greater than 4 km/sec, there is an abbreviated time window to perform ATR processing. As such, processing speed requirements and resulting throughputs must be strongly factored into the implementation of the MACH correlation filter algorithm. In particular, the MACH filter algorithm (for correlation filters) substantially reduces the information requirement into a few templates, and thereby eliminates the requirement for an extensive library of filters. The MACH correlation filter algorithm implementation has relatively modest memory requirements and is consequently suitable for real-time implementation.

The following section is comprised of a detailed statement of the problem, specifically, automatic target recognition (ATR) in the context of the Maximum Average Correlation Height (MACH) simulation. The ATR MACH algorithm is represented in Figure 1 and is summarized and then each method is described in detail in the subsequent section. The MACH filter algorithm is designed to optimize the amplitude (height) of the mean correlation peak with respect to expected distortions.

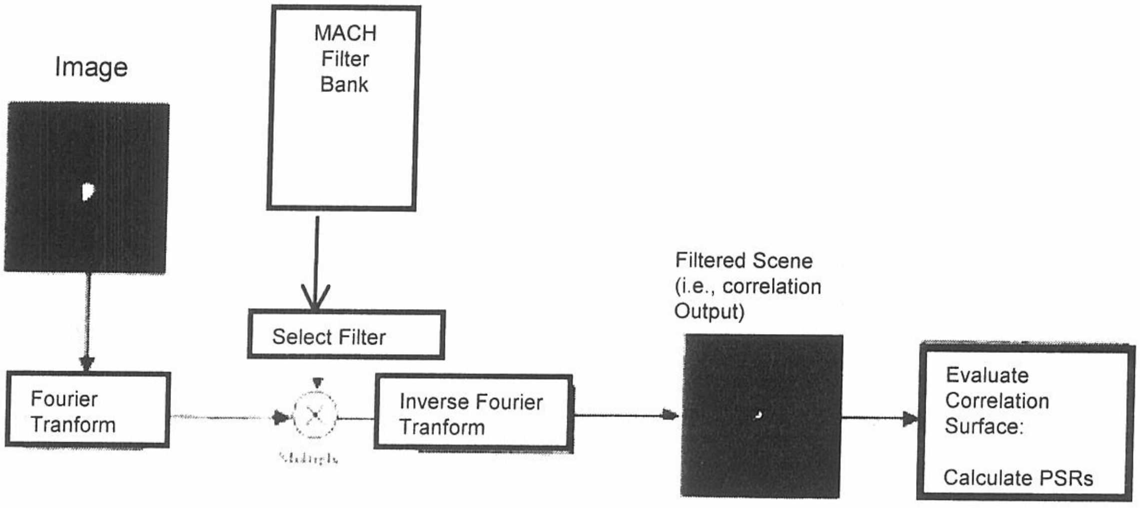

Figure 1: Overall MACH Process Flow Algorithm

As shown, the input image is cross-correlated with a set of carefully designed correlation templates that have been calculated and optimized with respect to the mean correlation peak. Specifically, cross-correlation means the two-dimensional spatial correlation for the large matrices. Ultimately, the Fast Fourier Transform (FFT) is utilized transform this spatial correlation into a Fourier-domain element-by-element multiplication. These spatial (or correlation) filters are used to detect and locate targets in observed scenes. The resulting correlation surfaces are then created and evaluated with an input image and the bank of filters. The resulting output is then searched for large peaks. The MACH algorithm is designed to optimize the amplitude (height) of the mean correlation peak with respect to expected distortions. [4]

Maximum Average Correlation Height (MACH) Algorithm Process: Input image transformation

The first method in the Maximum Average Correlation Height (MACH) algorithm process is the transformation of the input image. Specifically, each image is correlated with the relevant filters in the filter bank. As shown in Figure 1, each image is Fast Fourier Transformed (FFT) only one time; it is then correlated against all the aspect filters within the relevant filter bank. Consequently, the peaks in the resulting correlation surfaces will have peaks that show both the locations as well as the intensities of the best "matches" with regard to the corresponding filters.

Maximum Average Correlation Height (MACH) Algorithm Process: Filter Initialization

The filter bank is created for the designated target at various aspect angles.

The overarching design is to maximize the amplitude (height) of the mean correlation peak with respect to expected distortions. Prior to the first use of the filter banks, each of the MACH filters is transformed from its initial spatial format to a Fourier domain format. This method occurs prior to the initial use of the filter bank in the subsequent correlation. As such, this procedure helps to minimize the system throughput requirements.

