Math 424/524 — Hw 3
Sec 001
Spring 2010

Print Name ______________________

Signature ______________________

Date ______________________

1. The arm of the pendulum oscillates as $l = l_0 + a\cos(\Omega t)$. Find Lagrangian $L = \frac{1}{2}mv^2 - U(x,y)$ and derive the equation of motion in terms of angle $\varphi(t)$ (see your notes), here $x(t) = l(t)\sin\varphi(t)$, $y(t) = l(t)\cos\varphi(t)$, $v^2 = (x'(t))^2 + (y'(t))^2$ and the potential energy has following form: $U = -mgl(t)\cos\varphi(t)$

2. Solve the equation $x''(t) + \omega_0^2 x(t) = B\sin\Omega t$, ($\omega_0 \neq \Omega$) with following initial conditions: $x(0) = 0$, $x'(0) = 0$

3. Solve the equation $x''(t) + \omega_0^2 x(t) = B\sin\omega_0 t$, with following initial conditions: $x(0) = 0$, $x'(0) = 0$. Hint - use substitution $x(t) = At\sin(\omega_0 t + \varphi) + x_0(t)$, here $x_0(t)$ is the general solution of homogeneous equation (equation without right hand side term).

4. Find inhomogeneous solution of the equation: $x''(t) + \gamma x'(t) + 4x(t) = \sin\Omega t$, here $\gamma > 0$. Using Matlab plot dependance of the amplitude and phase φ of this solution on Ω $(1 \leq \Omega \leq 6)$ for $\gamma = 0.1,\ 02,\ 0.4,\ 0.6\ ,0.8,\ 1.0$