Maximum Average Correlation Height (MACH) Algorithm Process: Correlation Surface Method

From a historical context, two mathematicians of note contributed to the development of the Maximum Average Correlation Height (MACH) algorithm. Specifically, Jean Baptiste Fourier, a French mathematician and physicist and graduate from the Ecole Polytechnique, developed the Fourier domain of mathematical analysis. It is interesting to note that Fourier is also credited with the discovery in 1824 that gases in the atmosphere might increase the surface temperature of the Earth, what is now called the greenhouse effect – a prescient and relevant observation. In addition, John Tukey a mathematician from Princeton University, was one of the mathematician who developed the Fast Fourier Transform(FFT). The FFT is an efficient algorithm to compute the discrete Fourier transform and the FFT inverse. [6]

The correlations surface method of the Maximum Average Correlation Height (MACH) algorithm process can be extremely throughput intensive for image matrices, specifically the two-dimensional spatial correlation. Consequently, the “Fast” correlation (i.e., FFT) is employed to transform a spatial correlation into a Fourier-domain, element-by-element multiplication. With respect to the mathematical model, the correlation surfaces may be written in terms of the input image and the appropriate filter in the Fourier domain, specifically:

$$g_{k,c,\theta} = \hat{I}_k(u,v) \otimes h_{c,\theta} \qquad (1)$$

In this equation, $g_{k,c,\theta}$ represents the correlation image, $\hat{I}_k(u,v)$ represents the k^th pre-processed image. The symbol

$\otimes$ represents the two-dimensional spatial correlation. Moreover, $h_{c,\theta}$ represents the filter coefficients in the spatial domain.

With regard to the Matlab implementation, the functional realization is achieved by employing the FFT2 Matlab routine. The Matlab product called the Image Processing Toolbox, specifically the fft2(X) routine, is employed. This routine is employed to transform a spatial correlation into a Fourier domain element-by-element multiplication. With respect to the Matlab implementation, the correlation surfaces may be written in terms of the input image and the appropriate filter in the Fourier domain, specifically:

$$g_{k,c,\theta} = \left(\left(\Im_{2D}^{-1}\left(\Im_{2D}\left(\hat{I}_k(u,v)\right) \hat{\bullet} H^*{}_{c,\theta}\right)\right)\right) \qquad (2)$$

With respect to this equation, the variables for target class are specified as c. The aspect angle is represented by the variable θ. As shown, the function, $\Im_{2D}(\bullet)$, computes the two dimensional Fast Fourier Transform of its argument; the function, $\Im_{2D}^{-1}(\bullet)$ computes its inverse. The symbol $\hat{\bullet}$ denotes an element-by-element multiplication. $\hat{I}_k(u,v)$ is the kth pre-processed image, and H^* is the selected filter. As shown in this equation, the FFT of the image and the appropriate filter are combined via an element-by-element multiplication, and inverse FFT transformed.

Maximum Average Correlation Height (MACH) Algorithm Process: Correlation Surface PSR Post-processing

The correlation surface PSR post-processing of the Maximum Average Correlation Height (MACH) algorithm process is applied to the correlation surface in order to evaluate the relative strengths of the correlation peaks. This method computes the Peak-to-Sidelobe Ratio (PSR) for each point on the correlation surface. As each correlation surface is produced, the post-processing function transforms the correlation surface into a PSR surface. The PSR surface is then searched for the maximum value. The PSR is the key correlation performance metric with respect to the MACH filter optimization. The equation for the PSR is given by:

$$PSR = \frac{peak - \mu}{\sigma} \qquad (3)$$

where peak is a peak response in the correlation surface; μ is the mean response local to the peak; and σ is the standard deviation local to the peak. As each correlation surface is produced, the post-processing function transforms the correlation surface into a PSR surface. The PSR surface is then searched for the maximum value. As shown mathematically in the equation, the MACH filter minimizes the mean and variance of the response while maximizing the peak; this, in turn, maximizes the PSR.

For the simulation, the performance of the MACH algorithm with a simple model was evaluated utilizing synthetic imagery, as shown in Figure 2, Images and Correlation Surface Intensities.

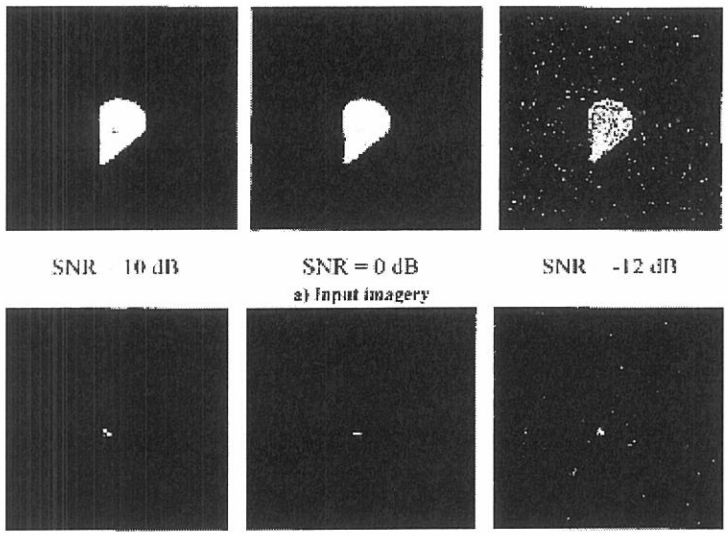

Figure 2: Images and Correlation Surface Intensities

In essence, the MACH filter algorithm seeks to strike a balance between robustness and simplicity in order to be realizable in the real world. The first image set depicts the results of a MACH filter on an image with very little system noise. Note that the resulting correlation surface is clean and the peak PSR value is relatively clear. Subsequently, as shown in the figure on the right (above), the same input image then had a few random noise pixels added. As shown in the figure on the right (below), the